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Review Article

Nonlinear Analysis of Isotropic Rectangular Thin Plates Using Ritz Method

Enem, J. I[1*]

[1]Department of Civil Engineering, Enugu State University of Science and Technology, P.M.B, Independence Layout 01660, Enugu, Nigeria

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***Corresponding author:** Enem, J. I
Department of Civil Engineering, Enugu State University of Science and Technology, P.M.B, Independence Layout 01660, Enugu, Nigeria

Abstract

This research work presents Nonlinear Analysis of Isotropic Thin Rectangular Plates using Energy Principle (Ritz method). Isotropic thin Rectangular plate having different twelve boundary conditions were analyzed and these boundary conditions were formed by combination of three major supports – Clamp, C; Simply supported, S; and Free, F.; General expressions for displacement and stress functions for large deflection of isotropic thin rectangular plate under uniformly distributed transverse loading were obtained by direct integration of Von karman's non-linear governing differential compatibility and equilibrium equations. Polynomial function instead of trigonometry function as was with previous researchers was used on the decoupled Von Karman's equations to obtain particular stress and displacement functions respectively. Non-linear total potential Energy was formulated using Von Karman equilibrium equation and Ritz method was deployed in this formulation. This equation was fully converted to potential energy by multiplying all the terms in it with displacement, w and the formed total potential energy, π consists of potential energy of internal forces and potential energy of external forces. This formulated total potential energy π, could give an accurate approximation of displacement field if the parameters were properly chosen. However, we assumed deflection, w to be ΔH_1, and stress function, ϕ to be $\Delta^2 H_2$ and substituted into the formulated potential energy. H_1 and H_2 are profiles of the deflection and stress function respectively, and Δ is deflection coefficient factor of the plate. Potential energy formulated contains deflection coefficient factor to the power of four. This potential energy was minimized by differentiating it partially with respect to coefficient factor reducing to cubic form.

Keywords: Nonlinear Analysis, Rectangular Thin Plates, Ritz Methods, Von Karman's Equation, Variational Principles, Boundary conditions, Large deflection.

1. INTRODUCTION

A thin plate is a flat structural element bounded by two parallel planes called faces, and a cylindrical surface, called an edge or boundary. The separation between the plane faces is referred to as the thickness (h) of the plate. When the thickness of a plate is divided into equal halves by a plane parallel to its faces, this plane is called the middle plane or midplane. In general, the plate thickness is small compared to other characteristic dimensions of the faces, be it length, width, or diameter. Plate could be bounded geometrically by either straight or curved boundaries where a, and b are principal dimensions, and h is the thickness [1].

Typically, the thickness dimension of the plate is much smaller than its planer dimensions yielding a "thin-walled" type of structure [2]. Among practical examples to describe the dimensions of these plates are roof, building windows, flat part of a table, manhole thin covering and panels. Plates are divided into two categories: thin plates with large deflections and thick plates [3].

Kirchhoff made some fundamental assumptions regarding thin plates. First of the assumptions states that plate is made of material that is elastic, homogeneous, and isotropic. The second assumption states that plate is initially flat, and that smallest lateral dimension of the plate is at least ten times larger than its thickness [4]. If the deflection of a thin plate is quite small in comparison with its thickness, ($\frac{w}{h} < 0.2$) Kirchhoff's theory of stiff plate will become valid.

The loads in this case are carried by bending action of the plate only. This theory neglects the deformation of the middle surface and its corresponding in-plane forces. The deformation in this is evaluated by a fourth-order linear partial differential equation which is simply known as the plate equation. There are so many authors to the plate equation who have worked out various loading and boundary conditions. Among the numerous literature that has dealt with deflections of plates, majority of them were based on the solutions of plate equation [4].

The major problem facing the analysis of large deflection of thin plate has been on how to solve the coupled equations of Von Karman. Previous researchers have tried to assume solutions of the equations. They have not tried to solve these coupled governing differential equations by direct integration. Previous researchers used trigonometric function as their shape function. This function can only be used effectively for SSSS and CCCC plates; apart from these boundary conditions its efficiency reduces. The interest of the previous researchers have been on determining the deflection only. No extension was made in determining stresses, bending moments, shear forces, and in-plane forces. The deflections determined by the past researchers were mainly central deflections. The deflection of the other parts of the plate were not considered. There is need to develop methods of solving the Von Karman equation that will also be applicable to different loading conditions and will be able to determine deflections at any part of the plate, the centre, inconclusive.

2. RELATED WORK

In related works, many researcher carried out noble works the on analysis of thine plates .Debabrata *et al.* [5] carried out research on large deflection analysis of skew plates under uniformly distributed load for mixed boundary condition. Das *et al.,* [6] analysed the deflection static behavior of isotropic skew plates under uniform pressure loading. Alwar and Rao [7, 8] investigated the static behaviour of orthotropic skew plates using dynamic relaxation. Duan and Mahendran [9] developed a new nonlinear quadrilateral hydride/mixed shell element using oblique coordinate system to investigate the large deflection of skew plates under uniformly distributed and concentrated loads. Singh and Elaghabash [10] developed a numerical method for geometrically nonlinear analysis of thin plates having quadrangular boundary with four straight edges. Bhattacharya [11] investigated the deflection of plates under static and dynamic loads by using a new finite difference analysis.

This approach gives the fourth order bi-harmonic equation which varies from node to node and found the time mode shape of the plate at each node. Defu and Sheikh [12] have presented the mathematical approach for large deflection of rectangular plates. Their analysis, based on the two fourth order and second degree partial differential von Karman equations, found lateral deflection to applied load. Bakker *et al.,* [13] have studied the approximate analysis method for large deflection of rectangular thin plate with simply supported boundary condition under action of transverse loads. Liew *et al.,* [14] developed the differential quadrature method and harmonic differential quadrature method for static analysis of three dimensional rectangular plates. Jain [15] presented analysis of stress concentration and deflection in isotropic and orthotropic rectangular plates with central circular hole under transverse static loading.

Von Karman, Levy, Volmir, Zienkiewitez [16-19] have made an in-road in the development of various numerical approaches that treated the analysis of large deflection with trigonometric and finite element. Levy [20] solved the bending of rectagular plates with large deflections by presenting a solution of Von Karman's equations in terms of trigonometric series. Gora *et al.,* [21] used trigonometric series as his shape function in the study of large deflection analysis of rhombic sandwich plates.

Okodi Allan *et al.,* [22] studied the exact large deflection analysis of thin rectangular plates under distributed lateral line load by assuming approximate cosine polynomial function. Levy [23] substituted a double Fourier series solution into the equations for rectangular plates and evaluated the coefficients. Wang [24] in his nonlinear large-deflection boundary value problems of rectangular plates and bending of rectangular plates with large deflection wrote the equations for rectangular plates in a finite difference form and solved them by the method of successive approximations. Way [25] in solving the circular plate equations substituted power series into the differential equations.

This research centered on thin rectangular plate with four edges. Each edge is supported independently and could be either clamped, simply supported or free. Therefore the boundary conditions are formed by combination of these supports. These three supports and the twelve boundary conditions obtained are idealized in Tables 1.1 and 1.2 respectively, whereas Table 1.3 shows the meaning of edge conditions.

Table 1.1: Symbolic description of three typical support

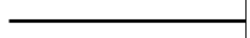
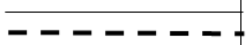
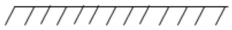
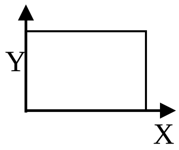
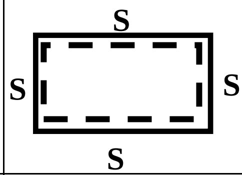
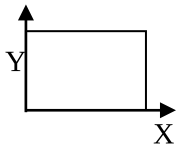
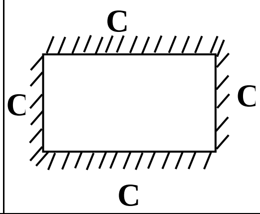
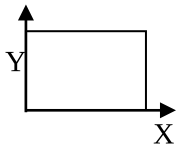
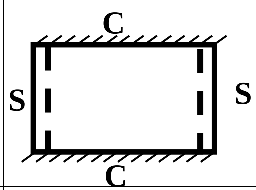
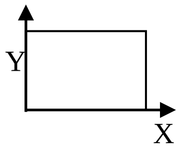
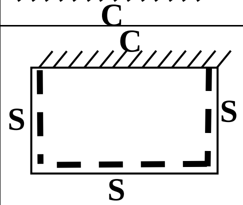
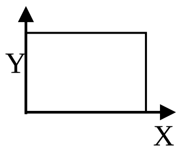
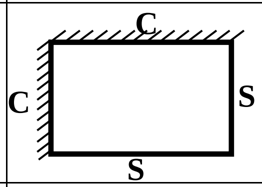
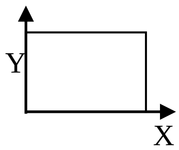
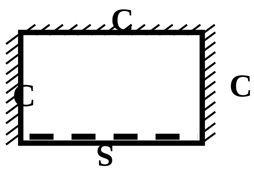
Description	Abbreviation	Section	Plan view
Free edge	F		
Simply supported	S		
Clamped edge	C		

Table 1.2: Symbolic representations of boundary conditions and list of cases analyzed

Proposed Plates to be analysed	**Boundary conditions**			
	Plate No	**Plates With Axes**	**Plate With Symbols**	**Abbreviation**
	PL 1			**SSSS**
	PL 2			**CCCC**
	PL 3			**CSCS**
	PL 4			**CSSS**
	PL 5			**CCSS**
	PL 6			**CCSC**

S

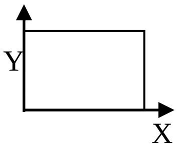
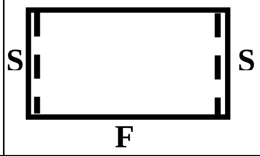
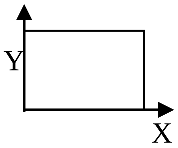
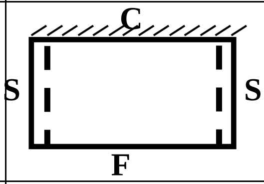
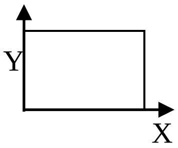
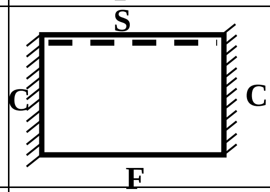
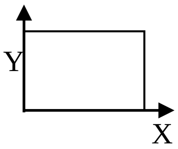
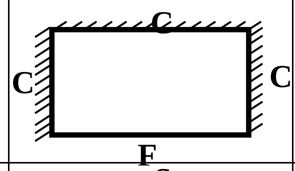
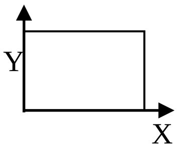
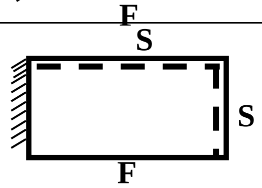
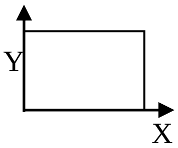
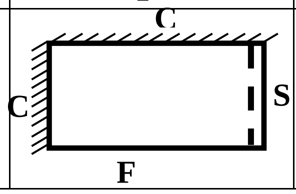
	PL 7			**SSFS**
	PL 8			**CSFS**
	PL 9			**SCFC**
	L 10			**CCFC**
	PL 11			**SCFC**
	PL 12			**CCFS**

Table 1.3: Description of the thin rectangular plates with twelve types of boundary conditions under consideration

Plate no	Edge condition	Description
PLATE 1	**SSSS**	Thin rectangular plate simply supported on the four edges.
PLATE 2	**CCCC**	Thin rectangular plate clamped on all edges.
PLATE 3	**CSCS**	Thin rectangular plate clamped on two opposite edges and simply supported on the other two opposite edges.
PLATE 4	**CSSS**	Thin rectangular plate clamped on the first edge and simply supported on the remaining three edges.
PLATE 5	**CCSS**	Thin rectangular plate clamped on the first two edges and simply supported on the remaining two edges.
PLATE 6	**CCCS**	Thin rectangular plate clamped on three edges and simply supported on the last edge.
PLATE 7	**CCFC**	Thin rectangular plate clamped on three edges and free on the third.
PLATE 8	**SSFS**	Thin rectangular plate simply supported on three edges and free on the third edge.
PLATE 9	**CCFS**	Thin rectangular plate clamped on two edges, free on the third edge, and simply supported on the last edge.
PLATE 10	**SCFC**	Thin rectangular plate simply supported on the first edge, free on the third edge and clamped on the two edges.
PLATE 11	**CSFS**	Thin rectangular plate clamped on the first edge, free on the third edge and simply supported on the other two edges.
PLATE 12	**SCFS**	Thin rectangular plate simply supported on two edges, clamped on the second edge and free on the third edge.

3. ANALYTICAL FORMULATION

3.1. General Equations for Large Deflections of Plate

The large deflection theory assumes that the deflections are rationally large with respect to the plate thickness but remain small compared to the other characteristic dimensions of the plate. When the deflection is of the order of magnitude of the thickness of the plate, it leads to a pair of coupled non-linear fourth order equations for the transverse displacement and the stress function for the in-plane stress resultants [26].

Those equations which are very significant to our study have been derived and are given by Equations 1, 2 and 3:

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \quad \ldots\ldots 1$$

$$\varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \quad \ldots\ldots 2$$

$$\Upsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \quad \ldots\ldots 3$$

These three equations are combined together after taking their second derivatives. That is:

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x dy} = 0 \quad \ldots\ldots 4$$

Where

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} = \frac{\partial^3 u}{\partial x d y^2} + \frac{1}{2}\frac{\partial^2}{\partial y^2}\left(\frac{\partial w}{\partial x}\right)^2 \quad \ldots\ldots 5$$

$$\frac{\partial^2 \varepsilon_x}{\partial x^2} = \frac{\partial^3 v}{d x^2 \partial y} + \frac{1}{2}\frac{\partial^2}{\partial x^2}\left(\frac{\partial w}{\partial y}\right)^2 \quad \ldots\ldots 6$$

$$\frac{\partial^2 \gamma_{xy}}{\partial x dy} = \frac{\partial^3 u}{\partial x d y^2} + \frac{\partial^3 v}{d x^2 \partial y} + \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \quad \ldots\ldots 7$$

Equations 5, 6 and 7 were substituted into Equation 4 to obtain

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x dy} = \frac{\partial^3 u}{\partial x d y^2} + \frac{1}{2}\frac{\partial^2}{\partial y^2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\partial^3 v}{d x^2 \partial y} + \frac{1}{2}\frac{\partial^2}{\partial x^2}\left(\frac{\partial w}{\partial y}\right)^2 - \frac{\partial^3 u}{\partial x d y^2} + \frac{\partial^3 v}{d x^2 \partial y} + \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \quad \ldots\ldots 8$$

Simplifying Equation 8 further gave:

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \Upsilon_{xy}}{\partial x dy} = \left(\frac{\partial^2 w}{\partial x dy}\right)^2 - \frac{\partial w^2}{\partial x^2}\frac{\partial^2}{\partial y^2} \quad \ldots\ldots 9$$

Using Hooke's law for linearly elastic materials. Stresses are related with strains as shown :

$$\sigma_2 = \frac{E}{1-v^2}(\varepsilon_x + v\varepsilon_y) \quad \ldots\ldots 10$$

$$\sigma_y = \frac{E}{1-v^2}(\varepsilon_y + v\varepsilon_x) \quad \ldots\ldots 11$$

$$\tau_{xy} = \frac{E}{1+v}(\gamma_{xy}) \quad \ldots\ldots 12$$

Inverse relationship to relate the strains with stresses are:

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y) \quad \ldots\ldots 13$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - v\sigma_x) \quad \ldots\ldots 14$$

$$\gamma_{xy} = 2\left(\frac{1+v}{E}\right)(\tau_{xy}) \quad \ldots\ldots 15$$

Resultant membrane forces in terms of stresses are given by:

$$N_x = \int_{-h/2}^{h/2} \sigma_x dz = \sigma_x [h]_{-h/2}^{h/2} = \sigma_x h \quad \ldots\ldots 16$$

$$N_y = \int_{-h/2}^{h/2} \sigma_y dz = \sigma_y [h]_{-h/2}^{h/2} = \sigma_y h \quad \ldots\ldots 17$$

$$N_{xy} = \int_{-h/2}^{h/2} \tau_{xy} = \tau_{xy} [h]_{-h/2}^{h/2} = \tau_{xy} h \quad \ldots\ldots 18$$

Stresses from Equations 16, 17 and 18 are given by:

$$\sigma_x = \frac{N_x}{h} \; ; \; \sigma_y = \frac{N_y}{h} \; ; \; \tau_{xy} = \frac{N_{xy}}{h} \quad \ldots\ldots 19$$

Substituting Equation 19 into Equations 13, 14 and 15 gave:

$$\varepsilon_x = \frac{1}{E}\left(\frac{N_x}{h} - \frac{vN_y}{h}\right) = \frac{1}{Eh}(N_x - vN_y) \quad \ldots\ldots 20$$

$\varepsilon_y = \frac{1}{E}\left(\frac{N_y}{h} - \frac{vN_x}{h}\right) = \frac{1}{Eh}\left(N_y - vN_x\right)$........................ 21

$\gamma_{xy} = \frac{1+v}{E}\left(\frac{N_{xy}}{h}\right) = 2\left(\frac{1+v}{Eh}\right)N_{xy}$ 22

Substituting Equations 20, 21 and 22 into the left hand side of Equation 9 gave:

$$\frac{\partial^2}{\partial y^2}\left[\frac{1}{Eh}\left(N_x - vN_y\right)\right] + \frac{\partial^2}{\partial x^2}\left[\frac{1}{Eh}\left(N_y - vN_x\right)\right] - \frac{\partial^2}{\partial x\partial y}\left[2\frac{(1+v)}{Eh}\left(N_{xy}\right)\right] = \left(\frac{\partial^2 \mathrm{w}}{\partial xdy}\right)^2 - \frac{\partial w^2}{\partial x^2}\frac{\partial^2}{\partial y^2} \quad \text{.................... 23}$$

Simplifying Equation 23 further gave:

$$\frac{1}{Eh}\left[\frac{\partial^2}{\partial y^2}\left(N_x - vN_y\right) + \frac{\partial^2}{\partial x^2}\left(N_y - vN_x\right) - \frac{\partial^2}{\partial x\partial y}\left(2(1+v)\,N_{xy}\right)\right] = \left(\frac{\partial^2 \mathrm{w}}{\partial xdy}\right)^2 - \frac{\partial w^2}{\partial x^2}\frac{\partial^2}{\partial y^2} \quad \text{......................... 24}$$

Airy's stress function, ϕ will be used to represent the stresses and this function for two-dimensional electrostatic problem without body forces are given as;

$$\sigma_x = \frac{\partial^2 \phi}{\partial y^2}\ ;\ \sigma_y = \frac{\partial^2 \phi}{\partial x^2}\ ;\ \tau_{xy} = \frac{\partial^2 \phi}{\partial xdy} \quad \text{.................... 25}$$

Equating Equation 25 with Equation 19 gave;

$$\frac{N_x}{h} = \frac{\partial^2 \phi}{\partial y^2} \Rightarrow N_x = \frac{h\partial^2 \phi}{\partial y^2} \quad \text{................................ 26}$$

$$\frac{N_y}{h} = \frac{\partial^2 \phi}{\partial x^2} \Rightarrow N_y = \frac{h\partial^2 \phi}{\partial x^2} \quad \text{................................ 27}$$

$$\frac{N_{xy}}{h} = \frac{\partial^2 \phi}{\partial x\partial y} \Rightarrow N_{xy} = \frac{h\partial^2 \phi}{\partial x\partial y} \quad \text{................................ 28}$$

Substitute Equations 26, 27 and 28 into equation 24 gave:

$$\frac{1}{Eh}\left[\frac{\partial^2}{\partial y^2}\left(\frac{h\partial^2\phi}{\partial y^2} - \frac{vh\partial^2\phi}{\partial x^2}\right) + \frac{\partial^2}{\partial x^2}\left(\frac{h\partial^2\phi}{\partial x^2} - \frac{vh\partial^2\phi}{\partial y^2}\right) - \frac{\partial^2}{\partial xdy}\left[-2(1+v)\frac{vh\partial^2\phi}{\partial xdy}\right]\right] = \left(\frac{\partial^2 \mathrm{w}}{\partial xdy}\right)^2 - \frac{\partial w^2}{\partial x^2}\frac{\partial^2}{\partial y^2} \quad \text{.................. 29}$$

Expanding Equation 29 and rearranging gave:

$$\frac{1}{E}\left(\frac{\partial^4\phi}{\partial y^4} + \frac{2\partial^4\phi}{\partial x^2\partial y^2} + \frac{\partial^4\phi}{\partial y^4}\right) = \left(\frac{\partial^2 \mathrm{w}}{\partial xdy}\right)^2 - \frac{\partial w^2}{\partial x^2}\frac{\partial^2}{\partial y^2} \quad \text{............................. 30}$$

Multiplying Equation 30 by E gave:

$$\frac{\partial^4\phi}{\partial x^4} + \frac{2\partial^4\phi}{\partial x^2\partial y^2} + \frac{\partial^4\phi}{\partial y^2} = E\left[\left(\frac{\partial^2 \mathrm{w}}{\partial x\partial y}\right)^2 - \frac{\partial^2 \mathrm{w}\,\partial^2 \mathrm{w}}{\partial x^2\partial y^2}\right] \quad \text{............................ 31}$$

Equation 32 was first derived by Von Karman and is known as geometrically nonlinear plate theory

$$\frac{\partial^4 w}{\partial x^4} + \frac{2\partial^4 w}{\partial x^2\partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D}\left(q + N_x\frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x\partial y} + N_y\frac{\partial^2 w}{\partial y^2}\right) \quad \text{.......... 32}$$

Expressing Equation 32 in terms of Airy's stress function by substituting the values of resultant stresses from Equations 26, 27 and 28 gave:

$$\frac{\partial^4 \mathrm{w}}{\partial x^4} + \frac{2\partial^4 \mathrm{w}}{\partial x^2\partial y^2} + \frac{\partial^4 \mathrm{w}}{\partial y^4} = \frac{1}{D}\left[q + h\left(\frac{\partial^2\phi}{\partial y^2}\frac{\partial^2 \mathrm{w}}{\partial x^2} + \frac{\partial^2\phi}{\partial x^2}\frac{\partial^2 \mathrm{w}}{\partial y^2} - \frac{2\partial^2\phi}{\partial x\partial y}\frac{\partial^2 \mathrm{w}}{\partial x\partial y}\right)\right] \quad \text{...... 33}$$

Equations 32 and 33 define a system of nonlinear, partial differential equations, and they are referred to as the governing differential equations for large deflections theory of plates. The first equation can be described as compatibility equation and, describing the second equation in the same tone as equilibrium equation.

With Equations 32 and 33, the stress function ϕ, and deflection w, can be calculated which in return facilitates the determination of the inplane forces N_x, N_y and N_{xy}, as well as the corresponding bending moments M_x, M_y and twisting moment M_{xy}. The major challenge lies in resolving Equations 32 and 33. It is a herculean and mathematical uphill task solving these equations. To lessen the cumbersomeness of the usual numerical procedures is to reduce the nonlinear differential Equations 32 and 33 to a system of nonlinear algebraic equations using the displacement methods. It is worthwhile to note that the appropriate boundary condition was used. Figure 2 shows a typical rectangular plate with its characteristic dimensions 'a' and 'b'.

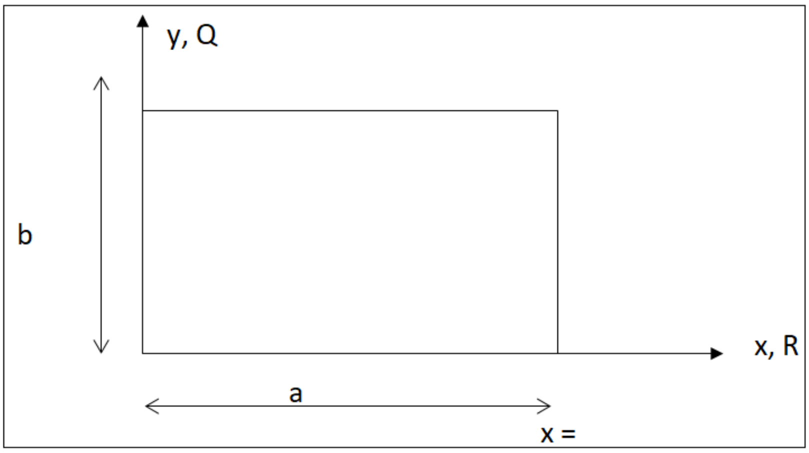


Figure 1: A typical idealized plate

$Let\ R = \frac{x}{a} : x = aR;\ 0 \le R \le 1; 0 \le x \le a$

$Q = \frac{y}{b} : y = bQ;\ 0 \le Q \le 1; 0 \le y \le b$

Where R and Q are non-dimensional parameters, and they are in the x and y-directions respectively. Expressing Equation 32 and 33 in terms of R and Q reduced:

$$\frac{\partial^4\phi}{a^4\partial R^4} + 2\frac{\partial^4\phi}{a^2b^2\partial R^2\partial Q^2} + \frac{\partial^4\phi}{b^4\partial Q^4} = E\left(\left(\frac{\partial^2 w}{ab\ \partial R\partial Q}\right)^2 - \frac{\partial^2 w\, \partial^2 w}{a^2\partial R^2 b^2\partial Q^2}\right) \quad \ldots\ldots 34$$

$$\frac{\partial^4 w}{a^4\partial R^4} + 2\frac{\partial^4 w}{a^2b^2\partial R^2 Q^2} + \frac{\partial^4 w}{b^4\partial Q^4} = \frac{1}{D}\left(q + h\left(\frac{\partial^2\phi}{b^2\partial Q^2}\frac{\partial^2 w}{a^2\partial R^2} + \frac{\partial^2\phi}{a^2\partial R^2}\frac{\partial^2 w}{b^2\partial Q^2} - \frac{2\partial^2 Q}{ab\ \partial R\ \partial Q}\frac{\partial^2 w}{ab\ \partial R\ \partial Q}\right)\right) \quad \ldots\ldots 35$$

The aspect ratio $\propto$ is equal to a/b ; $a = \propto b$ and substituting in Equations 34 and 35 respectively gave the following equations:

$$\frac{\partial^4\phi}{\propto^4 b^4\partial R^4} + \frac{2\partial^4\phi}{\propto^2 b^4\partial R^2\partial Q^2} + \frac{\partial^4\phi}{b^4\partial Q^4} = \frac{E}{\propto^2 b^4}\left(\left(\frac{\partial^2 w}{\partial R\ \partial Q}\right)^2 - \frac{\partial^2 w}{\partial R^2}\frac{\partial^2 w}{\partial Q^2}\right) \quad \ldots\ldots 36$$

$$\frac{\partial^4 w}{\propto^4 b^4\partial R^4} + \frac{2\partial^4 w}{\propto^2 b^4\partial R^2\partial Q^2} + \frac{\partial^4 w}{b^4\partial Q^4} = \frac{q}{D} + \frac{h}{\propto^2 D b^4}\left(\frac{\partial^2\phi}{\partial Q^2}\frac{\partial^2 w}{\partial R^2} + \frac{\partial^2\phi}{\partial R^2}\frac{\partial^2 w}{\partial Q^2} - \frac{2\partial^2\phi}{\partial R\ \partial Q}\frac{\partial^2 w}{\partial R\ \partial Q}\right) \quad \ldots\ldots 37$$

Multiplying both of Equations 3.153 and 3.154 by b^4 gave

$$\frac{\partial^4\phi}{\propto^4\partial R^4} + \frac{2\partial^4\phi}{\propto^2\partial R^2\partial Q^2} + \frac{\partial^4\phi}{\partial Q^4} = \frac{E}{\propto^2}\left(\left(\frac{\partial^2 w}{\partial R\ \partial Q}\right)^2 - \frac{\partial^2 w}{\partial R^2}\frac{\partial^2 w}{\partial Q^2}\right) \quad \ldots\ldots 38$$

$$\frac{\partial^4 w}{\propto^4\partial R^4} + \frac{2\partial^4 w}{\propto^2\partial R^2\partial Q^2} + \frac{\partial^4 w}{\partial Q^4} = \frac{qb^4}{D} + \frac{h}{\propto^2 D}\left(\frac{\partial^2\phi}{\partial Q^2}\frac{\partial^2 w}{\partial R^2} + \frac{\partial^2\phi}{\partial R^2}\frac{\partial^2 w}{\partial Q^2} - \frac{2\partial^2\phi}{\partial R\ \partial Q}\frac{\partial^2 w}{\partial R\ \partial Q}\right) \quad \ldots\ldots 39$$

Equations 38 and 39 are nonlinear differential equation for large deflection of plate under normal load represented in non-dimensional axes. To obtain the solution of these equations for plates, an approximate method is pertinent.

3.2. Shape Functions

Assuming a displacement function of:

$$W = w(x,y)\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} P_m B_n x^m y^n \quad \ldots\ldots 40$$

Displacement functions in Equation 40 was expressed in terms of non-dimensional parameters (Q and R).

Recall that.

X = aR, and y = Bq ……………………………………………... 41

Substituting Equation 3.158 into Equation 3.157 and terminating the series at m = n = 4 gave

$$W = W(R,Q) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} P_m B_n a^m R^m b^n Q^n \quad \ldots\ldots 42$$

$$Let\ a_m = P_m a^m \text{and } b_n = B_n b^n \quad \ldots\ldots 43$$

Thus:

$$W = W(R,Q) = \sum_{m=0}^{4}\sum_{n=0}^{4} a_m R^m b_n Q^n \quad \ldots\ldots 44$$

Equation 44 is displacement function represented in non-dimensional axes. The displacement function can be expanded by using expansion theorem over an orthogonal basis, an infinite sum, making the system easy to solve. The expansion was carried out over two bases, one in the x-direction and the other in y-direction. Normalizing the function makes the denominator equal to one. Hence, expanding Equation 44 to 4th series gave

$W(a_iR, b_iQ) = (a_0 + a_1R + a_2R^2 + a_3R^3 + a_4R^4)(b_0 + b_1Q + b_2Q^2 + b_3Q^3 + b_4Q^4)$ ……………………………. 45

This is polynomial approximation of the shape function. With the proper use of the boundary conditions of the plate, the deflection of the plate will be adequately defined.

3.3. Rationalization of Von Karman Equations

Equations 38 and 39 are nonlinear differential equation for large deflection of plate expressed in terms of non-dimensional axes. They are functions of deflection, w and, stress function, ϕ. Equation 45 describes the shape function, and with this function Equation 38 was integrated in a close domain and rationalized to generate a general (Model) solution for stress function. This was achieved by substituting Equation 45 into the right hand side of Equation 38.

Thus:

$$\frac{\partial^4\phi}{\propto^4\partial R^4}+\frac{2\partial^4\phi}{\propto^2\partial R^2\partial Q^2}+\frac{\partial^4\phi}{\partial Q^4}=\frac{E}{\propto^2}\Bigg[\Big[\frac{\partial^2}{\partial R\,\partial Q}\{(a_0+a_1R+a_2R^2+a_3R^3+a_4R^4)(b_0+b_1Q+b_2Q^2+b_3Q^3+b_4Q^4)\}\Big]^2-\left(\frac{\partial^2}{\partial R^2}\{(a_0+a_1R+a_2R^2+a_3R^3+a_4R^4)(b_0+b_1Q+b_2Q^2+b_3Q^3+b_4Q^4)\}\right)\left(\frac{\partial^2}{\partial Q^2}\{(a_0+a_1R+a_2R^2+a_3R^3+a_4R^4)(b_0+b_1Q+b_2Q^2+b_3Q^3+b_4Q^4)\}\right)\Bigg] \quad \ldots\ldots 46$$

Differentiating the right hand side of Equation 46 accordingly gave.

$$\frac{\partial^4\phi}{\propto^4\partial R^4}+\frac{2\partial^4\phi}{\propto^2\partial R^2\partial Q^2}+\frac{\partial^4\phi}{\partial Q^4}=\frac{E}{\propto^2}[(a_1^2+4a_1a_2R+(6a_1a_3+4a_2^2)R^2+(8a_1a_4+12a_2a_3)R^3+(16a_2a_4+9a_3^2)R^4+24a_3a_4R^5+16a_4^2R^6)(b_1^2+4b_1b_2Q+(6b_1b_3+4b_2^2)Q^2+(8b_1b_4+12b_2b_3)Q^3+(16b_2b_4+9b_3^2)Q^4+24b_3b_4Q^5+16b_4^2Q^6)-(2a_0a_2+(2a_1a_2+6a_0a_3)R+(2a_2^2+6a_1a_3+12a_0a_4)R^2+(8a_2a_3+12a_1a_4)R^3+(6a_3^2+14a_2a_4)R^4+18a_3a_4R^5+12a_4^2R^6)(2b_0b_2+(2b_1b_2+6b_0b_3)Q+(2b_2^2+6b_1b_3+12b_0b_4)Q^2+(8b_2b_3+12b_1b_4)Q^3+(6b_3^2+14b_2b_4)Q^4+18b_3b_4Q^5+12b_4^2Q^6)] \quad \ldots\ldots 47$$

Integrating Equation 47 four times with respect to non-dimensional coordinates R and Q gave

$$\int_{0,0}^{1,1}\int_{0,0}^{1,1}\int_{0,0}^{1,1}\int_{0,0}^{1,1}\frac{\partial^4\phi}{\propto^4\,\partial R^4}+\frac{2\partial^4\phi}{\propto^2\,\partial R^2\partial Q^2}+\frac{\partial^4\phi}{\partial Q^4}$$
$$=\frac{E}{\propto^2}\int\int\int\int[(a_1^2+4a_1a_2R+(6a_1a_3+4a_2^2)R^2+(8a_1a_4+12a_2a_3)R^3+(16a_2a_4+9a_3^2)R^4+24a_3a_4R^5+16a_4^2R^6)(b_1^2+4b_1b_2Q+(6b_1b_3+4b_2^2)Q^2+(8b_1b_4+12b_2b_3)Q^3+(16b_2b_4+9b_3^2)Q^4+24b_3b_4Q^5+16b_4^2Q^6)-(2a_0a_2+(2a_1a_2+6a_0a_3)R+(2a_2^2+6a_1a_3+12a_0a_4)R^2+(8a_2a_3+12a_1a_4)R^3+(6a_3^2+14a_2a_4)R^4+18a_3a_4R^5+12a_4^2R^6)(2b_0b_2+(2b_1b_2+6b_0b_3)Q+(2b_2^2+6b_1b_3+12b_0b_4)Q^2+(8b_2b_3+12b_1b_4)Q^3+(6b_3^2+14b_2b_4)Q^4+18b_3b_4Q^5+12b_4^2Q^6)]\,\partial R^4\partial Q^4 \quad \ldots\ldots 48$$

Evaluating Equation 48 in a closed domain gave

$$\emptyset\left[\frac{Q^4}{\propto^4}+\frac{2R^2Q^2}{\propto^2}+R^4\right]_{0,0}^{1,1}=\frac{E}{\propto^2}\Bigg[\left(\frac{a_1^2}{24}R^4+\frac{a_1a_2}{30}R^5+\frac{1}{180}(3a_1a_3+2a_2^2)R^6+\frac{1}{210}(2a_1a_4+3a_2a_3)R^7+\frac{1}{1680}(16a_2a_4+9a_3^2)R^8+\frac{a_3a_4}{126}R^9+\frac{a_4^2}{315}R^{10}+\frac{C_1R^3}{6}+\frac{C_2R^2}{2}+C_3R+C_4\right)\left(\frac{b_1^2}{24}Q^4+\frac{b_1b_2}{30}Q^5+\frac{1}{180}(3b_1b_3+2b_2^2)Q^6+\frac{1}{210}(2b_1b_4+3b_2b_3)Q^7+\frac{1}{1680}(16b_2b_4+9b_3^2)Q^8+\frac{b_3b_4}{126}Q^9+\frac{b_4^2}{315}Q^6+\frac{T_1Q^3}{6}+\frac{T_2Q^2}{2}+T_3Q+T_4\right)-\left(\frac{a_0a_2}{12}R^4+\frac{1}{60}(a_1a_2+3a_0a_3)R^5+\frac{1}{180}(a_2^2+3a_1a_3+6a_0a_4)R^6+\frac{1}{210}(2a_2a_3+3a_1a_4)R^7+\frac{1}{840}(3a_3^2+7a_2a_4)R^8+\frac{a_3a_4}{168}R^9+\frac{a_4^2}{420}R^{10}+\frac{J_1R^3}{6}+\frac{J_2R^2}{2}+J_3R+J_4\right)\left(\frac{b_0b_2}{12}Q^4+\frac{1}{60}(b_1b_2+3b_0b_3)Q^5+\frac{1}{180}(b_2^2+3b_1b_3+6b_0b_4)Q^6+\frac{1}{210}(2b_2b_3+3b_1b_4)Q^7+\frac{1}{840}(3b_3^2+7b_2b_4)Q^8+\frac{b_3b_4}{168}Q^9+\frac{b_4^2}{420}Q^6+\frac{P_1Q^3}{6}+\frac{P_2Q^2}{2}+P_3Q+P_4\right)\Bigg] \quad \ldots\ldots 49$$

Applying the boundary condition to Equation 49 and also assuming all the constant of integration to be zero gave.

$$\emptyset=\frac{E}{\propto^2\left(\frac{1}{\propto^4}+\frac{2}{\propto^2}+1\right)}\Bigg[\left(\frac{a_1^2}{24}R^4+\frac{a_1a_2}{30}R^5+\frac{1}{180}(3a_1a_3+2a_2^2)R^6+\frac{1}{210}(2a_1a_4+3a_2a_3)R^7+\frac{1}{1680}(16a_2a_4+9a_3^2)R^8+\frac{a_3a_4}{126}R^9+\frac{a_4^2}{315}R^{10}\right)\left(\frac{b_1^2}{24}Q^4+\frac{b_1b_2}{30}Q^5+\frac{1}{180}(3b_1b_3+2b_2^2)Q^6+\frac{1}{210}(2b_1b_4+3b_2b_3)Q^7+\frac{1}{1680}(16b_2b_4+9b_3^2)Q^8+\frac{b_3b_4}{126}Q^9+\frac{b_4^2}{315}Q^6\right)-\left(\frac{a_0a_2}{12}R^4+\frac{1}{60}(a_1a_2+3a_0a_3)R^5+\frac{1}{180}(a_2^2+3a_1a_3+6a_0a_4)R^6+\frac{1}{210}(2a_2a_3+3a_1a_4)R^7+\right.$$

$$\frac{1}{840}(3a_3^2 + 7a_2a_4)R^8 + \frac{a_3a_4}{168}R^9 + \frac{a_4^2}{420}R^{10}\Big)\Big(\frac{b_0b_2}{12}Q^4 + \frac{1}{60}(b_1b_2 + 3b_0b_3)Q^5 + \frac{1}{180}(b_2^2 + 3b_1b_3 + 6b_0b_4)Q^6 + \frac{1}{210}(2b_2b_3 + 3b_1b_4)Q^7 + \frac{1}{840}(3b_3^2 + 7b_2b_4)Q^8 + \frac{b_3b_4}{168}Q^9 + \frac{b_4^2}{420}Q^6\Big)\Big] \ldots\ldots\ldots\ldots 50$$

$$Let\ \beta = \frac{E}{\propto^2\left(\frac{1}{\propto^4} + \frac{2}{\propto^2} + 1\right)} \ldots\ldots\ldots\ldots 51$$

Hence, Equation 50 became;

$$\emptyset = \beta\Big[\Big(\frac{a_1^2}{24}R^4 + \frac{a_1a_2}{30}R^5 + \frac{1}{180}(3a_1a_3 + 2a_2^2)R^6 + \frac{1}{210}(2a_1a_4 + 3a_2a_3)R^7 + \frac{1}{1680}(16a_2a_4 + 9a_3^2)R^8 + \frac{a_3a_4}{126}R^9 + \frac{a_4^2}{315}R^{10}\Big)\Big(\frac{b_1^2}{24}Q^4 + \frac{b_1b_2}{30}Q^5 + \frac{1}{180}(3b_1b_3 + 2b_2^2)Q^6 + \frac{1}{210}(2b_1b_4 + 3b_2b_3)Q^7 + \frac{1}{1680}(16b_2b_4 + 9b_3^2)Q^8 + \frac{b_3b_4}{126}Q^9 + \frac{b_4^2}{315}Q^6\Big) - \Big(\frac{a_0a_2}{12}R^4 + \frac{1}{60}(a_1a_2 + 3a_0a_3)R^5 + \frac{1}{180}(a_2^2 + 3a_1a_3 + 6a_0a_4)R^6 + \frac{1}{210}(2a_2a_3 + 3a_1a_4)R^7 + \frac{1}{840}(3a_3^2 + 7a_2a_4)R^8 + \frac{a_3a_4}{168}R^9 + \frac{a_4^2}{420}R^{10}\Big)\Big(\frac{b_0b_2}{12}Q^4 + \frac{1}{60}(b_1b_2 + 3b_0b_3)Q^5 + \frac{1}{180}(b_2^2 + 3b_1b_3 + 6b_0b_4)Q^6 + \frac{1}{210}(2b_2b_3 + 3b_1b_4)Q^7 + \frac{1}{840}(3b_3^2 + 7b_2b_4)Q^8 + \frac{b_3b_4}{168}Q^9 + \frac{b_4^2}{420}Q^{10}\Big)\Big] \ldots\ldots\ldots\ldots 52$$

Equation 52 is the general stress function, Ø(R, Q) for a rectangular plate of any boundary condition. With that the specific (peculiar) stress functions for the various boundary conditions considered in this work were determined.

3.4 Formulation of non-linear total potential energy functional for large deflection

The principal function of variational principle is searching of unknown functions that give a maximum or minimum value of a functional. Functional is a quantity which can be handled as a function of an infinite number of independent variables.

3.4.1 Total potential energy

Consider Equation 39 as a functional expressing total potential energy, π of a deformed elastic body and load acting on it. Von Karman equilibrium Equation of 39 which is now taken as total potential energy, π consists of potential energy of internal forces and potential energy of external forces.

From the elementary physics, potential energy of a body is a measure of work done by external and internal forces in moving the body from its initial position to a final one. Since, all the terms in Equation 39 are in form of force. Equation 39 was therefore converted to full potential energy by multiplying all the terms in it by displacement, *w*, hence

$$\pi = \frac{1}{2}\int_0^1\int_0^1\Big(\frac{\partial^4 w}{\propto^4\partial R^4}.w + \frac{2\partial^4 w}{\propto^2\partial R^2\partial Q^2}.w + \frac{\partial^4 w}{\partial Q^4}.w\Big)\partial R\partial Q - \frac{1}{D}\int_0^1\int_0^1\Big[qb^4.w + \frac{h}{2\propto^2}\Big(\frac{\partial^2\phi}{\partial Q^2}\frac{\partial^2 w}{\partial R^2}.w + \frac{\partial^2\phi}{\partial R^2}\frac{2\partial^2 w}{\partial Q^2}.w - \frac{2\partial^2\phi}{\partial R\partial Q}.\frac{\partial^2 w}{\partial R\partial Q}.w\Big)\Big]\partial R\partial Q \ldots\ldots\ldots\ldots 53$$

Letting $w = \Delta H_1$ and $\emptyset = \Delta^2 H_2$

Where Δ is the coefficient factor of the plate. H_1 and H_2 are the profiles of the deflection and stress function respectively. Substituting for *w* and Ø into equation 53 gave:

$$\pi = \frac{1}{2}\int_0^1\int_0^1\Big(\frac{\partial^4\Delta H_1}{\propto^4\partial R^4}.\Delta H_1 + \frac{2\partial^4\Delta H_1}{\propto^2\partial R^2\partial Q^2}.\Delta H_1 + \frac{\partial^4\Delta H_1}{\partial Q^4}.\Delta H_1\Big)\partial R\partial Q - \frac{1}{D}\int_0^1\int_0^1\Big[qb^4.\Delta H_1 + \frac{h}{2\propto^2}\Big(\frac{\partial^2\Delta^2 H_2}{\partial Q^2}\frac{\partial^2\Delta H_1}{\partial R^2}.\Delta H_1 + \frac{\partial^2\Delta^2 H_2}{\partial R^2}\frac{\partial^2\Delta H_1}{\partial Q^2}.\Delta H_1 - \frac{2\partial^2\Delta^2 H_2}{\partial R\partial Q}.\frac{\partial^2\Delta H_1}{\partial R\partial Q}.\Delta H_1\Big)\Big]\partial R\partial Q \ldots\ldots\ldots\ldots 54$$

Factorizing coefficient factor Δ out reduced Equation 54 to

$$\pi = \frac{\Delta^2}{2}\int_0^1\int_0^1\Big(\frac{\partial^4 H_1}{\propto^4\partial R^4}.H_1 + \frac{2\partial^4 H_1}{\propto^2\partial R^2\partial Q^2}.H_1 + \frac{\partial^4 H_1}{\partial Q^4}.H_1\Big)\partial R\partial Q - \frac{1}{D}\int_0^1\int_0^1\Big[\Delta.qb^4.H_1 + \frac{\Delta^4 h}{2\propto^2}\Big(\frac{\partial^2 H_2}{\partial Q^2}\frac{\partial^2 H_1}{\partial R^2}.H_1 + \frac{\partial^2 H_2}{\partial R^2}\frac{\partial^2 H_1}{\partial Q^2}.H_1 - \frac{2\partial^2 H_2}{\partial R\partial Q}.\frac{\partial^2 H_1}{\partial R\partial Q}.H_1\Big)\Big]\partial R\partial Q \ldots\ldots\ldots\ldots 55$$

Equation 55 is Von Karman equilibrium equation expressed as a potential energy. To further reduce coefficient factor in Equation 55, minimization was carried out on it.

3.4.2 Minimization of Total Potential Energy

Minimization of total potential energy is a very vital aspect of variational principle in which the functional was decomposed. This was realized by differentiating total potential energy partially with respect to coefficient factor, which is unknown parameter of the shape function. Resultant partial derivative of the function was therefore equated to zero. This enabled determination of the unknown parameter.

$$\frac{\partial\pi}{\partial\Delta} = \Delta\int_0^1\int_0^1\left(\frac{\partial^4 H_1}{\propto^4\partial R^4}.H_1 + \frac{2\partial^4 H_1}{\propto^2\partial R^2\partial Q^2}.H_1 + \frac{\partial^4 H_1}{\partial Q^4}.H_1\right)\partial R\partial Q - \frac{1}{D}\int_0^1\int_0^1 qb^4.H_1\partial R\partial Q - \frac{2\Delta^3 h}{D\propto^2}\int_0^1\int_0^1\left(\frac{\partial^2 H_2}{\partial Q^2}\frac{\partial^2 H_1}{\partial R^2}.H_1 + \frac{\partial^2 H_2}{\partial R^2}\frac{\partial^2 H_1}{\partial Q^2}.H_1 - \frac{2\partial^2 H_2}{\partial R\partial Q}.\frac{\partial^2 H_1}{\partial R\partial Q}.H_1\right)\partial R\partial Q \quad \ldots\ldots 56$$

Equation 56 forms general minimized total potential energy upon which determination of coefficient factor (Amplitude) of various boundary conditions was based. Noteworthy, this amplitude determines the extent of deflection of the plate. The larger the amplitude (coefficient factor) the larger the deflection and vice versa.

4.1 DISCUSSION OF RESULTS

4.1.1 Discussion on integration of von Karman equations

The major challenge bedeviling the analysis of large deflection of thin rectangular plates has been on finding solution to von Karman equations. Previous researchers by passed the direct integration of these equations by assuming a solution. In this analysis the direct integration of von Karman equations was carried out successfully.

In arriving at this result, we assumed a polynomial function against the usual tradition of assuming a trigonometry function. The assumed shape function was substituted into von Karman equation and integrated in a close domain. Successful integration of von Karman equations is indeed a milestone because this result will serve as model for the determination of deflection and subsequently other stresses that are needed for the design of thin rectangular plates.

4.1.2 Discussion on formulated non-linear total potential energy functional for large deflection of plates and its minimization.

Non-linear total potential energy was formulated using Von Karman equilibrium equation and Ritz method was deployed in this formulation. In order to convert Von Karman second equation to full potential energy, all the terms in that equation was multiplied by displacement, *w* and that gave rise to Equation 55. This equation forms total potential energy, π and consists of the potential energy of internal forces and the potential energy of external forces. This formulated total potential energy π, could give an accurate approximation of the displacement field, *w* if the parameters were properly chosen.

However, we assumed deflection, *w* to be ΔH, and stress function, ϕ to be $\Delta^2 H_2$ and substituted into the formulated potential energy. H_1 and H_2 are the profiles of the deflection and stress function respectively, and Δ is the coefficient factor of the plate. The equation contains 4Δ coefficients factor.

Minimization process was carried out on this total potential energy by differentiating the Equation with respect to Δ and that gave rise Equation 56. Hence, we are left with three coefficients, Δ to be solved for.

5 CONCLUSIONS

In sincerity this thesis present a breakthrough in the analysis of large deflection of thin rectangular plate. The major problem facing the analysis of large deflection of thin plate has been on how to solve the coupled equation of Von Karman. The previous researchers assumed solutions for this equations but, this shortcoming has been overcome with the advance of this work which has successfully obtained the general expressions for deflection and stress function by direct integration of Von Karman compatibility and equilibrium equations.

Ritz method which is under direct method in approximate method was adopted as an analytical tool for the purpose of this research. With this tool, non-linear total potential energy functional for large deflection of plates were formulated which was later minimized to obtain a general equation for amplitude of deflection.

Noteworthy, the use of polynomial as a shape function has been successfully and effectively carried out in this work against the usual traditional trend of using trigonometric series as shape function.

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