OPEN ACCESS
Saudi Journal of Economics and Finance
Abbreviated Key Title: Saudi J Econ Fin
ISSN 2523-9414 (Print) |ISSN 2523-6563 (Online)
Scholars Middle East Publishers, Dubai, United Arab Emirates
Journal homepage: http://saudijournals.com/sjef/

Original Research Article

Convertible Bond Pricing Based on Variance Gamma Model

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DOI: 10.36348/sjef.2020.v04i06.015 | **Received:** 15.06.2020 | **Accepted:** 22.06.2020 | **Published:** 24.06.2020

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Abstract

Due to the 'spike and tail' phenomenon of asset returns, the applicability of the Black-Scholes model for pricing convertible bonds has been questioned, and the variance gamma model can cope well with this phenomenon and solve the 'volatility smile dilemma'. This paper combines the variance gamma model with the least squares Monte Carlo simulation method to empirically analyze the Everbright convertible bonds based on its high activity in the Chinese market. In this paper, the theoretical price and the actual price are compared, and the applicability of the variance gamma model in the Chinese convertible bond market is analyzed. The empirical results show that the fitting price obtained by the variance gamma model is consistent with the actual price trend, indicating that the method is applicable to the Chinese convertible bond market.

Keywords: Convertible Bonds; Pricing Method; Variance Gamma Model.

INTRODUCTION

Convertible bonds originated in the 1970s. At present, the United States, Europe, Japan and some Asian countries are the major issuers of it, and the size of the global convertible bond market has exceeded $1 trillion. The development of China's convertible bond market started late, and it has only been more than 20 years. However, with the continuous improvement of China's financial system, the development of convertible bonds has been relatively stable and gradually occupied an increasingly important position.

There are not many varieties of convertible bonds circulating in China, and the level of activity is far less than that of stocks or bonds. At the same time, its' complicated pricing principle make its research relatively less. However, the international research on the theory and practice of the convertible bond pricing model has been relatively mature, providing a useful reference basis for research in China. Due to the classic Black-Scholes model cannot solve the problem of 'spike and thick tail' phenomenon of stock returns and the related 'volatility smile dilemma', its accuracy of pricing convertible bond was doubted. However, the variance gamma model that emerged in recent years can solve these two key problems well. This paper will verify the applicability of the variance gamma pricing model in China's convertible bond market through empirical analysis. This paper will take Everbright convertible bonds as an example, and by comparing the fitted price that obtained based on the variance gamma pricing model and its actual price and calculating the error rate, this paper will study whether the method has superiority in the pricing accuracy in the Chinese convertible bond market.

LITERATURE REVIEW

According to different underlying assets, the theoretical research related to convertible bond pricing can be divided into two aspects, the pricing model based on company value and on stock price.

A. the pricing model based on company value. Ingersoll [1] assumes that the dynamic change process of the company's value follows the geometric Brown movement. By considering the optimal transformation strategy of the investor and the optimal redemption strategy of the issuer, the analytical solution of the non-redeemable and redeemable zero-interest convertible bond is obtained. Brennan and Schwartz [2] put the redemption clause of the convertible bonds into the model, and deduce the partial differential equation of the convertible bond price with the principle of no arbitrage. At the same time, the boundary and terminal conditions are clarified, which further perfected the convertible bond pricing model. Brennan and Schwartz [3] consider the interest rate randomization on the basis of the previous model and compare the pricing results under fixed and random interest rates, of which the empirical results show that the pricing is not significantly affected by the interest rate. Sirbu [4] converts the pricing of convertible bonds into a relatively simple optimal time-stopping decision. It is considered that when the product of interest rate and redemption value is lower than the dividend value, the redemption is superior to the exercise of equity. Otherwise, the exercise of equity is better than redemption. The China Securities Research Institute [5] is the first institution to use BS model to price convertible bonds in China. It advocates that directly add together the value of the option value and the common creditor's value directly can form the price of the convertible bond.

b. the pricing model based on stock price. McConnell and Schwartz [6] proposed a single factor pricing model based on stock value. This pricing model aims at the convertible bonds with zero-coupon and the right of redemption, resale and transfer of equity. Tsiveriotis and Fernandes [7] propose that the normative value of convertible bonds should be divided into two parts: pure debt value and partial stock value. Because of their different risks, the discount rate for the two part is different. This model is also called the TF98 credit risk model. Based on the long-term observation of market prices, Fama E *et al*. [8] suggest that the logarithmic return of the stock is usually accompanied by a Statistically 'thick tail'. Merton [9] believes that the logarithmic price distribution of assets is a random finite jump on the basis of a random normal diffusion, and a bounded jump diffusion model, that is, MJD model, is proposed. Madan *et al*. [10] constructed the variance gamma model, a generalized jump model, which assumes that the price of assets jumps indefinitely in a given time interval, and the small jump also occurs frequently. Duffie [11] argues that asset prices follow a specific jump diffusion model at random time. The Monte Carlo simulation method is improved by Longstaff and Schwartz [12], and the least square Monte Carlo simulation method is proposed to solve the situation of American option ahead of time. Pavlo Kovalov and Vadim Linetsky [13] use the Markov process, which including stock price, volatility, credit risk and interest rate, to deduce the corresponding partial differential equation in the pricing of convertible bonds and use the finite element method to solve it. Zhang Weiguo [14] uses random Faure sequence and variance reduction technique to effectively reduce the error of the model estimation results, and proposes a least square Quasi Monte Carlo pricing model for convertible bonds.

METHOD INTRODUCTION

Variance Gamma Process

The main variance gamma process takes three variables: volatility, kurtosis and skewness. Considering a Brown movement, of which the drift rate and volatility are presented by the following stochastic process:

$$b(t;\theta,\sigma) = \theta t + \sigma W(t) \tag{1}$$

W (t) is a standard Brown movement. The variance gamma process is essentially evaluated by the Brown movement in random time variations under given gamma process. The given gamma process G (t;μ,ν) refers to the process of independent gamma increment in the (t, t+h), where μ is the mean rate and ν is the variance. Through the variable substitution for Brown movement process, the the Brown movement b(t; θ,σ) and the gamma process G(t;μ,ν) will be used to definite variance gamma process:

$$X(t;\sigma,v,\theta) = b(G(t;1,v);\theta,\sigma) \tag{2}$$

σ = the volatility of Brown movement (volatility)
V = variance rate of gamma time variation (kurtosis)
θ = the drift rate of Brown movement (skewness)

The first four moments of the variance gamma process are:

$$E(X(t)) = \theta t \tag{3}$$

$$E[X(t) - E(X(t))^2] = (\theta^2 v + \sigma^2)t \tag{4}$$

$$E[X(t) - E(X(t))^3] = (2\theta^3 v^2 + 3\sigma^2\theta v)t \tag{5}$$

$$E[X(t) - E(X(t))^4] = (3\sigma^4 v + 12\sigma^2\theta^2 v^2 + 6\theta^4 v^3)t + (3\sigma^4 + 6\sigma^6\theta^2 v + 3\theta^4 v^2)t^2 \tag{6}$$

Under the risk neutral process, the dynamic change of asset prices follows the following VG process:

$$S(t) = S(0)\exp(rt + X(t;\sigma,v,\theta) + \omega t),\ \omega = \ln(1-\theta v-0.5\sigma^2 v)/v \tag{7}$$

Therefore, the index return of an asset can be modeled according to the following formula:

$$R(t) = \log S(t) - \log S(0) = rt + X((t;\sigma,v,\theta) + \omega t = ct + X((t;\sigma,v,\theta),\ c = r + \omega \tag{8}$$

Here, r= risk-free rate， σ = volatility rate

Option Pricing Under the Variance Gamma Model

European Option Pricing

Chinese convertible bonds will carry out special downward revision clauses to adjust the price. This behavior has the characteristics of European option. Assuming that the asset price process follows the formula (7), and then the conditional distribution can be obtained from it, where the condition is G (T) = x.

log(S(T)/S(0))|G(T) = x follows a normal distribution, where the mean is cT+θx, and the variance is $\sigma^2 x$, which means that log(S(T))|G(T) = x also obeys normal distribution, where the mean islogS(0) + cT + θx, and the variance is $\sigma^2 x$.In other words, (S(T)|G(T) = x)obeys a lognormal distribution, where the location parameter is m(x) = log(S(0)) + cT + θx and the scaling parameter is $s(x) = \sigma x^{(1/2)}$.

Then, calculating the mean value of S (T) under specific conditions G (T) = x by following method:

$$E(S(T)|G(T) = x) = S(0)\exp(cT + \theta x + \sigma^2 x/2) \quad (9)$$

At this point, we can calculate the price of call option at the price of K by the following points.

$$c = \exp(-rT)\, E(\max(S(T) - K), 0)$$
$$= \exp(-rT)\int_0^{\infty}\int_k^{\infty} (y - K)\, f_Y(y; m(x), s(x))\, g_X(x; T/v, v)\,dy\,dx \quad (10)$$

Among them, $f_Y(y; m(x), s(x))$is a probability density function of lognormal distribution; $g_X(x; T/v, v)$ is a probability density function of gamma distribution. The price of put option can be obtained through the right to trade parity.

$$p = c - \exp(-rT)[K - E(S(T))],\ \ E(S(T)) = \int_0^{\infty} E(S(T)|G(T) = x)\, g_X(x; T/v, v)\,dx \quad (11)$$

American Option Pricing

Since the convertible bond holder has the right to exercise the transfer of equity at any time point before convertible bond maturity date, and this exercise is similar to the American option.

The method starts with generating a finite sample of the R paths of the underlying asset. Each path has N time points. Suppose that the k^{th} path in the sample is presented in the following order:

S(0), S(k,1), S(k,2), ..., S(k,N)

After reordering the asset price paths according to the asset price, those paths will divided into Q different bundling intervals, in which there are P paths in each interval.

Defining the intrinsic value of options I(k,t) through the k^{th} path and the time t as follows:

$$I(k,t)=\begin{cases}\max\{0,S(T)-K\}, \text{ for call option}\\ \max\{0,K-S(T)\}, \text{ for put option}\end{cases} \quad (12)$$

The value H(k, t) is the discounted expected value of the options that in the intervals containing the k^{th} path. For the same interval, holding value should be equal.

The key to this algorithm is to determine the path index $k_*(t)$ as a clear boundary for deciding whether to continue to hold or exercise options. When k is more than $k \geq k_*(t)$, the value of Y (k, t) is determined to be 1, and when $k \leq k_*(t)$, the value of the variable is 0. Therefore, the indicator variable z(k,t) can be estimated in the following way:

$$z(k,t)=\begin{cases}1, \text{ when } y(k,t)= 1,\ y(k,s) = 0, \text{ for all } s<t\\ 0, \text{ others}\end{cases} \quad (13)$$

When the option is exercised, the present value of the option is equal to the intrinsic value; on the other hand, when the holder chooses to continue holding the option, the present value of the option is equivalent to the holding value. The option value at time 0 can be estimated by reverse derivation. It is worth noting that the accuracy of this estimation method is based on the chosen path partitioning method.

EMPIRICAL RESEARCH

Sample Selection

Based on Wind, this paper sorts the daily turnover and cumulative turnover of convertible bonds, and selects the Everbright convertible bonds with the highest activity as the research object. The basic information of Everbright convertible bond is shown in figure 1.

Bond Name	China Everbright Bank Co., Ltd. publicly issues A-share convertible corporate bonds		
Short Name	Everbright convertible bond	Bond Code	113011
Issue Amount (100 million yuan)	300	Issue Price (yuan)	100
Issuance Method	Outlets are publicly issued; Online pricing is issued	Term (year)	6
Issue Coupon Rate (%)	0.5	Listing Place	Shanghai Stock Exchange
Interest-bearing Day	2017/3/17	Expiry Date	2023/3/17
Release Date	2017/3/17	Release Deadline	2017/3/17
Issue Unit	China Everbright Bank Co., Ltd.	Repayment Method	Annual interest payment
Yield to Maturity (%)	0.97	Remaining Period (year)	4.6438
Issue Object	**Online** : holding the first public investor in the A, B, D, and F securities accounts opened by the registered company (except those prohibited by laws and regulations); **Offline** : holding a registered company to open a qualified securities account Institutional investors (except those prohibited by laws and regulations).		

Fig-1: The Basic Information of Everbright Convertible Bond
(Source: Wind)

The price chart of China Everbright Bank from August 18, 2010 to April 25, 2018 is shown in figure 2.

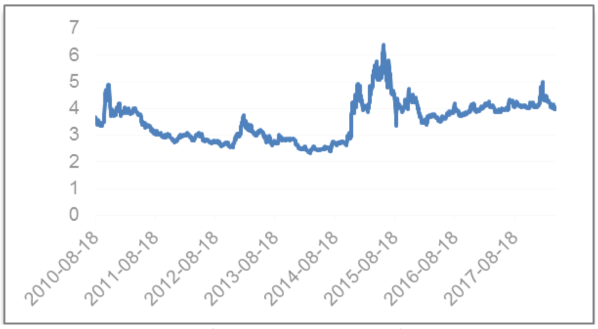

Fig-2: The Trend of Close Stock Price of Everbright Bank
(Source: Wind)

Empirical Analysis

Empirical Thinking

The main idea of the pricing is to assume that the underlying asset of the convertible bond is the variance gamma distribution, based on which the least-squares Monte Carlo model is used to price the convertible bonds.

First, based on the closing price of the Everbright Bank from March 13, 2017 to March 12, 2018 (a total of 245 trading days) as a historical data sample, this paper constructs an exponential variance gamma process, and determine the volatility =0.18332, kurtosis v=0.3995 and skewness theta =0.1692 with the help of historical volatility and moment estimation.

Secondly, generating multiple paths of underlying asset prices. The trend of stock price generally has the characteristics of 'peak and thick tail', so the variance gamma model can better describe its characteristics and improve the problem of the smile curve of the classic BS model. In the risk-neutral process, the stock price follows the previous formula (7), in which the parameter of risk-free interest rate should be determined. To simplify the empirical process, the three-year interest rate of 2017 Treasury bonds is used to replace the risk-free interest rate approximately.

Finally, the LSM method is used to calculate the optimal execution time and return of each path, so that the fitting price of convertible bonds is obtained after discounting.

EMPIRICAL RESULTS

First, a simple comparison is made between the variance gamma model and the random distribution of geometric Brown movement, as shown in figures 3 and 4.

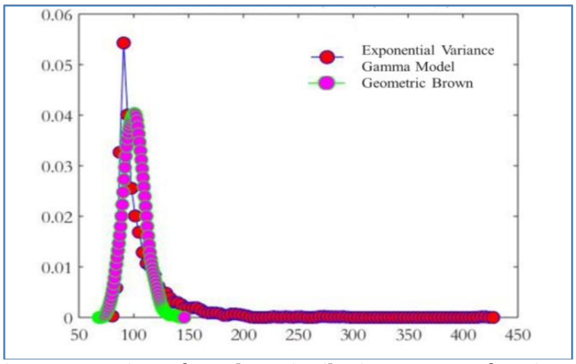

Fig-3: Comparison of Random Distribution Patterns of Variance Gamma Model and Geometric Brownian motion (1)

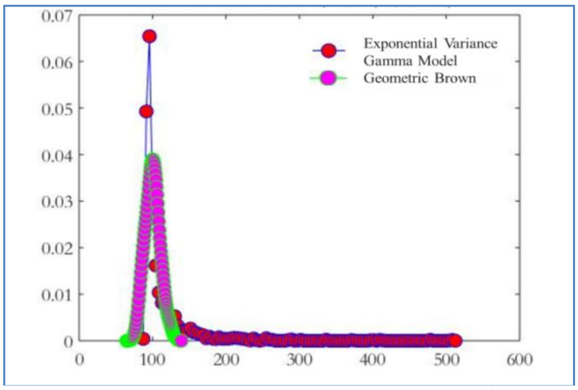

Fig-4: Comparison of Random Distribution Patterns of Variance Gamma Model and Geometric Brownian Motion (2)

The difference in Figure 3 and 4 lies in its variance and skewness; the variance and skewness of Figure 3 are 0.5 and 0.1 respectively, while the variance and bias of figure 4 are 0.75 and 0.05 respectively. From the two figures, it is obvious that the random sample under the variance gamma model has higher kurtosis than the geometric Brown movement, and the tail is longer, which is characterized as the 'peak and thick tail', so it is more in line with the price distribution characteristics of the convertible bonds in reality. Therefore, the variance gamma model can be applied to the pricing of convertible bonds, and the results should be more realistic and accurate.

Based on the above empirical operation, this paper uses MATLAB to fit the price of 30 trading days from March 13, 2018 to April 2018, and draws a theoretical price, which is compared with the actual price of the same period, as shown in Figure 5:

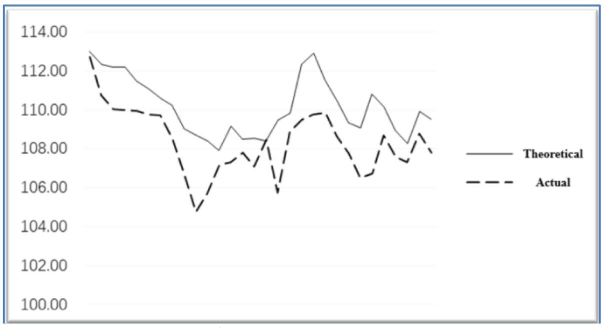

Fig-5: Comparison of the Theoretical Price and Actual Price

According to figure 5, the theoretical price and the actual price of Everbright convertible bonds are basically have the same trend based on the variance gamma model. In order to further measure the accuracy of the pricing method based on variance gamma model, the error rate is introduced.

Suppose the theoretical price of the convertible bond fitting is T and the actual market price is P. The error rate can be defined as

$$E=(T-P)/P*100\% \qquad (14)$$

The maximum error rate is -3.67%, and the average error rate is -1.64%. The empirical data and error rate are shown in table 1 below. Obviously, the theoretical price fitted by the index variance gamma pricing model is consistent with the actual price trend of the market.

Table-1: Theoretical Price, Actual Price and Error Rate of Everbright Convertible bond

Date	Theoretical Price	Actual Price	Error Rate	Date	Theoretical Price	Actual Price	Error Rate
3/13	112.71	113	-0.26%	4/3	108.48	108.4	0.07%
3/14	110.74	112.3	-1.39%	4/4	105.75	109.46	-3.39%
3/15	110.03	112.21	-1.94%	4/9	108.86	109.8	-0.86%
3/16	109.98	112.18	-1.96%	4/10	109.49	112.31	-2.51%
3/19	109.95	111.49	-1.38%	4/11	109.77	112.89	-2.76%
3/20	109.74	111.09	-1.22%	4/12	109.86	111.53	-1.50%
3/21	109.71	110.59	-0.80%	4/13	108.62	110.45	-1.66%
3/22	108.52	110.21	-1.53%	4/16	107.75	109.34	-1.45%
3/23	106.76	109	-2.06%	4/17	106.48	109.08	-2.38%
3/26	104.76	108.7	-3.62%	4/18	106.74	110.81	-3.67%
3/27	105.69	108.39	-2.49%	4/19	108.69	110.12	-1.30%
3/28	107.15	107.91	-0.70%	4/20	107.56	108.95	-1.28%
3/29	107.29	109.17	-1.72%	4/23	107.29	108.26	-0.90%
3/30	107.79	108.5	-0.65%	4/24	108.75	109.92	-1.06%
4/2	107.07	108.54	-1.35%	4/25	107.79	109.5	-1.56%
Max. Error Rate		3.67%		Avg. Error Rate		1.64%	

CONCLUSION

This paper introduces variance gamma model to discuss the pricing of convertible bonds in China. This paper introduces the basic concept of variance gamma model, and briefly derives the analytical solution of European option and American option under variance gamma model. Based on this, the variance gamma model is introduced into the pricing method of convertible bonds. Further, based on the convertible bond pricing method under the variance gamma model; this paper fits the theoretical price of the convertible bond. By comparing the theoretical price and the actual price, it verify the advantage of the variance gamma model on the accuracy of the convertible bond pricing from the point of view of the data support. The empirical results show that the variation trend of theoretical price and actual price based on variance gamma model is basically the same, and the fitting degree is good. At the same time, the error rate is small according to the calculated error rate. Therefore, from the empirical level, the accuracy of the variance gamma model is very good.

In view of the better theoretical and empirical performance of the variance gamma model, the future China's convertible bonds pricing can use more variance gamma model to replace the Black-Scholes model to improve the pricing accuracy. Meanwhile, due to the complexity of convertible bonds, the convertible bond pricing method based on the variance gamma model still needs to be further revised and developed. In combination with the actual conditions of China, the in-depth study of the it can be developed as a follow-up topic.

REFERENCES

1. Ingersoll Jr, J. E. (1977). A contingent-claims valuation of convertible securities. *Journal of Financial Economics*, *4*(3), 289-321.
2. Brennan, M. J., & Schwartz, E. S. (1977). Convertible bonds: Valuation and optimal strategies for call and conversion. *The Journal of Finance*, *32*(5), 1699-1715.
3. Brennan, M. J., & Schwartz, E. S. (1980). Analyzing convertible bonds. *Journal of Financial and Quantitative analysis*, *15*(4), 907-929.
4. Sîrbu, M., & Shreve, S. E. (2006). A two-person game for pricing convertible bonds. *SIAM Journal on Control and Optimization*, *45*(4), 1508-1539.
5. Huaxia Securities Research Institute.(1998). Convertible bond pricing theory analysis [J], China Securities Journal, 2.21.
6. McConnell, J. J., & Schwartz, E. S. (1986). LYON taming. *The Journal of Finance*, *41*(3), 561-576.
7. Tsiveriotis, K., & Fernandes, C. (1998). Valuing convertible bonds with credit risk. *The Journal of Fixed Income*, *8*(2), 95.
8. Fama, E. (1965). The behavior of stock market prices[J], Journal of Business, 1965(38):34-105.
9. Merton, R. C. (1975). Option pricing when underlying stock returns are discontinuous.
10. Madan, D. B., Carr, P. P., & Chang, E. C. (1998). The variance gamma process and option pricing. *Review of Finance*, *2*(1), 79-105.
11. Duffie, D., & Singleton, K. J. (1999). Modeling term structures of defaultable bonds. *The review of financial studies*, *12*(4), 687-720.
12. Yigitbasioglu, A. B. (2002, August). Pricing convertible bonds with interest rate, equity, credit, and FX risk. In *EFMA 2002 London Meetings* (pp. 2001-14).
13. Kovalov, P., & Linetsky, V. (2008). Valuing convertible bonds with stock price, volatility, interest rate, and default risk. *FDIC Center for Financial Research Working Paper Series*, (2008-02).
14. Zhang, W. (2011). Research on the pricing of convertible bonds based on total least squares Quasi Monte Carlo method [J]. Management, 1.
15. Du, Chengkai. (2008). An empirical study of the double exponential jump diffusion model in China[D]. Xiamen: Xiamen University.
16. Zheng, Z.(2004). Lin Hai. Study on the pricing of convertible bonds in China [J]. Journal of Xiamen University: Philosophy and Social Sciences Edition, (2): 93-99.