Convertible Bond Pricing Based on Variance Gamma Model

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Abstract

Due to the ‘spike and tail’ phenomenon of asset returns, the applicability of the Black-Scholes model for pricing convertible bonds has been questioned, and the variance gamma model can cope well with this phenomenon and solve the ‘volatility smile dilemma’. This paper combines the variance gamma model with the least squares Monte Carlo simulation method to empirically analyze the Everbright convertible bonds based on its high activity in the Chinese market. In this paper, the theoretical price and the actual price are compared, and the applicability of the variance gamma model in the Chinese convertible bond market is analyzed. The empirical results show that the fitting price obtained by the variance gamma model is consistent with the actual price trend, indicating that the method is applicable to the Chinese convertible bond market.

Keywords: Convertible Bonds; Pricing Method; Variance Gamma Model.

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INTRODUCTION

Convertible bonds originated in the 1970s. At present, the United States, Europe, Japan and some Asian countries are the major issuers of it, and the size of the global convertible bond market has exceeded $1 trillion. The development of China’s convertible bond market started late, and it has only been more than 20 years. However, with the continuous improvement of China’s financial system, the development of convertible bonds has been relatively stable and gradually occupied an increasingly important position.

There are not many varieties of convertible bonds circulating in China, and the level of activity is far less than that of stocks or bonds. At the same time, its’ complicated pricing principle make its research relatively less. However, the international research on the theory and practice of the convertible bond pricing model has been relatively mature, providing a useful reference basis for research in China. Due to the classic Black-Scholes model cannot solve the problem of ‘spike and thick tail’ phenomenon of stock returns and the related ‘volatility smile dilemma’, its accuracy of pricing convertible bond was doubted. However, the variance gamma model that emerged in recent years can solve these two key problems well. This paper will verify the applicability of the variance gamma pricing model in China’s convertible bond market through empirical analysis. This paper will take Everbright convertible bonds as an example, and by comparing the fitted price that obtained based on the variance gamma pricing model and its actual price and calculating the error rate, this paper will study whether the method has superiority in the pricing accuracy in the Chinese convertible bond market.

LITERATURE REVIEW

According to different underlying assets, the theoretical research related to convertible bond pricing can be divided into two aspects, the pricing model based on company value and on stock price.

A. the pricing model based on company value.

Ingersoll [1] assumes that the dynamic change process of the company’s value follows the geometric Brown movement. By considering the optimal transformation strategy of the investor and the optimal redemption strategy of the issuer, the analytical solution of the non-redeemable and redeemable zero-interest convertible bond is obtained. Brennan and Schwartz [2] put the redemption clause of the convertible bonds into the model, and deduce the partial differential equation of the convertible bond price with the principle of no arbitrage. At the same time, the boundary and terminal conditions are clarified, which further perfected the convertible bond pricing model. Brennan and Schwartz [3] consider the interest rate randomization on the basis of the previous model and compare the pricing results under fixed and random interest rates, of which the empirical results show that the pricing is not...
significantly affected by the interest rate. Sirbu [4] converts the pricing of convertible bonds into a relatively simple optimal time-stopping decision. It is considered that when the product of interest rate and redemption value is lower than the dividend value, the redemption is superior to the exercise of equity. Otherwise, the exercise of equity is better than redemption. The China Securities Research Institute [5] constructed the model assumes that the price of assets jumps will be considered to essentially reduce the lognormal distribution of the index return of an asset can be modeled according to the following formula:

\[ R(t) = \log S(t) - \log S(0) = rt + X(t; \mu, \nu, \theta) + \omega t = ct + X(t; \mu, \theta, \nu) \]

\[ \sigma = \text{volatility of Brown movement (volatility)} \]

\[ V = \text{variance rate of gamma time variation (kurtosis)} \]

\[ \theta = \text{drift rate of Brown movement (skewness)} \]

The first four moments of the variance gamma process are:

\[ E(X(t)) = \theta t \] \hspace{1cm} (3)

\[ E[X(t) - E(X(t))]^2 = (\theta^2 \nu + \sigma^2) t \] \hspace{1cm} (4)

\[ E[X(t) - E(X(t))]^3 = (2 \theta^3 \nu^2 + 3 \sigma^2 \theta \nu) t \] \hspace{1cm} (5)

\[ E[X(t) - E(X(t))]^4 = (3 \theta^4 \nu^4 + 12 \sigma^4 \theta^2 \nu^2 + 6 \sigma^4 \nu^2 + 3 \sigma^4 \theta^4 + 3 \sigma^4 \theta^2 \nu^2) t^2 \] \hspace{1cm} (6)

Under the risk neutral process, the dynamic change of asset prices follows the following VG process:

\[ S(t) = S(0) \exp(rt + X(t; \mu, \nu, \theta) + \omega t), \omega = \ln(1-0.5\sigma^2)/\nu \]

Therefore, the index return of an asset can be modeled according to the following formula:

\[ R(t) = \log S(t) - \log S(0) = rt + X(t; \mu, \nu, \theta) + \omega t = rt + X(t; \mu, \theta, \nu, \omega) \]

Here, \( r \) = risk-free rate, \( \sigma \) = volatility rate.
Option Pricing Under the Variance Gamma Model

**European Option Pricing**

Chinese convertible bonds will carry out special downward revision clauses to adjust the price. This behavior has the characteristics of European option. Assuming that the asset price process follows the formula (7), and then the conditional distribution can be obtained from it, where the condition is \( G(T) = x \).

Then, calculating the mean value of \( S(T) \) under specific conditions \( G(T) = x \) by following method:

\[
\begin{align*}
E(S(T) | G(T) = x) &= S(0) \exp(cT + \theta x + \sigma^2 x/2) \\
\end{align*}
\]

At this point, we can calculate the price of call option at the price of \( K \) by the following points.

\[
\begin{align*}
c &= \exp(-rT) \left[ \max(S(T), K) \right] \\
&= \exp(-rT) \int_0^\infty \int_0^\infty (y - K) f_Y(y; m(x), s(x)) g_X(x; T/v, v) dy dx
\end{align*}
\]

Among them, \( f_Y(y; m(x), s(x)) \) is a probability density function of lognormal distribution; \( g_X(x; T/v, v) \) is a probability density function of gamma distribution. The price of put option can be obtained through the right to trade parity.

\[
\begin{align*}
p &= c - \exp(-rT)[K - E(S(T))], \quad E(S(T)) = \int_0^\infty E(S(T) | G(T) = x) g_X(x; T/v, v) dx
\end{align*}
\]

**American Option Pricing**

Since the convertible bond holder has the right to exercise the transfer of equity at any time point before convertible bond maturity date, and this exercise is similar to the American option.

The method starts with generating a finite sample of the \( R \) paths of the underlying asset. Each path has \( N \) time points. Suppose that the \( k^{th} \) path in the sample is presented in the following order:

\[
S(0), S(k,1), S(k,2), ..., S(k,N)
\]

After reordering the asset price paths according to the asset price, those paths will divided into \( Q \) different bundling intervals, in which there are \( P \) paths in each interval.

Defining the intrinsic value of options \( I(k,t) \) through the \( k^{th} \) path and the time \( t \) as follows:

\[
I(k,t) = \begin{cases} 
\max(0, S(T) - K), & \text{for call option} \\
\max(0, K - S(T)), & \text{for put option}
\end{cases}
\]

The value \( H(k, t) \) is the discounted expected value of the options that in the intervals containing the \( k^{th} \) path. For the same interval, holding value should be equal.

The key to this algorithm is to determine the path index \( k^*(t) \) as a clear boundary for deciding whether to continue to hold or exercise options. When \( k \) is more than \( k \geq k^*(t) \), the value of \( Y(k, t) \) is determined to be 1, and when \( k \leq k^*(t) \), the value of the variable is 0. Therefore, the indicator variable \( z(k,t) \) can be estimated in the following way:

\[
z(k,t) = \begin{cases} 
1, & \text{when } y(k,t) = 1, y(k,s) = 0, \text{ for all } s < t \\
0, & \text{others}
\end{cases}
\]

**EMPIRICAL RESEARCH**

Sample Selection

Based on Wind, this paper sorts the daily turnover and cumulative turnover of convertible bonds, and selects the Everbright convertible bonds with the highest activity as the research object. The basic information of Everbright convertible bond is shown in figure 1.
The price chart of China Everbright Bank from August 18, 2010 to April 25, 2018 is shown in figure 2.

The main idea of the pricing is to assume that the underlying asset of the convertible bond is the variance gamma distribution, based on which the least-squares Monte Carlo model is used to price the convertible bonds.

First, based on the closing price of the Everbright Bank from March 13, 2017 to March 12, 2018 (a total of 245 trading days) as a historical data sample, this paper constructs an exponential variance gamma process, and determine the volatility =0.18332, kurtosis v=0.3995 and skewness theta =0.1692 with the help of historical volatility and moment estimation.

Secondly, generating multiple paths of underlying asset prices. The trend of stock price generally has the characteristics of "peak and thick tail", so the variance gamma model can better describe its characteristics and improve the problem of the smile curve of the classic BS model. In the risk-neutral process, the stock price follows the previous formula (7), in which the parameter of risk-free interest rate should be determined. To simplify the empirical process, the three-year interest rate of 2017 Treasury bonds is used to replace the risk-free interest rate approximately.

Finally, the LSM method is used to calculate the optimal execution time and return of each path, so that the fitting price of convertible bonds is obtained after discounting.

**EMPIRICAL RESULTS**

First, a simple comparison is made between the variance gamma model and the random distribution of geometric Brown movement, as shown in figures 3 and 4.
The difference in Figure 3 and 4 lies in its variance and skewness; the variance and skewness of Figure 3 are 0.5 and 0.1 respectively, while the variance and bias of figure 4 are 0.75 and 0.05 respectively. From the two figures, it is obvious that the random sample under the variance gamma model has higher kurtosis than the geometric Brown movement, and the tail is longer, which is characterized as the ‘peak and thick tail’, so it is more in line with the price distribution characteristics of the convertible bonds in reality. Therefore, the variance gamma model can be applied to the pricing of convertible bonds, and the results should be more realistic and accurate.

Based on the above empirical operation, this paper uses MATLAB to fit the price of 30 trading days from March 13, 2018 to April 2018, and draws a theoretical price, which is compared with the actual price of the same period, as shown in Figure 5:

\[
E = \frac{(T - P)}{P} \times 100\%
\]

The maximum error rate is -3.67%, and the average error rate is -1.64%. The empirical data and error rate are shown in table 1 below. Obviously, the theoretical price fitted by the index variance gamma pricing model is consistent with the actual price trend of the market.

### Table-1: Theoretical Price, Actual Price and Error Rate of Everbright Convertible bond

<table>
<thead>
<tr>
<th>Date</th>
<th>Theoretical Price</th>
<th>Actual Price</th>
<th>Error Rate</th>
<th>Date</th>
<th>Theoretical Price</th>
<th>Actual Price</th>
<th>Error Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/13</td>
<td>112.71</td>
<td>113</td>
<td>-0.26%</td>
<td>4/3</td>
<td>108.48</td>
<td>108.4</td>
<td>0.07%</td>
</tr>
<tr>
<td>3/14</td>
<td>110.74</td>
<td>112.3</td>
<td>-1.39%</td>
<td>4/4</td>
<td>105.75</td>
<td>109.46</td>
<td>-3.39%</td>
</tr>
<tr>
<td>3/15</td>
<td>110.03</td>
<td>112.21</td>
<td>-1.94%</td>
<td>4/9</td>
<td>108.86</td>
<td>109.8</td>
<td>-0.86%</td>
</tr>
<tr>
<td>3/16</td>
<td>109.98</td>
<td>112.18</td>
<td>-1.96%</td>
<td>4/10</td>
<td>109.49</td>
<td>112.31</td>
<td>-2.51%</td>
</tr>
<tr>
<td>3/19</td>
<td>109.95</td>
<td>111.49</td>
<td>-1.38%</td>
<td>4/11</td>
<td>109.77</td>
<td>112.89</td>
<td>-2.76%</td>
</tr>
<tr>
<td>3/20</td>
<td>109.74</td>
<td>111.09</td>
<td>-1.22%</td>
<td>4/12</td>
<td>109.86</td>
<td>111.53</td>
<td>-1.50%</td>
</tr>
<tr>
<td>3/21</td>
<td>109.71</td>
<td>110.59</td>
<td>-0.80%</td>
<td>4/13</td>
<td>108.62</td>
<td>110.45</td>
<td>-1.66%</td>
</tr>
<tr>
<td>3/22</td>
<td>108.52</td>
<td>110.21</td>
<td>-1.53%</td>
<td>4/16</td>
<td>107.75</td>
<td>109.34</td>
<td>-1.45%</td>
</tr>
<tr>
<td>3/23</td>
<td>106.76</td>
<td>109</td>
<td>-2.06%</td>
<td>4/17</td>
<td>106.48</td>
<td>109.08</td>
<td>-2.38%</td>
</tr>
<tr>
<td>3/26</td>
<td>104.76</td>
<td>108.7</td>
<td>-3.62%</td>
<td>4/18</td>
<td>106.74</td>
<td>110.81</td>
<td>-3.67%</td>
</tr>
<tr>
<td>3/27</td>
<td>105.69</td>
<td>108.39</td>
<td>-2.49%</td>
<td>4/19</td>
<td>108.69</td>
<td>110.12</td>
<td>-1.30%</td>
</tr>
<tr>
<td>3/28</td>
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<td>-0.70%</td>
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<td>107.56</td>
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<td>-1.28%</td>
</tr>
<tr>
<td>3/29</td>
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<td>109.17</td>
<td>-1.72%</td>
<td>4/23</td>
<td>107.29</td>
<td>108.26</td>
<td>-0.90%</td>
</tr>
<tr>
<td>3/30</td>
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<td>108.5</td>
<td>-0.65%</td>
<td>4/24</td>
<td>108.75</td>
<td>109.92</td>
<td>-1.06%</td>
</tr>
<tr>
<td>4/2</td>
<td>107.07</td>
<td>108.54</td>
<td>-1.35%</td>
<td>4/25</td>
<td>107.79</td>
<td>109.5</td>
<td>-1.56%</td>
</tr>
</tbody>
</table>

| Max. Error Rate | 3.67% |
| Avg. Error Rate | 1.64% |

### CONCLUSION

This paper introduces variance gamma model to discuss the pricing of convertible bonds in China. This paper introduces the basic concept of variance gamma model, and briefly derives the analytical solution of European option and American option under variance gamma model. Based on this, the variance gamma model is introduced into the pricing method of convertible bonds. Further, based on the convertible bond pricing method under the variance gamma model; this paper fits the theoretical price of the convertible bond. By comparing the theoretical price and the actual price, it verify the advantage of the variance gamma model on the accuracy of the convertible bond pricing from the point of view of the data support. The empirical results show that the variation trend of theoretical price and actual price based on variance gamma model is basically the same, and the fitting degree is good. At the same time, the error rate is small according to the calculated error rate. Therefore, from the empirical level, the accuracy of the variance gamma model is very good.
In view of the better theoretical and empirical performance of the variance gamma model, the future China’s convertible bonds pricing can use more variance gamma model to replace the Black-Scholes model to improve the pricing accuracy. Meanwhile, due to the complexity of convertible bonds, the convertible bond pricing method based on the variance gamma model still needs to be further revised and developed. In combination with the actual conditions of China, the in-depth study of the it can be developed as a follow-up topic.

REFERENCES