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Probabilistic Assessment of a Doubly Symmetric I-Steel Beam

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Abstract

This paper presents the findings of a probabilistic evaluation of a doubly symmetric I-steel beam's bending, shear, and deflection limit states. The design adhered to BS 5950, Part 1, 2000. Failure equations for flexure, shear, and deflection were derived, while random variable probabilistic models were sourced from the literature. Optimization using the First-Order Reliability Method (FORM) yielded design points, reliability indices, and sensitivity analyses. The results revealed that the reliability index decreased as beam span increased, with negative indices observed at a load ratio of 1.0 and beam span of 8.5m. Moreover, increasing the beam span to an overall depth ratio above 42 compromised reliability. The design achieved material savings in the plastic section modulus for a target reliability index of 3.0 but increased the modulus for a target index of 3.80 over a 50-year period. The design proved critical in bending, safe in deflection, and satisfactory in shear.

Keywords: Failure analysis, sensitivity analysis, reliability level.

1.0 INTRODUCTION

In recent times, the Nigerian populace has been witness to an unprecedented and alarming rise in the collapse of building structures, resulting in the destruction of properties worth billions of naira. This distressing trend necessitates the implementation of reliability analysis for structures or their components at every stage of their service lives, rather than adopting a passive approach and observing their eventual collapse, Sule *et al.*, [1]. According to the esteemed scholars Mosley and Bungey [2], engineered structures must fulfill the requirements of both the ultimate and serviceability limit states. The strength of any engineered structure inevitably undergoes degradation over time, making condition assessment of utmost importance [3, 4].

It is imperative to acknowledge that the design of structures or structural members based solely on codes cannot guarantee absolute safety, as the design may prove inadequate due to poor estimation of loading and may even be uneconomical due to an overestimation of loads. The root cause of poor loading estimation lies in the inherent variability of the design parameters employed in the design equations. The presence of variability in these design parameters renders it exceedingly challenging to accurately predict the safety of engineered structures and cost implications during the design phase [5]. Given the catastrophic consequences that result from structural failure, it is essential for structural engineers to intervene promptly at every stage of a structure's service life, in order to avert the devastating effects that failure and subsequent collapse can inflict.

The implementation of a probabilistic framework has proven immensely beneficial in the condition assessment of various civil engineering facilities, as it effectively addresses the uncertainties associated with design parameters [6]. In light of this, the present study aims to conduct a probabilistic assessment of a doubly symmetric I-steel beam, focusing on the limit state of bending, shear, and deflection, respectively. To achieve this objective, the First-Order reliability method was employed, and a bespoke MATLAB code was developed, utilizing the derived failure functions. This code enables estimation of the reliability indices for different values of the random variables, thereby facilitating an investigation into the impact of these variables on the beam's reliability levels.

Esmaeil, *et al.*, [7] conducted an extensive investigation on reliability index, with a specific focus on optimizing self-centering structures to attain the minimum weight possible by utilizing metaheuristic algorithms. This study encompassed not only linear but nonlinear reliability problems, thereby providing a comprehensive analysis. The findings of their research unveiled significant results, indicating a noteworthy decrease in weight of 36%, 30%, and 32% for buildings with 10, 15, and 20 storeys, respectively, when uncertainties were not accounted for. However, when uncertainties were factored in, a remarkable weight reduction of 23% was achieved for the same buildings. This implies that the consideration of uncertainties can lead to an increase in failure probability of up to 23%. In addition, the authors made an interesting observation regarding the performance of the charged system search and colliding bodies optimization algorithms, noting their effectiveness in the context of this study. Consequently, it can be concluded that the incorporation of a reliability index, while leading to the construction of heavier structures, ultimately enhances the overall safety of these structures.

Junho [8] extensively examined the concept of "Reliability-Based Design Optimization (RBDO) of Structures Using Complex-Step Approximation with Sensitivity Analysis". In the study, he conducted a thorough examination of the application of reliability analysis in the field of structural design. Through the meticulous experimentation and examination of various structural optimization problems, encompassing a wide spectrum of statistical variations, he successfully showcased the potential of this method in achieving optimal performance while adhering to highly precise probabilistic constraints. By employing complex-step approximation, the accuracy of RBDO was significantly enhanced, leading to a notable improvement in the overall performance benefits associated with structural optimization.

Jerez *et al.*, [9] in their studies conducted a comprehensive survey to explore the most recent advancements in reliability-based design optimization of structures subjected to stochastic excitation. In their study, they examined various approaches, including the search-based technique, sequential optimization approach, and scheme-based approach. An intriguing observation made by the authors was the significant influence of computational aspects in successfully addressing optimization problems. Furthermore, their comprehensive overview suggests that the methods employed for achieving optimal design in stochastic structural dynamics are no longer confined to academic scenarios but can also serve as valuable tools in solving a wide range of engineering design problems.

The investigation conducted by [10] focused on the examination of reliability analysis and design optimization for nonlinear structures. In order to accomplish this, they employed the Kriging based method and the First-order reliability method (FORM). The Kriging based method, showed greater levels of efficiency and accuracy when compared to the FORM based method and the Monte Carlo Simulation (MCS) method. Interestingly, the Kriging based method did not require the determination of the response sensitivity, thereby enhancing its adaptability for various scenarios.

An examination of the reliability analysis of steel rack frames using the Direct Design Method was carried out by [11]. Furthermore, they developed curves that illustrate the relationship between the system reliability index (β) and the system resistance factors (ϕs) for these steel rack frames and compared their findings to those of a traditional design approach based on elastic analysis. To carry out their assessment, the researchers employed a combination of the DDM, a formulation of the limit state function, and probabilistic modelling through Monte Carlo simulation. This allowed them to thoroughly investigate the reliability of steel rack frames and derive the system reliability indices. The results they obtained indicated that the utilization of the DDM offers more advantages than the traditional design approach based on elastic analysis. Specifically, when the system reliability indices fell within the range of ≤3, they found that similar structural reliability was achieved for both unweighted and weighted unit pallet loading. This observation demonstrated the consistency between the load combination factors and their corresponding coefficients of variation. Overall, the researchers concluded that incorporating sectional imperfections in the analysis model did not yield any discernible benefits in terms of the adopted system resistance factor.

2.0 METHODOLOGY

2.1 Development of Limit State Functions

A limit state function is a representation of a specific failure mode and it establishes a connection between various parameters. Its development is a fundamental aspect of structural engineering and plays a vital role in the design and construction of safe and efficient structures. In this study, the failure functions were developed according to the provisions of [12] for the design of steel structures.

2.1.1 Bending Limit State Function

The limit state of bending happens when the bending moments or tensile stresses are more than what is necessary for the structure to be safe and functional before failing. Equation 1 illustrates its function as follows:

$$G(X) = P_y S_x - 0.125 * 1.6 * q_k * (0.875\alpha + 1) * L^2 \qquad (1)$$

2.1.2 Shear Limit State Function

Shear forces that are too great for the structure's safety and serviceability standards before failure cause the shear limit condition. The shear resistance of the I-section according to [12] is given in equation 2 as:

$$P_v = 0.60 P_y A_v \quad (2)$$

Where,

$$A_v = Dt \quad (3)$$

The maximum shear force is given as:

$$Fmax = \frac{5wL}{8} = 0.625 * 1.6qk(0.875\alpha + 1) * L \quad (4)$$

The failure function in shear is generated by subtracting equation 4 from equation 2. This is given as shown in equation 5 and equation 6:

$$G(X) = 0.60 P_y Dt - 0.625 * 1.6 * q_k * (0.875\alpha + 1) * L \quad (5)$$

$$G(X) = 0.60 P_y t - 0.625 * 1.6 * q_k * (0.875\alpha + 1) * \frac{L}{D} \quad (6)$$

Let,

$$\frac{L}{D} = \lambda \quad (6b)$$

Equation 6 now becomes:

$$G(X) = 0.60 P_y t - 0.625 * 1.6 * q_k * (0.875\alpha + 1) * \lambda \quad (7)$$

Where g_k = Characteristic dead load; q_k = Characteristic live load; P_y = Bending strength of steel

Equation 7 is the failure function in shear of the doubly symmetrical I-steel beam.

2.1.3 Deflection Limit State Function

The deflection limit state is a conception that focuses on the gravity of keeping the deflection of a structural member below a certain boundary to ensure the structure's performance and safety, alleviating the risk of failure, and maintaining stability, functionality, and durability. The allowable deflection is shown in equation 8 as:

$$\delta_{all} = \frac{L}{360} \quad (8)$$

Eqution 9 presents the maximum value of deflection for a uniformly loaded beam as:

$$\delta \max = \frac{0.0052wL^4}{EI} \quad (9)$$

The limit state function in deflection is developed by subtracting Eq. (9) from Eq. (8) and is given in Eq. (10) as:

$$G(X) = \frac{L}{360} - \frac{0.0052wL^4}{EI} \quad (10)$$

Substituting the value of *w* from equation 4 into the equation 10 gives:

$$G(X) = \frac{L}{360} - \frac{0.0052 * 1.6 * q_k * (0.875\alpha + 1) * L^4}{EI} \quad (11)$$

Eq. (11) is the failure function in deflection of the doubly symmetrical I-steel beam.

2.2 Probabilistic Design of Steel Beam in Bending

The limit state function in bending is given as:

$$G(X) = P_y S_x - M_D - M_L \quad (12)$$

Where M_D and M_L are induced moment due to dead and live loads respectively.

Induced moment due to dead and live loads are shown in equation 13 and equation 14 respectively.

$$M_D = \frac{g_k L^2}{8} \quad (13)$$

$$M_L = \frac{q_k L^2}{8} \quad (14)$$

Let:

$$P_y = X_1; g_k = X_2; q_k = X_3 \quad (14b)$$

Equuation 12 now becomes:

$$G(X) = X_1 * S_x - \frac{X_2 L^2}{8} - \frac{X_3 L^2}{8} \quad (15)$$

Or

$$G(X) = \frac{8 S_x X_1}{L^2} - X_2 - X_3 \quad (16)$$

Let the coefficient of X_1 be b. Therefore,

$$\frac{8 S_x}{L^2} = b \quad (17)$$

Eq. (16) now becomes:

$$G(X) = bX_1 - X_2 - X_3 \quad (18)$$

The statistics of the design parameters in equation 18 are used as input variables in the MATLAB code and the value of b corresponding to the target reliability index of 3.0 recommended for beams in flexure is obtained by optimization.

2.3 Reliability Analysis

The design process for a doubly symmetric I-steel beam, subject to uncertain dead and live loads of 20KN/m and 10KN/m respectively, was conducted in accordance with the design specifications outlined in [12] A UB section with dimensions of 406*140*46Kg/m was determined to meet the necessary criteria for bending, shear, and deflection.

The characteristic live load value is kept constant at 20KN/m while the varying load ratio values considered are 0.5, 0.75, 1.0, 1.25, 1.50, 1.75, 2.0 and 2.25 respectively. The values of the characteristic dead load corresponding to the above load ratios are 10KN/m, 15KN/m, 20KN/m, 25KN/m, 30KN/m, 35KN/m, 40KN/m and 45KN/m respectively. The deterministic design for the plastic section modulus of the I-beam corresponding to the characteristic dead and live loads was carried out and the results obtained are compared with the results of the probabilistic design at target reliability indices of 3.0 and 3.80 respectively. The probabilistic models for the basic random variables are presented in Table 1.

Table 1: Probabilistic models of the basic random variables

S/N	Variables	Unit	Type of probability distribution	Mean	Standard deviation	Coefficient of variation
1	P_y	N/mm^2	Normal	275	27.5	0.10
2	q_k	kN/m	Normal	20	5	0.25
3	I_x	mm^4	Normal	475400000	23770000	0.05
4	E	N/mm^2	Normal	205000	10250	0.05
5	L	mm	Normal	8000	400	0.05
6	D	mm	Normal	528.3	26.415	0.05
7	S_x	cm^3	Normal	2059000	102950	0.05
8	α	-	Fixed	Varying		
9	t	mm	Normal	9.6	0.48	0.05
10	D/L	-	Fixed	Varying	-	-
11	g_k	kN/m	Normal	Varying	-	0.10

2.3.1 First Order Reliability Analysis

The limit state function $G(X)$ is a function of the basic random variables. $G(X)$ is the limit state function such that $G(X) < 0$ represents unsafe state of a structure, $G(X) > 0$ represents the safe state of a structure and $G(X) = 0$ represents the demarcation between the safe and unsafe state of the structure respectively.

Let the limit state function in the space of n-dimensional input variables $X_1, X_2, \ldots, X_n$ be given by:

$$G = g(X_1, X_2, \ldots, X_n) = 0 \quad (19)$$

Also,
Let the vector of the be random variables with second moment statistics $E(X)$ and $Cov(X, X')$ be $X = [X_1, X_2, \ldots, X_n]'$.

The normalized random variables $y_1, y_2, \ldots, y_n$ are introduced by a suitable one to one linear mapping $X = L(y)$such that $y = L^{-1}(X)$. The corresponding space of y is then defined by the transformation:

$$X = L(y), y = L^{-1}(X) \quad (20)$$

Applying equation 20 maps equation 19 into:

$$h(y_1, y_2, \ldots, y_n) = 0 \quad (21)$$

Where the function h is defined by:

$$h(y) = g[L(y)] \quad (22)$$

Equation (22) represents the failure function in normalized coordinate. The mean value of y is the origin and the projection of y on the arbitrary straight line through the origin is the random variable with the standard deviation of unity.

The reliability index β is the distance between the origin and the failure surface in the normalized coordinate. It is given by:

$$\beta = min\left\langle \sqrt{\Sigma(y_1{}^2 + y_2{}^2 + \ldots + y_n{}^2)} \,\middle|\, h(y_1, y_2, \ldots, y_n) \right\rangle = 0 \quad (23)$$

Equation (23) is minimized subject to the constraint that $h(y_1, y_2, ..., y_n) = 0$. The design points on the failure surface are obtained by optimization.

In First-Order reliability method, all non-normal random variables must first be transformed to their equivalent normal random variables before they can be used. This requires that the distribution function of the basic variable and the equivalent normal variable are equated at the design point as:

$$\Phi\left(\frac{x_i{}^* - \mu_{xi}^N}{\sigma_{xi}^N}\right) = Fx_i(x_i{}^*) \quad (24)$$

Where $\Phi(\cdot)$ = cumulative distribution function of the standard normal variable at the design point; μ_{xi}^N, σ_{xi}^N = mean and standard deviation of the equivalent normal variable at the design point respectively; $Fx_i(x_i{}^*)$ = cumulative distribution function of the original non-normal variables.

The mean of the equivalent normal variable at the design point is given by:

$$\mu_{xi}^N = x_i{}^* - \Phi^{-1}[Fx_i(x_i{}^*)]\sigma_{xi}^N \quad (25)$$

The distribution function of the basic variable and the equivalent normal variable are equal
at the design point:

$$\frac{\varphi}{\sigma_{xi}^N}\left(\frac{x_i{}^* - \mu_{xi}^N}{\sigma_{xi}^N}\right) = fx_i(x_i{}^*) \quad (26)$$

Where $\varphi(\cdot)$ and $fx_i(x_i{}^*)$ = probability distribution function of the equivalent standard normal and the original non-normal random variable respectively.

Applying equation 25;

$$\Phi^{-1}[Fx_i(x_i{}^*)]\sigma_{xi}^N = x_i{}^* - \mu_{xi}^N \quad (27)$$

Applying Equation (26), the standard deviation of the equivalent normal variables are given as:

$$\sigma_{xi}^N = \varphi \frac{(\Phi^{-1}[F_{xi}(x_i^*)])}{fx_i(x_i^*)} \quad (28)$$

3.0 RESULTS AND DISCUSSION

The results of the MATLAB program automated reliability analysis of a doubly symmetric I-section steel beam are presented in Figure 1 to Figure 6 respectively and Table 2.

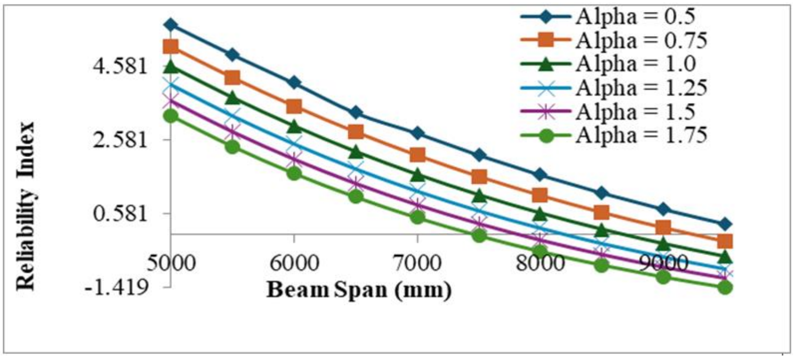

Figure 1: Relationship between reliability index and beam span or varying load ratio (Bending limit state)

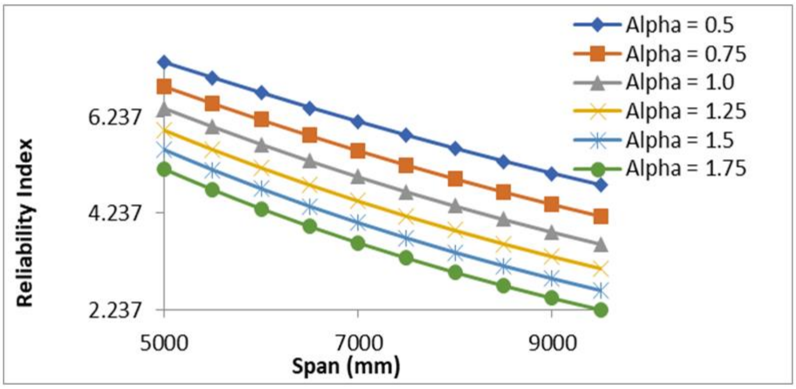

Figure 2: Relationship between reliability index and beam span for varying load ratio (Shear limit state)

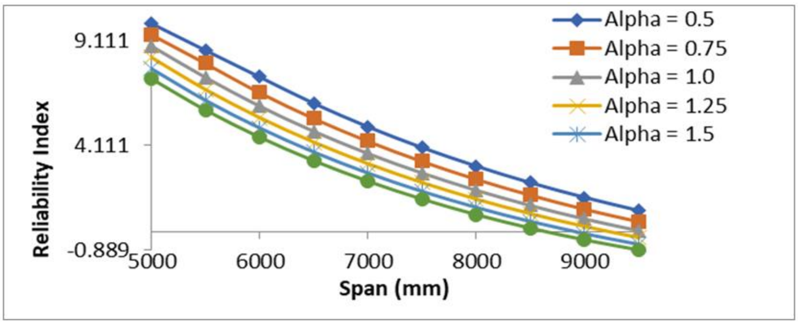

Figure 3: Relationship between reliability index and beam span for varying load ratio (Deflection limit state)

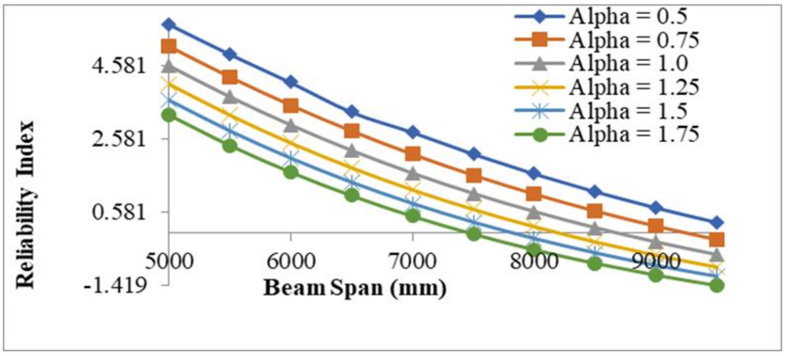

Figure 4: Relationship between reliability index and beam span for varying load ratio (Bending, shear and deflection limit state)

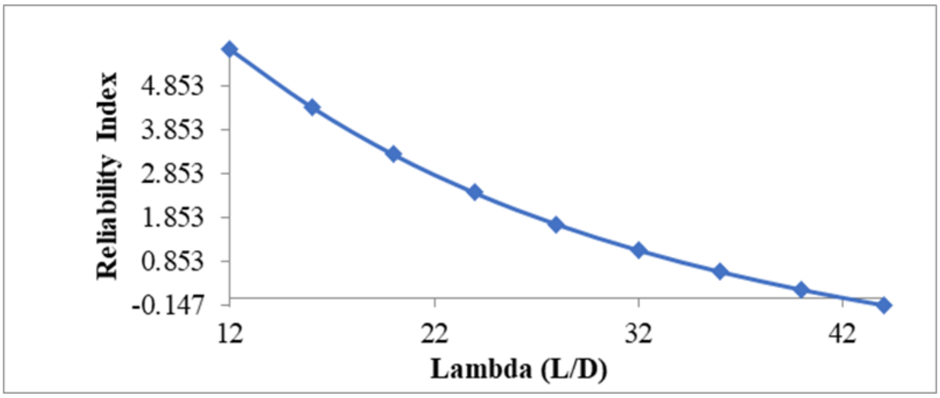

Figure 5: Relationship between reliability index and lambda at alpha = 1.0 (Shear limit state)

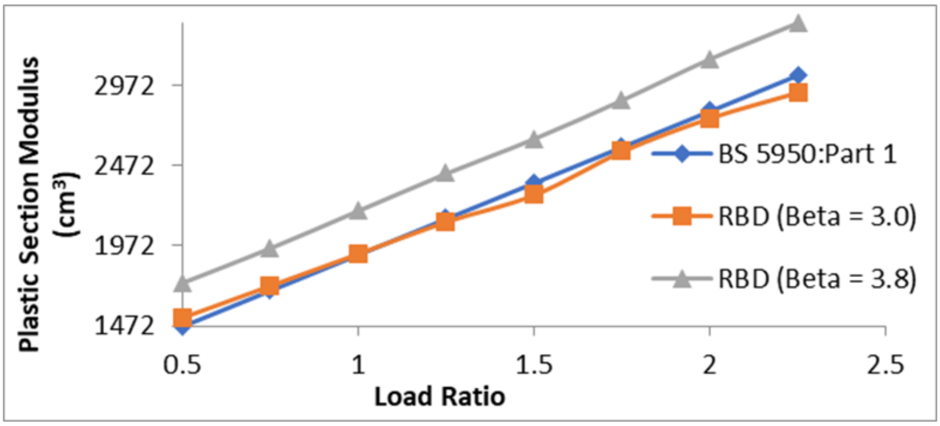

Figure 6: Relationship between load ratio and plastic section modulus values for deterministic and reliability-based design

Table 2: Values of Plastic section modulus (m^3) for varying load ratios obtained from code-based design and reliability-based design at target reliability indices of 3.0 and 3.80

Load Ratio	**0.5**	**0.75**	**1.0**	**1.25**	**1.5**	**1.75**	**2.0**	**2.25**
BS 5950: Part 1 (1985)	1472	1696	1919.90	2143.90	2368	2591.60	2816	3040
RBD (Beta = 3.0)	1520	1720	1920	2120	2288	2560	2768	2928
RBD (Beta = 3.8)	1744	1960	2192	2424	2640	2880	3136	3360

The MATLAB code employed in this study yielded the reliability indices for the bending, shear, and deflection limit states. These indices were determined through the utilization of the first-order reliability method. The obtained results are showcased in Figure 1 to Figure 3 which demonstrate the relationship between the reliability indices and varying load ratios for each limit state. It is evident from the figures that an increase in beam span and load ratio led to a decrease in the reliability indices for the bending, shear, and deflection failure modes. This observation aligns with the conclusions drawn by [13] who found that safety levels decline with an increase in beam span and load ratio.

Based on the data presented in Figure 1, the safety indices ranged from -1.419 to 5.708, with an average value of 3.145. Similarly, Figure. 2 showcases the implied safety indices from 2.237 to 7.357, with an average value of 4.798. Furthermore, Figure 3 demonstrates that the implied safety indices vary from -0.889 to 9.98, with an average value of 4.546. Notably, Figure 1 to Figure 3 shows that the average values of the implied safety indices for shear and deflection limit states exceed the recommended range of the target safety index, which is 3.3 to 3.7, for structures with minor to large consequences of failure [14]. In addition, for beam spans beyond 8.5m and load ratios surpassing 1.0, the safety of the beam cannot be ensured as indicated by the negative values of the safety indices [13, 15].

Figure 4 illustrates the range of implied safety indices for the bending limit state, which extends from -0.582 to 4.561, with an average value of 1.990. Similarly, for the shear limit state, the implied safety indices vary from 3.601 to 6.412, with an average value of 5.007. Additionally, the values of the implied safety indices for the deflection limit state range from 0.0399 to 8.87, with an average value of 4.455, as portrayed in Figure 4. Figure 5 depicts the correlation between the safety indices and the beam span-overall depth ratio (Lamda) at a constant load ratio of 1.0. The data from Figure 5 reveals a clear decrease in the reliability index as the beam span-overall depth ratio increases. It is crucial to emphasize that exceeding a beam span-overall depth ratio of 42 would compromise the safety of the beam, as indicated by the negative value of the reliability index. These findings align with the conclusions drawn by [16] which underscore the threat posed to the beam's safety by the negative values of the reliability indices. Hence, when the load ratio is 1.0 and the beam span measures 8.5m, it can be deduced that the design is crucial in terms of bending, secure in deflection, and satisfactory in shear. Additionally, the probabilistic design of the doubly symmetric beam was executed under the bending limit state, considering predefined reliability indices of 3.0 and 3.8, correspondingly. The deterministic values of the plastic section modulus were compared with the design values based on reliability, yielding the outcomes displayed in Table 2. It can be clearly seen from the Table, that the beam's deterministic and probabilistic design was carried out to establish the plastic section modulus at various load ratios, namely 0.5, 0.75, 1.0, 1.25, 1.5, 1.75, 2.0, and 2.5. It is apparent that for load ratios of 0.50 and 0.75, with a target reliability index of 3.0, the plastic section modulus experiences an increase of 3.3% and 1.42% correspondingly.

At load ratio of 1.0, the values of the plastic section modulus are almost identical. However, at load ratios of 1.25, 1.5, 1.75, 2.0 and 2.25, the readings of the plastic section modulus reduce by 1.13%, 3.49%, 1.23%, 1.73% and 3.83% respectively. This results to savings in the quantity of materials of I-steel beam. As the beam was designed for a target reliability index of 3.80 to reflect the consequences of failure for 50 years design period, the values of the plastic section modulus of the beam increases by 18.5% for 0.5 load ratio, 15.6% for 0.75 load ratio, 14.2% for 1.0 load ratio, 13.1% for 1.25 load ratio, 11.5% for 1.5 load ratio, 11.1% for 1.75 load ratio, 11.4% for 2.0 load ratio and 10.5% for 2.25 load ratio respectively.

4.0 CONCLUSIONS

The findings of the probabilistic evaluation of a doubly symmetric I-steel beam in relation to the limit state of bending, shear, and deflection, as per the design requirements outlined in [12] have been presented. The reliability estimates were obtained using a MATLAB automated program developed based on the First-Order Reliability Method. It was observed that the reliability indices decreased as the load ratio and beam span increased for the bending, shear, and deflection failure modes under consideration. It is not advisable to exceed a beam span of 8.5m and a load ratio of 1.0, as these values yielded negative safety indices.

When the load ratio was kept constant at 1.0, the reliability index decreased as the beam span-overall depth ratio increased. A beam span-overall depth ratio exceeding 42 would compromise the safety of the beam. The analysis showed that the design is critical in terms of bending, safe in terms of deflection, and satisfactory in terms of shear. The probabilistic design results for the plastic section modulus of the beam in bending, targeting a reliability index of 3.0 and a constant load ratio of 1.0, indicated material savings for the I-steel beam by considering different beam section choices based on the plastic section modulus values. However, when the beam was designed for a target reliability index of 3.80 to account for failure consequences over a 50-year design period, the values of the plastic section modulus of the beam increased.

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