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Review Article

Steady State MHD Free Convection Slip Flow of an Exothermic Fluid in a Convectively Heated Vertical Channel

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Abstract

In this study, the influence of Magnetohydrodynamics on free convection slip flow of an exothermic fluid in a convectively heated vertical channel is analyzed. The problem is solved analytically using perturbation series method and expression for velocity, temperature, skin friction, Nusselt Number are obtained and also the influence of some physical parameters such as Hartmann number(Ha), Biot number(Br), Navier slip parameter(γ) and Frank-Kamenetskii parameter(λ) are discussed. It is observed that both velocity fluid and skin friction decreases with increasing value of Hartmann number.

Keywords: Magnetohydrodynamics, skin friction, velocity fluid, Navier slip parameter(γ).

1.0. INTRODUCTION

Magnetohydrodynamics (MHD) is the study of magnetic properties and behavior of electrically conducting fluid. Many experimental and theoretical studies on convectional electrically conducting fluids indicate that magnetic field markedly changes their transport and heat transfer characteristics. MHD has many important applications such as cooling of nuclear reactors by induction flow meter and liquid sodium which depends on the potential difference perpendicular to the motion and to the magnetic field. Moreover, applications of MHD in polymer industry attracted several researchers' attention to investigate the MHD flow. The first researcher to study the effectiveness of MHD flow in fluid was Sarpkaya [1]. After that, recently some few studies concerning MHD heat and mass transfer with different conditions were conducted [2-9].

Jha and Bello [10] investigate MHD free convection flow in a vertical slit micro-channel with super-hydrophobic slip and temperature jump. Ali *et al.*, [11] analyzed numerical solution of MHD free convection flow and heat transfer characteristics. The combine effects of heat and mass transfer on free convection MHD flow of viscous reactive fluid embedded in a porous medium has been analyzed by Farhad *et al.*, [12]. Jayabalan *et al.*, [13] discussed the effect of fully developed MHD mixed convection flow in a vertical channel with first order chemical reaction and observed that dual solutions exist for both velocity and temperature. The influences of velocity slip and temperature jump parameters on temperature distribution and heat transfer on free convective flow of viscous reactive fluid in vertical micro-channel was examined by Ahmad and Jha [14]. Also, the steady free convection flow of viscous reactive fluid in a vertical channel tube and annulus was examine by Jha *et al.*, [15-18].

On the other hand, slip flow sensation has received a special deliberation, due to it attainable applications in micro/nano-channels and mother science and technology [19]. Many researches have been conducted on natural convection slip flow in a vertical channel with different boundary conditions. Mattews and Hills [20] investigate the effect of slip boundary condition on momentum boundary layer thickness on a flat plate. Pal and Talukdar [21] explore analytically the combined effect of heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction. Makinde [22] dispense the join effect of Navier slip and Newtonian heating on MHD unsteady flow and heat transfer toward a flat plate. Also, Hamza[23] analyze the combine influence of Navier slip and Newtonian heating on transient natural convection flow of a viscous reactive fluid in a vertical channel and observed that flow formation is strongly dependent on Newtonian heating, Navier slip and Frank-Kamenetskii parameter. Kamran and Benchowan [24] discussed the effects of chemical reaction and Newtonian heating on steady convection flow of a micro polar fluid with second order slip at the boundary and found that the shrinking sheet produces a wider concentration boundary layer thickness by a small change in the chemical reaction parameter. In contrast to the stretching sheet, the Newtonian heating parameter raises the thermal boundary layer thickness by 39.93% for the shrinking sheet.

The main objective of the present paper is to examine the influence of Magnetohydrodynamics (MHD) on steady state natural convection flow of viscous reactive fluid in a convectively heated vertical channel.

2.0. Mathematical Analysis

Consider a steady state MHD natural convection flow of an exothermic fluid of Arrhenius kinetic with heat transfer and Navier slip condition in a channel formed by two infinite vertical parallel plates separated by a distance H as shown in Fig 1. The flow is induced by the convective heating introduced on the lower surface of the channel wall as well as the reactive property of the fluid. Following Jha *et al.*, [15] and Hamza [23] the non-dimensional governing equations under the Boussinesq's

Approximation can be written as

$$v\frac{d^2u'}{dy'^2} + g\beta(T' - T'_0) - \frac{\nabla B_0^2 u'}{\rho} = 0 \qquad (1)$$

$$\frac{k}{\rho C_p}\frac{d^2u\prime}{dy\prime^2} + \frac{QC_0^*A}{\rho C_p}\, e^{\left(\frac{-E}{RT'}\right)} = 0 \qquad (2)$$

The boundary conditions for the present problem are

$$\left.\begin{aligned} u' = \gamma^*\frac{du'}{dy'}, -k\frac{dT'}{dy'} = h[T'_0 - T'], at\ y' = 0 \\ u' = 0, T' = T'_0\ as\ y' \to H \end{aligned}\right\} \qquad (3)$$

where β is the coefficient of thermal expansion, *Q* is the heat of reaction, *A* is the rate constant, *E* is the activation energy, *R* is the universal constant, *v* is the kinematic viscosity, C_0 is the initial concentration of the reactant species, *g* is the gravitational force, C_p is the specific heat at constant pressure and *k* is the thermal conductivity of the fluid, while *p* is the density of the fluid.

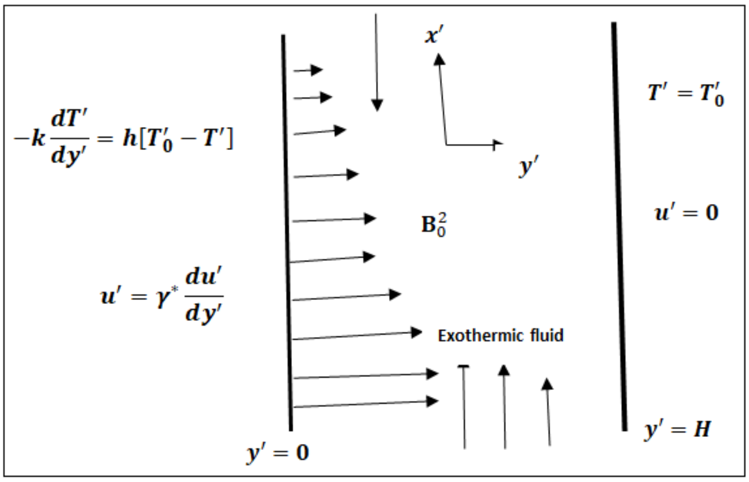

Figure 1: Geometry of the problem

To solve equations (1) and (2), we employ the following dimensionless variables and parameters

$$\left.\begin{aligned} y = \frac{y'}{H}, \theta = \frac{E(T'-T_0)}{RT_0^2}, \varepsilon = \frac{RT_0}{E}, U = \frac{u'\mu_0 E}{g\beta H^2 RT_0^2}, \lambda = \frac{QC_0^*AEH^2}{RT_0^2}\, e^{\left(\frac{-E}{RT_0}\right)}, Pr = \frac{\mu_0\rho C_p}{k} \\ \gamma = \frac{\gamma^*}{H}, \theta_a = \frac{E(T_a - T_0)}{RT_0^2}, Br = \frac{hH}{k} \end{aligned}\right\} \qquad (4$$

Using (4), Equations (1)-(3) take the following form:

$$\frac{d^2U}{dy^2} + \theta = 0 \qquad (5)$$

$$\frac{1}{Pr}\frac{d^2\theta}{dy^2} + \frac{\lambda}{Pr}e^{\left(\frac{\theta}{1+\varepsilon\theta}\right)} = 0 \quad (6)$$

The initial and boundary conditions in dimensionless form are

$$\left.\begin{array}{c} U = \gamma\frac{dU}{dy}, \frac{d\theta}{dy} = Br[\theta - \theta_a], at\ y = 0 \\ U = 0, \theta = 0\ as\ y = 1 \end{array}\right\} \quad (7)$$

Where γ, Br, Pr, λ, θ and ε are Navier slip parameter, Biot number, Prandtl number, Frank-Kamenetskii parameter, ambient temperature and activation energy parameter.

3.0. Method of Solution

The governing equations highlighted in the previous section are nonlinear and exhibit no analytical solutions. The state governing equations together with boundary condition can be written as follows:

$$\frac{d^2U}{dy^2} - Ha^2U = -\theta \quad (8)$$

$$\frac{d^2\theta}{dy^2} + \lambda e^{\left(\frac{\theta}{1+\varepsilon\theta}\right)} = 0 \quad (9)$$

$$U - \gamma\frac{dU}{dy} = 0, \frac{d\theta}{dy} - Br[\theta - \theta_a] = 0\ at\ y = 0 \quad (10)$$

$$U = 0, \theta = 0, at\ y = 1$$

To obtain the approximate solutions of equations (8) and (9) subject to (10), we take the power series expansion in the Frank-Kamenetskii parameter λ of the form:

$$\left.\begin{array}{c} U = U_0 + \lambda U_1 + \lambda^2 U_2 + o(\lambda) \\ \theta = \theta_0 + \lambda\theta_1 + \lambda^2\theta_2 + o(\lambda) \end{array}\right\} \quad (11)$$

Substituting (11) into (8)-(10) and equating the coefficient of like powers of λ, the resulting solutions of the momentum and energy balance equations are as follows:

$$U = A_1\cosh(Hay) + B_1\sinh(Hay) + py + q + \lambda[A_2\cosh(Hay) + B_2\sinh(Hay) + p_1y^5 + p_2y^4 + p_3y^3 + p_4y^2 + p_5y + p_6] + \lambda^2[A_3\cosh(Hay) + B_3\sinh(Hay) + q_1y^5 + q_2y^4 + q_3y^3 + q_4y^2 + q_5y + q_6] \quad (12)$$

$$\theta = (Ay + B) + \lambda\left[\frac{l_1}{2}y^2 + \frac{l_2}{6}y^3 + \frac{l_3}{12}y^4 + \frac{l_4}{20}y^5 + Cy + D\right] + \lambda^2\left[\frac{m_1}{2}y^2 + \frac{m_2}{6}y^3 + \frac{m_3}{12}y^4 + \frac{m_4}{20}y^5 + Gy + H\right] \ldots \quad (13)$$

From (12), the steady state skin frictions on the boundaries are as follows:

$$\frac{dU}{dy}\Big|_{y=0} = (B_1Ha + p) + \lambda[B_2Ha + p_5] + \lambda^2[B_3Ha + q_5] \quad (14)$$

$$\frac{dU}{dy}\Big|_{y=1} = [Ha(A_1\sinh(Ha) + B_1\cosh(Ha)) + p] + \lambda[Ha(A_2\sinh(Ha) + B_2\cosh(Ha) + P_e] + \lambda^2[Ha(A_3\sinh(Ha) + B_3\cosh(Ha) + Q_e)] \quad (15)$$

Also, from (13) the rates of heat transfer on the boundaries in terms of Nusselt number are as follows:

$$\frac{d\theta}{dy}\Big|_{y=0} = A + \lambda[C] + \lambda^2[G] \quad (16)$$

$$\frac{d\theta}{dy}\Big|_{y=1} = A + \lambda\left[l_1 + \frac{l_2}{2} + \frac{l_3}{3} + \frac{l_4}{4} + C\right] + \lambda^2\left[m_1 + \frac{m_2}{2} + \frac{m_3}{3} + \frac{m_4}{4} + G\right] \quad (17)$$

The constants, $A, B, C, D, G, H, A_1, A_2, A_3, B_1, B_2, B_3, K_1, K_2, p_1, p_2, p_3, p_4, p_5, p_6, q_1, q_2, q_3, q_4, q_5, q_6, l_1, l_2, l_3, l_4, m_1, m_2, m_3, m_4, p, q, P_e, Q_e$ are defined in the appendix section.

4.0. RESULT AND DISCUSSION

MHD free convections flow of exothermic fluid has been realized in a convectively heated vertical channel with Navier slip boundaries condition. The governing parameters controlling the systems of flow are as follows: Hartman Number *(Ha)*, Biot Number *(Br)*, Navier slip parameter *(γ)*, activation energy *(ε)*, Frank-Kamenetskii parameter*(λ)*. The following values are fixed for the governing parameters. *Br*=0.01, *γ*= 0.1, *ε*= 0.01, *Ha*=0.1, λ=0.01, θ_a=1.

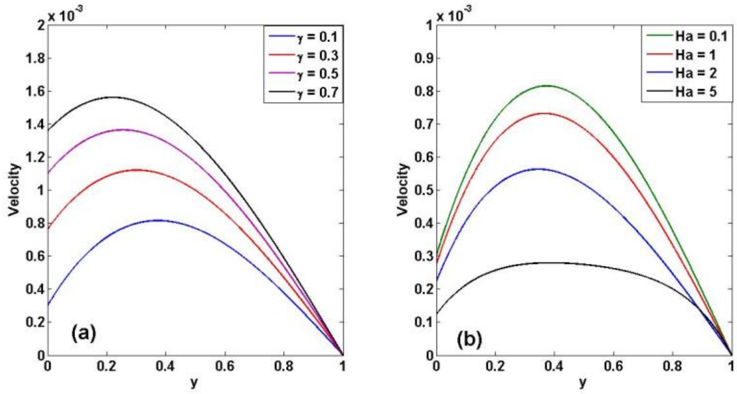

Figure 2: Velocity profile for different value of γ and *Ha*

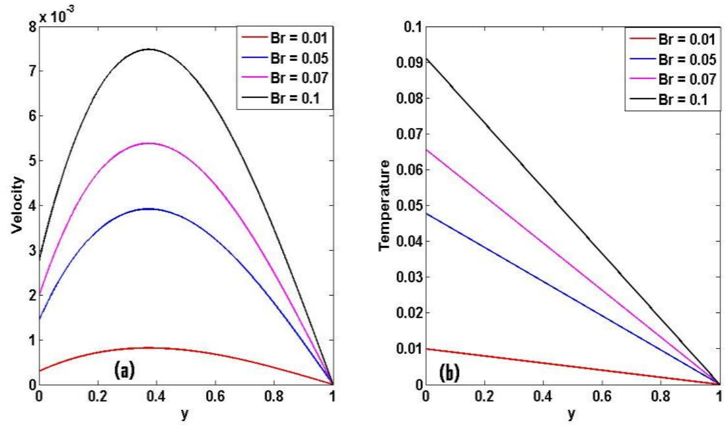

Figure 3: Velocity and Temperature for different value of *Br*

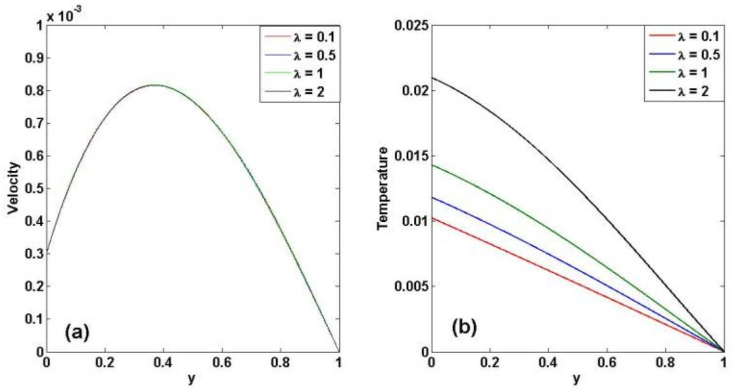

Figure 4: Velocity and temperature for different value of λ

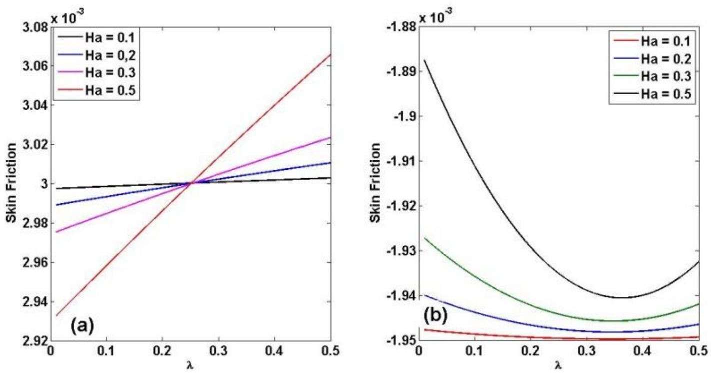

Figure 5: Skin friction against λ for different value of *Ha*

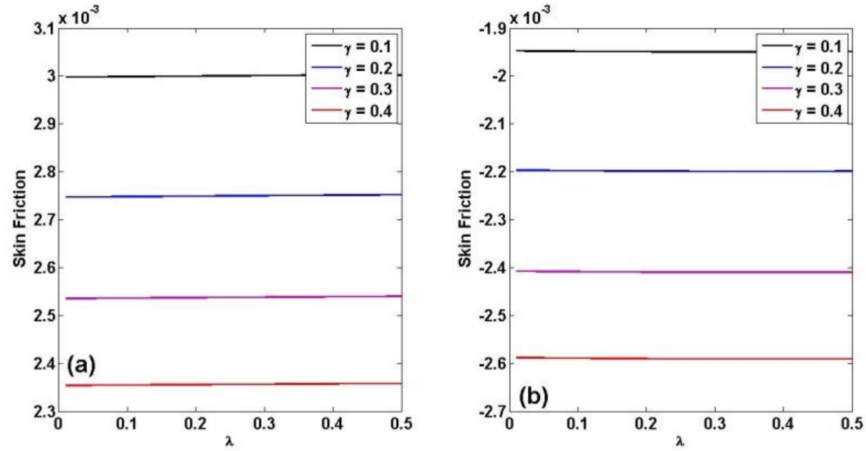

Figure 6: Skin friction against λ for different value of γ

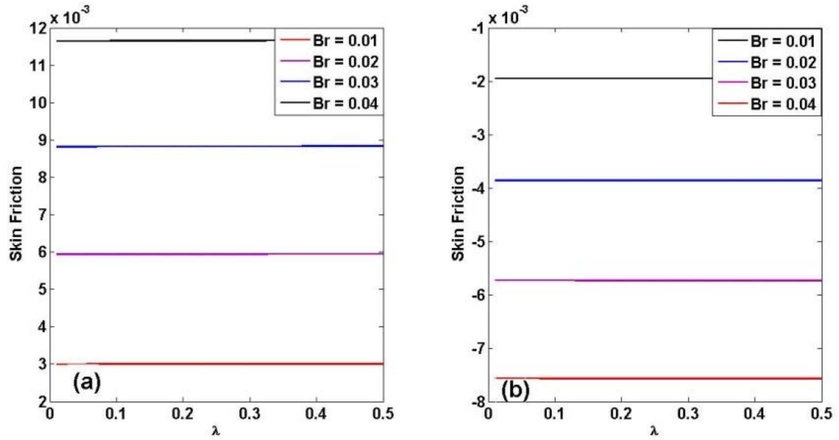

Figure 7: Skin friction against λ for different value of *Br*

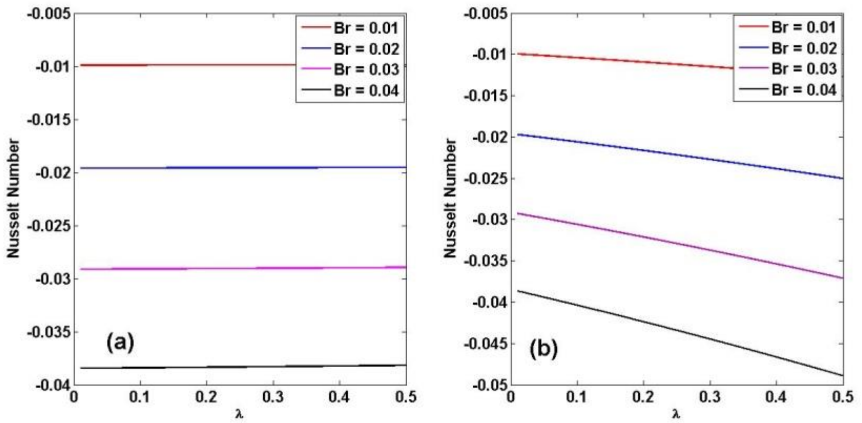

Figure 8: Nusselt number against λ for different value of *Br*

The effect of Navier slip parameter (γ) and Magnetohydrodynamic (*Ha*) on velocity profile is showed in Fig 2a and b respectively, from Fig 2a, it is observed that fluid velocity profile increases with increasing value of γ. i.e. higher value of γ means increase in the reaction and slipperiness of the lower plate surface. In Fig 2b, we realized decrease in velocity profile with increase in *Ha*.

The influence of Biot number (*Br*) on velocity and temperature profiles is shown in Fig. 3a and b. it is clearly seen that the velocity and temperature of fluid increases with increase in *Br*. Meaning higher *Br* equally higher degree of convective heating at lower channel wall lead to high velocity and temperature at lower plate., increasing the *Br* lead to a stronger convective heating at lower channel wall see Hamza [23].

Fig 4a and b show the effect of Frank-Kamenetskii parameter (λ) in velocity and temperature. Fig 4a shows that there is no significant increase or decrease in velocity with increase or decrease in λ, that is to say λ has no effect on fluid velocity, but there is temperature rises with increase in λ as seen in Fig.4b. In addition, maximum value of fluid temperature is attained at lower plate surface and decayed to the upper plate channel.

The variation of skin friction is revealed in Fig.5a and b at y=0 and y=1 respectively. It is observed that in Fig 5a, skin friction decreases with an increase in *Ha* and also increases with increase in λ to a point between λ=0.25 where graphs meet and the trend changes at y=0, but a different case occur with skin friction increasing with increase in *Ha* at y=1 as seen in Fig. 5b. Also, Fig 6a and b show the influence of Navier slip parameter (γ) in skin friction at y=0 and y=1 respectively. It is observed that both at plate y=0 and y=1, skin friction decrease with increase in γ.

The effect of Biot number (*Br*) in wall shear stress and rate of heat transfer is shown in Fig 1, 7 and 8. In Fig 1 & 7a, the skin friction increases with increase in Biot number (*Br*) at y=0 but decreases with increase in *Br* as seen in Fig 7b at y=1. The rate of heat transfer is decreasing with increase in *Br* for both y=0 and y=1 as seen in Fig 1, 8a and b.

5.0. CONCLUSION

The present paper investigates the effect of MHD on natural convection flow of an exothermic fluid in a vertical channel. The approximated solution of the problem is obtained using perturbation method. Summary of the major findings are as follows;

i. The velocity decreases with an increase in *Ha*, but skin friction decreases with increase in *Ha* to a point where the trend changes at y=0 and increase in skin friction with increase in *Ha* at y=1.
ii. The fluid velocity increases with increase in γ while skin friction decreases with increase in γ for both y=0 and y=1.
iii. The velocity and temperature of fluid increases with increase in *Br* while skin friction is high with increase in *Br* at y=0 and decrease at y=1.
iv. The Nusselt number decrease with increase in *Br* at y=0 and y=1
v. No significant increase or decrease in velocity with different value of λ but temperature rises with increase in λ.

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Appendix A

$$B = \frac{Br\theta_a}{1+Br}, A = -B,\ l_1 = B\left(-1+\varepsilon B - \frac{\varepsilon^2 B}{2}\right), l_2 = \left[-A + 2\varepsilon AB - \frac{3AB^2\varepsilon^2}{2}\right]$$

$$l_3 = \varepsilon A^2\left(1-\frac{3\varepsilon B}{2}\right), l_4 = -\left(\frac{\varepsilon^2 A^3}{2}\right), D = \frac{-1}{(1+Br)}\left[\frac{l_1}{2}+\frac{l_2}{6}+\frac{l_3}{12}+\frac{l_4}{20}\right]$$

$$C = BrD,\ m_1 = \frac{D}{2}[-2+4\varepsilon B - 3\varepsilon^2 B^2], m_2 = \left[\frac{C}{2}(-2+4\varepsilon B - 3\varepsilon^2 B^2) + AD(2\varepsilon - 3B)\right]$$

$$m_3 = \frac{-l_1}{4}(2-4\varepsilon B + 3\varepsilon^2 B^2) + \frac{A\varepsilon}{2}(4C - 3\varepsilon AD - 3\varepsilon BC)$$

$$m_4 = \frac{l_1\varepsilon A}{2}(2-3\varepsilon B) + \frac{l_2}{12}[2+\varepsilon B(4-3\varepsilon B)] - \frac{3\varepsilon^2 A^2 C}{2}$$

$$H = \frac{-1}{(1+Br)}\left[\frac{m_1}{2}+\frac{m_2}{6}+\frac{m_3}{12}+\frac{m_4}{20}\right], G = BrH, p = \frac{A}{Ha^2}, q = \frac{B}{Ha^2}$$

$$B_1 = \frac{-[(\gamma p - q)\cosh(Ha) + p + q]}{\gamma Ha\cosh(Ha) + \sinh(Ha)}, A_1 = \frac{-[B_1\sinh(Ha) + p + q]}{\cosh(Ha)}$$

$$p_1 = \frac{l_4}{20Ha^2}, p_2 = \frac{l_3}{12Ha^2}, p_3 = \frac{l_2 + 60p_1}{6Ha^2}, p_4 = \frac{l_1 + 24p_2}{2Ha^2}, p_5 = \frac{C + 6p_3}{Ha^2}, p_6 = \frac{D + 2p_4}{Ha^2}$$

$$K_1 = (p_1+p_2+p_3+p_4+p_5+p_6), B_2 = \frac{-[(\gamma p_5 - p_6)\cosh(Ha) + K_1]}{\gamma Ha\cosh(Ha) + \sinh(Ha)}, A_2 = \frac{-B_2\sinh(Ha) - K_1}{\cosh(Ha)}$$

$$q_1 = \frac{m_4}{20Ha^2}, q_2 = \frac{m_3}{12Ha^2}, q_3 = \frac{m_2 + 60q_1}{6Ha^2}, q_4 = \frac{m_1 + 24q_2}{2Ha^2}, q_5 = \frac{G + 6q_3}{Ha^2}, q_6 = \frac{H + 2q_4}{Ha^2}$$

$$K_2 = [q_1+q_2+q_3+q_4+q_5+q_6],\ B_3 = \frac{-[(\gamma q_5 - q_6)\cosh(Ha) + K_2]}{\gamma Ha\cosh(Ha) + \sinh(Ha)},\ A_3 = \gamma[B_3 Ha + q_5] - q_6$$