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Review Article

Maximum Likelihood Channel Estimation for Space Time Block Code - Orthogonal Frequency Division Multiplexing

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Abstract

Multiple Input Multiple Output -Orthogonal Frequency Division Multiplexing (MIMO-OFDM) is a promising technique for broad - band communication over mobile wireless channels. This paper presents channel estimation methods for Space Time Block Code - Orthogonal Frequency Division Multiplexing (STBC-OFDM) system. Expectation – Maximization based on an iterative algorithm and Decoupled Maximum Likelihood method are used to reduce computational complexity without loss of estimation accuracy. Simulation results show that the effectiveness of the Decoupled Maximum Likelihood method is improved.

Keywords: Channel estimation; Expectation-Maximization (EM) algorithm; Decoupled maximum likelihood (DEML) algorithm; Low complexity.

1. INTRODUCTION

High-data-speed is essential for future wireless communication systems to provide multimedia services such as data, voice, video, however, the problem of bandwidth limit and multipath fading channel are disadvantages of these systems. An effective method to reduce the effect of multipath fading is to use OFDM modulation [1, 2]. Multiple Input Multiple Output (MIMO) technology uses multiple antennas on both transmitter and receiver to provide gains in channel robustness and throughput [3]. Space Time Coding (STC) and Basic Local Alignment Search Tool (BLAST) [3-6] are usually designed for flat fading channel. However, there are many cases of frequency selective fading channels in the real environment. The solution to this problem is to use OFDM modulation, which separates the frequency selective fading channel into multiple narrow flat fading channels. Thus, the combination of MIMO and OFDM is a reliable solution for future radio communication systems.

Channel estimation for mobile communication systems is an important technique because the performance and capacity of mobile communication systems are directly influenced by the speed and precision of channel estimation [7-10]. Blind channel estimation technique based on symbol-pilot has high reliability, low complexity, but poor spectrum efficiency [11, 12]. LS, MMSE algorithm [13] and interpolation technique [14] in the time and frequency domain are also used for the blind estimation method. Recently, with advanced digital signal processing techniques, it is possible to obtain better channel estimation. For this purpose, other methods have also been studied to improve the efficiency of channel estimation for different wireless communication systems [15-17].

Multiple-Input and Multiple-Output (MIMO) is a method using multiple transmission and receiving antennas to exploit multipath propagation. Therefore, the problem of channel estimation for a MIMO system is very complex because many channels have to be estimated at the same time leading to increase the computational complexity of the system. In order to reduce the computational complexity, this paper presents the method of Maximum Likelihood (ML) based on the iterative Expectation Maximization (EM) algorithm for SFBC-OFDM system then applied to the STBC-OFDM and proposes Decoupled Maximum Likelihood (DEML) algorithm for the STBC-OFDM system.

The remainder of the paper is organized as follows. Section 2 describes the theoretical basis of the Alamouti STBC-OFDM system. Section 3 introduces ML channel estimation methods. Simulations are given

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in Section 4 and conclusions are summarized in Section 5.

2. ALAMOUTI STBC-OFDM SYSTEM

2.1 System modeling

The Alamouti STBC-OFDM system model is presented in Figure 1. Where, S is input data that passes through STBC encoder to generate the signals X_1 and X_2. A Fast Fourier transform (FFT) is used as the controller that converts signals from time domain to frequency domain propagating through transmitting antennas to transmit radio signals. On the receiver side, a receiving antenna is used to receive the signals on the transmitter side. The received signals are sent to the converter IFFT to convert into time domain signals. The STBC decoder and the channel estimator are applied to reconstruct the original signal $\hat{S}$.

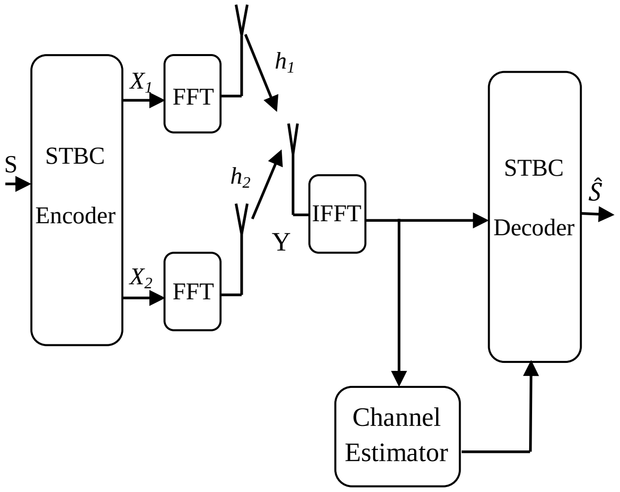

Figure 1. **STBC-OFDM system model**

2.2 STBC Decoder

From Fig. 1, the received signals is given as a matrix by

$$\left[\mathbf{Y}[k] = \mathbf{H}[k]\mathbf{X}[k] + \mathbf{Z}[k]\right] \quad (1)$$

with

$$\mathbf{H}[k] \square \begin{bmatrix} H_{1,1}[k] & H_{2,1}[k] \\ H_{2,2}^*[k] & -H_{1,2}^*[k] \end{bmatrix} \quad (2)$$

$$\mathbf{X}[k] \square \left[X_1[k] \quad X_2[k]\right]^T \quad (3)$$

$$\mathbf{Y}[k] \square \left[Y_1[k] \quad Y_2^*[k]\right]^T \quad (4)$$

$$\mathbf{Z}[k] \square \left[Z_1[k] \quad Z_2^*[k]\right]^T \quad (5)$$

where $\square$ are denoted as estimated value of matrix, $*$ is complex conjugate, Y is received signals, Z is the noise on the channel, and H is channel coefficient.

Assuming that the channel is a slow fading channel, we have

$$H_{1,1}[k] = H_{1,2}[k] \square \alpha_1[k] \quad (6)$$

$$H_{2,1}[k] = H_{2,2}[k] \square \alpha_2[k] \quad (7)$$

By substituting (6) and (7) into (2), we get the channel matrix as follow

$$\mathbf{H}[k] \square \begin{bmatrix} \alpha_1[k] & \alpha_2[k] \\ \alpha_2^*[k] & -\alpha_1^*[k] \end{bmatrix} \quad (8)$$

Consider the channel $\mathbf{H}[k]$ has been estimated, the STBC decoder can be presented as

$$\begin{aligned} \tilde{\mathbf{X}}[k] &= \mathbf{H}^H[k]\mathbf{Y}[k] \\ &= \mathbf{H}^H[k]\mathbf{H}[k]\mathbf{X}[k] + \mathbf{H}^H[k]\mathbf{Z}[k] \\ &= \left(|\alpha_1[k]|^2 + |\alpha_2[k]|^2\right)\mathbf{X}[k] + \mathbf{H}^H[k]\mathbf{Z}[k] \end{aligned} \quad (9)$$

with

$$\tilde{\mathbf{X}}[k] \square \left[\tilde{X}_1[k], \quad \tilde{X}_2[k]\right]^T$$

$\mathbf{H}^H$ is Hermitian the matrix of $\mathbf{H}$

Finally, we estimate the signals $\hat{X}_i[k]$ by

$$\hat{X}_t[k] = \arg\min_{X\in\chi}\left\{\left|\tilde{X}_t[k] - \rho[k]X\right|^2\right\}, \quad t = \{1,2\} \quad (10)$$

$$\rho \square |\alpha_1[k]|^2 + |\alpha_2[k]|^2$$

3. MAXIMUM LIKELIHOOD CHANNEL ESTIMATON

3.1. Maximum likelihood channel estimation based EM algorithm

The Expectation Maximization (EM) algorithm is an iterative method to find maximum likelihood in statistical models, which bases on unobserved latent variables.

Considering the SFBC-OFDM system, the received signals at time t are given by

$$Y = \mathbf{X}_1\mathbf{W}_L h_1 + \mathbf{X}_2\mathbf{W}_L h_2 + Z \quad (11)$$

with

W_L is FFT matrix, to apply the EM algorithm for estimating channel coefficient in the SFBC-OFDM system, we assume N_1 and N_2 are data signals separated by observed data Y as follows

$$N_i = \mathbf{X}_i\mathbf{W}_L h_i + Z_i, \quad i = 1,2 \quad (12)$$

with

$Z_i, i = 1,2$ is the component noise obtained by splitting random process the total noise Z . Thus, the relationship between complete data (N_1, N_2) and incomplete data Y are given by

$$Y = N_1 + N_2 \quad (13)$$

EM algorithm is used to estimate the channel h from incomplete data $Y(m) = \sum_{i=0}^{L-1} N_i(m)$ such that

- *Expectation step*

$$Q\left(N \mid h^{(l)}\right) = E_N\left\{\log f(N \mid h) \mid Y, h^{(l)}\right\} \quad (14)$$

$$Q\left(N \mid h^{(l)}\right) = K_1 - E\left\{\sum_{i=1}^{2}\frac{1}{2\sigma_i^2}\left\|N_i - \mathbf{X}_i\mathbf{W}_L h_i\right\|^2 Y, h^{(l)}\right\}$$

$$= K_2 - \sum_{i=1}^{2}\frac{1}{2\sigma_i^2}\left\|\hat{N}_i^{(l)} - \mathbf{X}_i\mathbf{W}_L h_i\right\|^2$$

and K_2 are constants independent of h, and the complete data is estimated by

$$\hat{N}_i^{(l)} = E\left\{N_i \middle| Y, h^{(l)}\right\} \qquad (16)$$

Here, we use the coefficients N_i depend only on Y. When N_i and Y are Gaussian distributions and $Y = \sum_i N_i$, we can estimate complete data N_i as

$$\hat{N}_i^{(l)} = N_i^{(l)} + \rho_i \frac{\sigma_i}{\sigma}\left(Y - \sum_{j=1}^{2} N_j^{(l)}\right), i = 1,2 \qquad (17)$$

where

$$N_i^{(l)} = \mathbf{X}_i\mathbf{W}_L h_i^{(l)} \qquad (18)$$

and ρ_i is the cross-correlation between N_i and Y

$$\rho_i = \frac{E\left\{Z_i(m)Z(m)\right\}}{\sqrt{Var(Z_i)Var(Z)}} = \frac{\sigma_i}{\sigma} \qquad (19)$$

Substituting (19) int to (17), we can calculate N_i such that

$$\hat{N}_i^{(l)} = N_i^{(l)} + \beta_i\left(Y - \sum_{j=1}^{2} N_j^{(l)}\right) \qquad (20)$$

with

$$\beta_i = \frac{\sigma_i^2}{\sigma^2}$$

- *Maximization step*

The channel coefficients can be calculated by

$$h^{(l+1)} = \arg\max_h Q(N \mid h^{(l)})$$

$$= \arg\min_h \sum_{i=0}^{L-1}\left\|\hat{N}_i^{(l)} - \mathbf{X}_i\mathbf{W}_L h_i\right\|^2 \qquad (21)$$

$$= \mathbf{W}_L^H \mathbf{X}_i^{-1}\hat{N}_i^{(l)} \quad i = 1,2$$

The results in (21) are applicable only to the case of observable signal at the receiver. For the unobservable signals excepting pilot symbols, the estimation of X_1 and X_2 at lth times are accepted as $X_1^{(l)}$ and $X_2^{(l)}$, respectively. Therefore, we can estimate the channel as follows

$$h_i^{(l+1)} = \mathbf{W}_L^H\left(\mathbf{X}_i^{(l)}\right)^{-1}\hat{N}_i^{(l)} \quad i = 1,2 \qquad (22)$$

with

$$\hat{N}_i^{(l)} = N_i^{(l)} + \rho_i\frac{\sigma_i}{\sigma}\left(Y - \sum_{j=1}^{2} N_j^{(l)}\right) \qquad (23)$$

(15)

$$N_i^{(l)} = \mathbf{X}_i^{(l)}\mathbf{W}_L h_i^{(l)} \qquad (24)$$

The above method using for SFBC-OFDM system can be easily applied to STBC-OFDM system, where the EM algorithm will be performed two iterations in one OFDM frame instead of finding the channel inverse matrix $\mathbf{H}^{-1}$.

3.2 DEML channel estimation

The schematic of STBC-OFDM with two transmitting antennas and one receiving antenna is presented in Fig. 1.

From Fig 1, the received signal is given by

$$\mathbf{Y} = \mathbf{HX} + \mathbf{Z} \qquad (25)$$

with

$\mathbf{Y}$ is the matrix of received signals, $\mathbf{X}$ is the matrix of the transmitted signals and $\mathbf{H}$ is the matrix of channel coefficient.

The covariance of the noise can be given as:

$$E\left[z(t_i)z^*(t_j)\right] = \mathbf{Q}\delta_{i,j} \qquad (26)$$

with

$\delta_{i,j}$ is the function of Kronecker delta, $\mathbf{Q}$ is the unknown covariance matrix. Assuming the signal $\mathbf{X}$ and the noise $\mathbf{Z}$ are not correlated, we have

$$\lim_{L\to\infty}\frac{1}{L}\sum_{l=1}^{L}\mathbf{X}(t_l)\mathbf{Z}^*(t_l) = \lim_{L\to\infty}\frac{1}{L}\sum_{l=1}^{L}\mathbf{XZ}^* = 0 \qquad (27)$$

The DEML is a channel estimation method, which calculates the channel matrix H and the noise covariance matrix coefficients. From (25) and (26), it shows that the calculation of the matrices H and Q is very complicated.

The log-likelihood function of $\mathbf{Y}$ given $\mathbf{X}$ is proportional to (within an additive constant) [18]

$$F_1 = -\ln|\mathbf{Q}| - \frac{1}{L}tr\left\{\mathbf{Q}^{-1}(\mathbf{Y}-\mathbf{HX})(\mathbf{Y}-\mathbf{HX})^H\right\} \qquad (28)$$

It is easy to find that the maximum of F1 with respect to $\mathbf{Q}$ is

$$\hat{\mathbf{Q}} = \frac{1}{L}(\mathbf{Y}-\mathbf{HX})(\mathbf{Y}-\mathbf{HX})^H \qquad (29)$$

with

$\hat{\mathbf{Q}}$ is the estimation of $\mathbf{Q}$.

Substituting (29) into (28), we have the maximum of the log-likelihood function F is the minimum of the function $\ln\left|\hat{\mathbf{Q}}\right|$. Let

$$F_1 = \left| \frac{1}{L} (\mathbf{Y} - \mathbf{HX})(\mathbf{Y} - \mathbf{HX})^H \right| \tag{30}$$

Let $\mathbf{R}_{XY}, \mathbf{R}_{XX}$ and $\mathbf{R}_{YY}$ are the covariance matrices, which are given as follows

$$\mathbf{R}_{XY} \triangleq \frac{1}{L} \sum_{l=1}^{L} \mathbf{X}(t_l) \mathbf{Y}^H(t_l) = \frac{1}{L} \mathbf{XY}^H \tag{31}$$

$$\mathbf{R}_{XX} \triangleq \frac{1}{L} \sum_{l=1}^{L} \mathbf{X}(t_l) \mathbf{X}^H(t_l) = \frac{1}{L} \mathbf{XX}^H \tag{32}$$

$$\mathbf{R}_{YY} \triangleq \frac{1}{L} \sum_{l=1}^{L} \mathbf{Y}(t_l) \mathbf{Y}^H(t_l) = \frac{1}{L} \mathbf{YY}^H \tag{33}$$

and define

$$\begin{aligned} \mathbf{F} &\triangleq \frac{1}{L} (\mathbf{Y} - \mathbf{HX})(\mathbf{Y} - \mathbf{HX})^H \\ &= \mathbf{R}_{YY} - \mathbf{HR}_{XY} - \mathbf{R}_{XY}^H \mathbf{H}^H + \mathbf{HR}_{XX} \mathbf{H}^H \\ &= (\mathbf{H} - \mathbf{R}_{XY}^H \mathbf{R}_{XX}^{-1}) \mathbf{R}_{XX} (\mathbf{H} - \mathbf{R}_{XY}^H \mathbf{R}_{XX}^{-1}) + \mathbf{R}_{YY} - \mathbf{R}_{XY}^H \mathbf{R}_{XX}^{-1} \mathbf{R}_{XY} \end{aligned} \tag{34}$$

Since $\mathbf{R}_{XX}$ is positive definite and the conditions in (34) do not depend on **H**, it follows that

$$\mathbf{F} \geq \mathbf{F} \Big|_{\mathbf{H} = \mathbf{R}_{XY}^H \mathbf{R}_{XX}^{-1}} \tag{35}$$

Therefore, since the whole sample covariance matrix **F** is minimized, the estimate $\hat{\mathbf{H}} = \mathbf{R}_{XY}^* \mathbf{R}_{XX}^{-1}$ of $\mathbf{H}$ will minimize to any non-decreasing function of **F** including the determine of $|\mathbf{F}|$. Thus, the ML estimation of **H** is given as following

$$\hat{\mathbf{H}} = \mathbf{R}_{XY}^H \mathbf{R}_{XX}^{-1} \tag{36}$$

Substituting (36) into (29), we obtain the estimation of **Q** as follows

$$\hat{\mathbf{Q}} = \mathbf{R}_{YY} - \mathbf{R}_{XY}^H \mathbf{R}_{XX}^{-1} \mathbf{R}_{XY} \tag{37}$$

4. SIMULATIONS RESULT

In this section, we compare the performance of two channel estimation methods for STBC-OFDM system.

(1) Maximum likelihood channel estimation based EM algorithm, here we use the training sequence to estimate the initial channel coefficients according to (44) and (45), then by using the EM algorithm to re-estimate.

(2) Decoupled maximum likelihood Channel Estimation, here we use the training sequence to estimate the channel coefficient directly.

Based on the Alamouti STBC-OFDM model as shown in Figure 1, the received signal matrix is given as follows

$$\mathbf{Y} = \mathbf{HX} + \mathbf{Z} \tag{38}$$

with

$$\mathbf{X} = \begin{pmatrix} X_1 & X_2 \\ -X_2^* & X_1^* \end{pmatrix} \tag{39}$$

$$\mathbf{Y} = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \tag{40}$$

$$\mathbf{Z} = \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} \tag{41}$$

Substituting (39), (40) and (41) into (38), we have

$$Y_1 = X_1 H_1 + X_2 H_2 + Z_1 \tag{42}$$

$$Y_2 = -X_2^* H_1 + X_1^* H_2 + Z_2 \tag{43}$$

From (42) and (43), we can be estimated channel **H** as follows

$$\hat{H}_1 = \frac{Y_1 X_1^* - Y_2 X_2^*}{|X_1|^2 + |X_2|^2} \tag{44}$$

$$\hat{H}_2 = \frac{Y_1 X_2^* + Y_2 X_1}{|X_1|^2 + |X_2|^2} \tag{45}$$

Case 1). In this case, the channel is used flat fading channel, the modulation method is selected BPSK, the number of FFT points is selected as N = 64, and the CP = 16. The noise on the channel is used white noise AWGN.

Fig. 2, Fig. 3 and Fig. 4 and Fig. 5 show BER of STBC-OFDM system with different channel estimation methods corresponding to the maximum doppler shift frequency $f_{d\max} = 1Hz$, $f_{d\max} = 10Hz$ $f_{d\max} = 100Hz$ and $f_{d\max} = 1000Hz$. From Fig. 2, Fig. 3 and Fig. 4 and Fig. 5, we can conclude that the performance of the EM channel estimation method depends on the accuracy of the initial training sequence. Besides, they also illustrate that the performance of the DEML estimation method is better than the EM estimation method.

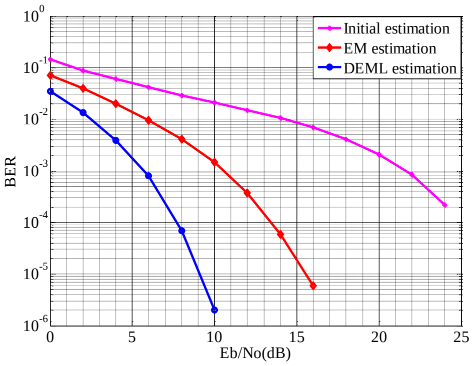

Figure 2. **BER of STBC-OFDM system with** $f_{d\max} = 1Hz$

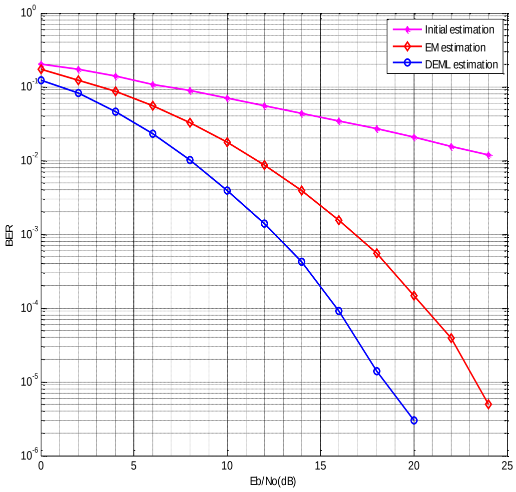

Figure 3. BER of STBC-OFDM system with $f_{d\max} = 10Hz$

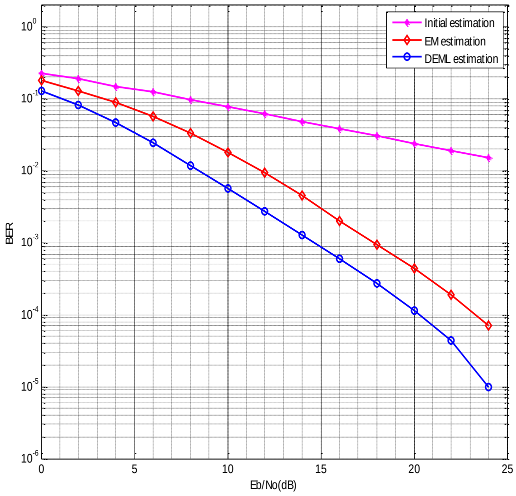

Figure 4. BER of STBC-OFDM system with $f_{d\max} = 100Hz$

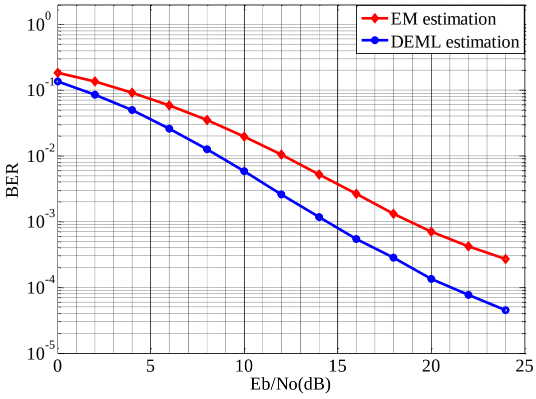

Figure 5. BER of STBC-OFDM system with f_{dmax}=1000Hz

Case 2). In this case, the channel is used multipath fading channel, where the multipath channel is selected as 2 with time delays and the gain factors of $\begin{bmatrix}0 & 0.1\end{bmatrix}\mu s$ and $\begin{bmatrix}0 & -12\end{bmatrix}dB$, respectively. Other simulation parameters are the same as case 1.

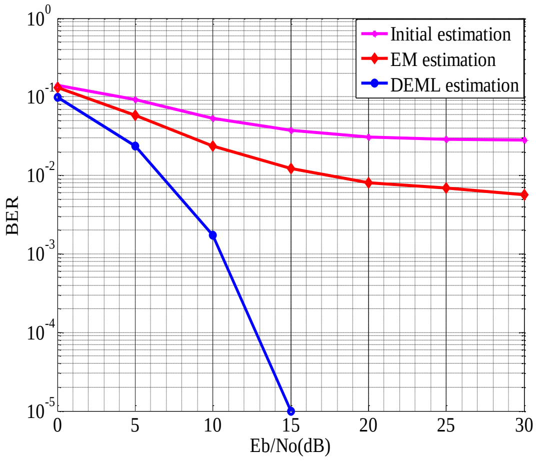

Figure 6. BER of STBC-OFDM system with multipath channel, $f_{d\max} = 1Hz$

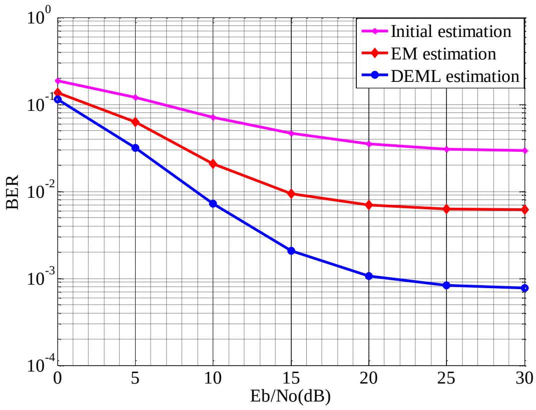

Figure 7. BER of STBC-OFDM system with multipath channel, $f_{d\max} = 10Hz$

Fig. 6 and Fig. 7 indicate the performance BER of the STBC-OFDM system using the EM algorithm and DEML method on the multipath channel. From Fig. 6 and Fig. 7, they also show that the performance of the DEML estimation method is reliable.

5. CONCLUSIONS

In this paper, we introduce channel estimation methods for STBC-OFDM systems with the aim of reducing the computational complexity. EM algorithm is first used to estimate channel matrices based on initial training sequence; this method avoids the problems of calculating the inverse matrix by applying an iterative algorithm. Then, by using the DEML method based on separating the log-likelihood of the received signals. It also reduces computational complexity. The simulation results show that the BER performance of the DEML method is much better than the EM algorithm on both flat fading channel and multipath fading channel.

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