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Original Research Article

Free-Vibration Study of Thick Rectangular Plates using Polynomial Displacement Functions

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Abstract

This paper applied polynomial displacement functions for free-vibration study of isotropic thick rectangular plates. The theory uses a third-order polynomial to describe how the transverse shear stress varies across the thickness of the plate and has no need for a shear correction factor. Polynomial expressions were used as the displacement functions (u, v and w) and also as shear deformation function f(z) in obtaining the general governing equations for the plate. For numerical illustrations, a thick rectangular plate whose four edges have simple supports (SSSS) was studied. The requirements for the edge conditions of the simply supported plate were satisfied and used in solving the general governing equation. This resulted to a linear equation used to generate for the plate, the non-dimensional fundamental natural frequency parameters for any value of span-thickness relation (a/t) and in-plane relation (b/a). To validate this study, results obtained herein were compared with related previous works in literature.

Keywords: Fundamental natural frequency, Shear deformation, Polynomial function, Displacement functions, plate, governing equation.

INTRODUCTION

Plates have numerous applications in Engineering. Apart from static loads, time-dependent loads or dynamic loads could also act on plate structures. Thus, it is not adequate to design plate structures to withstand only static loads [1, 2]. There are several number of discrete frequencies of the plate at which when time-dependent loads of equal frequencies act on the plate will result to large-amplitude vibrations, which could lead to structural failure. This phenomenum is known as resonance [3]. Thus, to rectify the above problem, it becomes necessary to conduct a free-vibration analysis of the rectangular plate so as to determine these frequencies that could cause resonance [1].

The Classical Plate Theory (CPT) did not consider the effects of the transverse shear strains and stresses and thus, restrict the model to thin plates [4]. Nevertheless, thick and moderately thick plates in which substantial transverse shear strains occur have numerous applications in engineering such as; bridge decks, cylinders for nuclear reactors, etc., The theory under-estimates of deflections and over-estimates the natural frequencies and buckling loads [5]. The first theory of beams that included the effects of shear deformation was developed by Timoshenko and this theory is now widely referred to as Timoshenko beam theory or First Order Shear Deformation Theory (FSDT) [6]. FSDT assumes a uniform shear strain distribution throughout the thickness of the plate. However, this assumption cannot be true and in order to correct this, a shear coefficient is introduced into the analysis [7]. As a result of these shortcomings of the (CPT) and (FSDT), the plate theories were further refined to circumvent the need for a shear correction factor and to get a convincing variation of the transverse shear strains within the plate's thickness. Such refined theories are called Higher Order Shear Deformation Theories (HSDT). Sayyad, I. I *et al.*, [5] applied trigonometric shear deformation functions in their study on thick plates. Hashemi, S. H *et al.*, [8] obtained exact characteristic equations for moderately thick plates using trigonometric displacement functions. Rajesh, K *et al.*, [9] carried out vibration analysis of thick plates using trigonometric displacement functions. The present work applied a polynomial shear deformation function and polynomial displacement functions in free vibration analysis of isotropic thick rectangular plates with simple supports at the four edges.

Mathematical Formulations

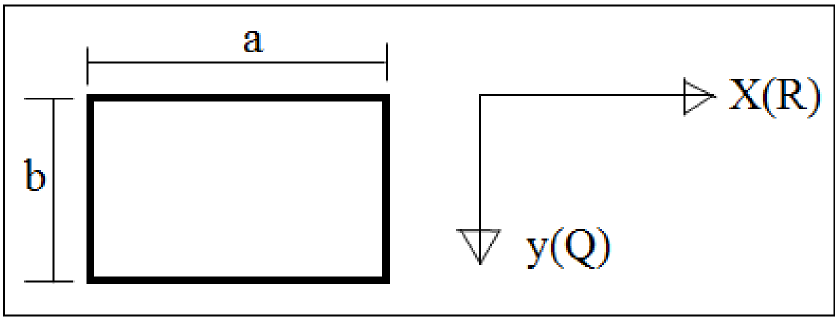

Fig-1: A Rectangular Plate

An isotropic rectangular thick plate of length 'a' in the horizontal axis, width 'b' in the vertical axis, and thickness 't' in the transverse axis is shown in Fig-1.

The displacement functions of the present theory were derived in [10] as;

$$u = -z\frac{\partial w}{\partial x} + f(z).\emptyset_x \text{ .... (2.1)}$$

$$v = -z\frac{\partial w}{\partial y} + f(z).\emptyset_y \text{ .... (2.2)}$$

$$w(x,y) = w_x.w_y = (a_0 + a_1R + a_2R^2 + a_3R^3 + a_4R^4) * (b_0 + b_1Q + b_2Q^2 + b_3Q^3 + b_4Q^4) \text{ .... (2.3a)}$$

Where;

u, v and w are the displacements in the x, y and z-directions respectively. *f(z)* is the shear deformation function $\emptyset_x$ *and* $\emptyset_y$ are the shear rotations in x and y-axis respectively given as;

$$\emptyset_x = C_x.\frac{\partial w}{\partial x}, \emptyset_y = C_y.\frac{\partial w}{\partial y} \text{ .... (2.3b)}$$

Where; C_x, C_y, a_i and b_i are constants.

R and Q are non-dimensional variables in x and y-axis respectively given as;

$$x = aR\ , y = bQ \text{ .... (2.3c)}$$

Such that; $0 \leq x \leq a, 0 \leq R \leq 1, 0 \leq y \leq b, 0 \leq Q \leq 1$

Kinematics

Using strain-displacement relationship of theory of elasticity, the shear strains are given as;

$$\varepsilon_x = \frac{du}{dx} = -z\frac{\partial^2 w}{\partial x^2} + f(z).\frac{\partial \emptyset_x}{\partial x} \text{ .... (2.4)}$$

$$\varepsilon_y = \frac{dv}{dy} = -z\frac{\partial^2 w}{\partial y^2} + f(z).\frac{\partial \emptyset_y}{\partial y} \text{ .... (2.5)}$$

$$\gamma_{xy} = \frac{du}{dy} + \frac{dv}{dx} = -2z\frac{\partial^2 w}{\partial x \partial y} + f(z).\frac{\partial \emptyset_x}{\partial y} + f(z).\frac{\partial \emptyset_y}{\partial x} \text{ .... (2.6)}$$

$$\gamma_{xz} = \frac{du}{dz} + \frac{dw}{dx} = f(z).\frac{\partial \emptyset_x}{\partial z} \text{ .... (2.7)}$$

$$\gamma_{yz} = \frac{dv}{dz} + \frac{dw}{dy} = f(z).\frac{\partial \emptyset_y}{\partial z} \text{ .... (2.8)}$$

The stress-strain relation of the isotropic plate can be written as follows;

$$\sigma_x = \frac{E}{1-\mu^2}\left[\varepsilon_x + \mu\varepsilon_y\right] \text{ .... (2.9)}$$

$$\sigma_y = \frac{E}{1-\mu^2}\left[\mu\varepsilon_x + \varepsilon_y\right] \text{ .... (2.10)}$$

$$\tau_{xy} = \frac{E(1-\mu)}{2(1-\mu^2)}\gamma_{xy} \text{ .... (2.11)}$$

$$\tau_{xz} = \frac{E(1-\mu)}{2(1-\mu^2)}\gamma_{xz} \ldots\ldots\ldots\ldots\ldots\ldots (2.12)$$

$$\tau_{yz} = \frac{E(1-\mu)}{2(1-\mu^2)}\gamma_{yz} \ldots\ldots\ldots\ldots\ldots\ldots (2.13)$$

E and μ respectively represent the elastic modulus and the Poisson's ratio of the material.

Strain Energy, Ψ.

The Strain energy stored in a continuum of the thick plate under study is defined as;

$$\Psi = \frac{1}{2}\int_x \int_y \left[\int_{-t/2}^{t/2} \left(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz}\right) dz\right] dx\, dy \ldots\ldots\ldots\ldots\ldots\ldots (2.14a)$$

Let;

$$\Psi_1 = \sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz} \ldots\ldots\ldots (2.14b)$$

Substituting Eqs. (2.5) - (2.13) into Eq. (2.14b) gives;

$$\Psi_1 = \frac{E}{1-\mu^2}\Bigg[\left[z^2\left(\frac{\partial^2 w}{\partial x^2}\right)^2 - 2zf(z).\frac{\partial \emptyset_x}{\partial x}\left(\frac{\partial^2 w}{\partial x^2}\right) + f^2(z).\left(\frac{\partial \emptyset_x}{\partial x}\right)^2\right] + \left[2z^2\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - 2Zf(z).\frac{\partial \emptyset_x}{\partial y}\frac{\partial^2 w}{\partial x \partial y} - 2Zf(z).\frac{\partial \emptyset_y}{\partial x}\frac{\partial^2 w}{\partial x \partial y}\right] + \left[z^2\left(\frac{\partial^2 w}{\partial y^2}\right)^2 - 2zf(z).\frac{\partial \emptyset_y}{\partial y}\frac{\partial^2 w}{\partial y^2} + f^2(z)\left(\frac{\partial \emptyset_y}{\partial y}\right)^2\right] + (1+\mu)\left[f^2(z).\frac{\partial \emptyset_y}{\partial x}\frac{\partial \emptyset_x}{\partial y}\right] + \frac{(1-\mu)}{2}\left[f^2(z)\left(\frac{\partial \emptyset_x}{\partial y}\right)^2 + f^2(z)\left(\frac{\partial \emptyset_y}{\partial x}\right)^2\right] + \frac{(1-\mu)}{2}.\left(\frac{df(z)}{dz}\right)^2.\left[\emptyset_x^2 + \emptyset_y^2\right]\Bigg] \ldots\ldots\ldots (2.15)$$

Let;

$$\Gamma = \int_{-t/2}^{t/2} z^2 dz = \frac{t^3}{12}, \Gamma H_1 = \int_{-t/2}^{t/2} z^2 dz\ , \Gamma H_2 = \int_{-t/2}^{t/2}[zf(z)]dz\ , \Gamma H_3 = \int_{-t/2}^{t/2}[(f(z))^2]dz\, , \Gamma\frac{\propto^2}{a^2}H_4 = \int_{-t/2}^{t/2}\left[\frac{df(z)}{dz}\right]^2 dz\, , D = \frac{\Gamma E}{1-\mu^2} = \frac{Et^3}{2(1-\mu^2)} . \ldots\ldots\ldots\ldots\ldots (2.16)$$

The shear deformation function $f(z)$ used in the present study is a third order polynomial function derived in [10] and presented here as;

$$f(z) = z - \frac{7z^3}{5t^2} \ldots\ldots\ldots\ldots\ldots\ldots (2.17)$$

Thus, from Eq. (2.17), Eqs. (2.18a) - (2.18e) are obtained;

$$\int_{-t/2}^{t/2}(z^2)dz = \left[\frac{z^3}{3}\right]_{-t/2}^{t/2} = \left(\frac{1}{3}\right)\left[\frac{t^3}{8} - -\frac{t^3}{8}\right] = 2\left(\frac{1}{3}\right).\left(\frac{t^3}{8}\right) = \frac{t^3}{12} \ldots\ldots\ldots\ldots\ldots\ldots (2.18a)$$

$$\left(f(z)\right)^2 = z^2 - \frac{14z^4}{5t^2} + \frac{49z^6}{25t^4}, \int_{-\frac{t}{2}}^{\frac{t}{2}}\left(f(z)\right)^2 dz = \left[\frac{z^3}{3} - \frac{14z^5}{25t^2} + \frac{7z^7}{25h^4}\right]_{-\frac{t}{2}}^{\frac{t}{2}} = \frac{253t^3}{4800} \ldots\ldots (2.18b)$$

$$zf(z) = z^2 - \frac{7z^4}{5t^2}\ , \int_{-\frac{t}{2}}^{\frac{t}{2}} zf(z)dz = \frac{t^3}{12} - \frac{7t^3}{400} = \frac{79t^3}{1200} \ldots\ldots\ldots\ldots\ldots (2.18c)$$

$$\left(\frac{df(z)}{dz}\right)^2 = 1 - \frac{42z^2}{5t^2} + \frac{441z^4}{25t^4}\ , \int_{-\frac{t}{2}}^{\frac{t}{2}}\left(\frac{df(z)}{dz}\right)^2 dz = \left[z - \frac{14z^3}{5t^2} + \frac{441z^5}{125t^4}\right]_{-\frac{t}{2}}^{\frac{t}{2}} = \frac{1041t}{2000} \ldots\ldots\ldots\ldots (2.18d)$$

Substituting Eq. (2.18a – 2.18d) into Eq. (2.16), Eq. (2.19a) - (2.19d) were obtained.

$$H_1 = 1, H_2 = 0.79\, , H_3 = 0.6325, H_4 = 6.246 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.19)$$

D is the flexural rigidity of the plate, and $\propto = {}^{a}/_{t}$ is the span-depth ratio.

Substituting Eq. (2.16) into Eq. (2.15) and integrating yields;

$$\int_{-\frac{t}{2}}^{\frac{t}{2}}(\boldsymbol{\Psi}_1)dz = \frac{\boldsymbol{E\Gamma}}{\boldsymbol{1-\mu^2}}\left[\left[H_1\left(\frac{\partial^2 w}{\partial x^2}\right)^2 - 2H_2.\frac{\partial \emptyset_x}{\partial x}\left(\frac{\partial^2 w}{\partial x^2}\right) + H_3.\left(\frac{\partial \emptyset_x}{\partial x}\right)^2\right] + \left[2H_1\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - 2H_2.\frac{\partial \emptyset_x}{\partial y}\frac{\partial^2 w}{\partial x \partial y} - 2H_2.\frac{\partial \emptyset_y}{\partial x}\frac{\partial^2 w}{\partial x \partial y}\right] + \left[H_1\left(\frac{\partial^2 w}{\partial y^2}\right)^2 - 2H_2\frac{\partial \emptyset_y}{\partial y}\frac{\partial^2 w}{\partial y^2} + H_3\left(\frac{\partial \emptyset_y}{\partial y}\right)^2\right] + (1+\mu)\left[H_3.\frac{\partial \emptyset_y}{\partial x}\frac{\partial \emptyset_x}{\partial y}\right] + \frac{(1-\mu)}{2}\left[H_3\left(\frac{\partial \emptyset_x}{\partial y}\right)^2 + H_3\left(\frac{\partial \emptyset_y}{\partial x}\right)^2\right] + \frac{(1-\mu)}{2}.\frac{\propto^2}{a^2}H_4\left[{\emptyset_x}^2 + {\emptyset_y}^2\right]\right] \ldots\ldots\ldots\ldots\ldots (2.20)$$

$$\boldsymbol{\Psi} = \frac{\boldsymbol{1}}{\boldsymbol{2}}\int_x\int_y\left[\int_{-t/2}^{t/2}(\boldsymbol{\Psi}_1)\,\boldsymbol{dz}\right]\boldsymbol{dx\,dy} = \frac{D}{2}\int_x\int_y\left[\left[H_1\left(\frac{\partial^2 w}{\partial x^2}\right)^2 - 2H_2.\frac{\partial \emptyset_x}{\partial x}\left(\frac{\partial^2 w}{\partial x^2}\right) + H_3.\left(\frac{\partial \emptyset_x}{\partial x}\right)^2\right] + \left[2H_1\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - 2H_2.\frac{\partial \emptyset_x}{y}\frac{\partial^2 w}{\partial x \partial y} - 2H_2.\frac{\partial \emptyset_y}{\partial x}\frac{\partial^2 w}{\partial x \partial y}\right] + \left[H_1\left(\frac{\partial^2 w}{\partial y^2}\right)^2 - 2H_2\frac{\partial \emptyset_y}{\partial y}\frac{\partial^2 w}{\partial y^2} + H_3\left(\frac{\partial \emptyset_y}{\partial y}\right)^2\right] + (1+\mu)\left[H_3.\frac{\partial \emptyset_y}{\partial x}\frac{\partial \emptyset_x}{\partial y}\right] + \frac{(1-\mu)}{2}\left[H_3\left(\frac{\partial \emptyset_x}{\partial y}\right)^2 + H_3\left(\frac{\partial \emptyset_y}{\partial x}\right)^2\right] + \frac{(1-\mu)}{2}.\frac{\propto^2}{a^2}H_4.\left[{\emptyset_x}^2 + {\emptyset_y}^2\right]\right]dxdy \ldots\ldots\ldots\ldots\ldots\ldots (2.21)$$

The average total work performed by perturbation on the plate is given by;

$$\Lambda = -\frac{m}{2}.\lambda^2\int_x\int_y (w^2)\,\partial x\partial y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.22)$$

Total Potential Energy

Total potential energy '$\mathcal{G}$', of the thick plate is the sum of strain energy, $\boldsymbol{\Psi}$ and external work, Λ.

$$\mathcal{G} = \boldsymbol{\Psi} + \Lambda = \frac{D}{2}\int_x\int_y\left[\left[H_1\left(\frac{\partial^2 w}{\partial x^2}\right)^2 - 2H_2.\frac{\partial \emptyset_x}{\partial x}\left(\frac{\partial^2 w}{\partial x^2}\right) + H_3.\left(\frac{\partial \emptyset_x}{\partial x}\right)^2\right] + \left[2H_1\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - 2H_2.\frac{\partial \emptyset_x}{\partial y}\frac{\partial^2 w}{\partial x \partial y} - 2H_2.\frac{\partial \emptyset_y}{\partial x}\frac{\partial^2 w}{\partial x \partial y}\right] + \left[H_1\left(\frac{\partial^2 w}{\partial y^2}\right)^2 - 2H_2\frac{\partial \emptyset_y}{\partial y}\frac{\partial^2 w}{\partial y^2} + H_3\left(\frac{\partial \emptyset_y}{\partial y}\right)^2\right] + (1+\mu)\left[H_3.\frac{\partial \emptyset_y}{\partial x}\frac{\partial \emptyset_x}{\partial y}\right] + \frac{(1-\mu)}{2}\left[H_3\left(\frac{\partial \emptyset_x}{\partial y}\right)^2 + H_3\left(\frac{\partial \emptyset_y}{\partial x}\right)^2\right] + \frac{(1-\mu)}{2}.\frac{\propto^2}{a^2}H_4\left[{\emptyset_x}^2 + {\emptyset_y}^2\right]\right]dxdy - \frac{m}{2}.\lambda^2\int_x\int_y (w^2)\,\partial x\partial y \ldots\ldots\ldots\ldots\ldots\ldots (2.23)$$

Let;

$$w = J_1 h \ldots\ldots\ldots\ldots\ldots\ldots (2.24a)$$

Substituting Eq. (2.24a) into Eq. (2.3b) gives;

$$\emptyset_x = C_a.J_1\frac{\partial h}{\partial x} = J_2\frac{\partial h}{\partial x},\ \emptyset_y = C_b.J_1\frac{\partial h}{\partial y} = J_3\frac{\partial h}{\partial y} \ldots\ldots\ldots\ldots (2.24b)$$

Where; $J_2 = C_a.J_1, J_3 = C_b.J_1, J_1, J_2, J_3$ are constants.

Substituting Eq. (2.24a) and (2.24b) into Eq. (2.23) and multiplying each term by $\frac{a^4}{a^4}$ gives;

$$\mathcal{G} = \frac{Dab}{2a^4}\int_0^1\int_0^1\left[\left({J_1}^2H_1 - 2J_1J_2H_2 + {J_2}^2H_3\right)\left(\frac{d^2h}{dR^2}\right)^2 + \left({J_1}^2H_1 - 2J_1J_3H_2 + {J_3}^2H_3\right)\frac{1}{P^4}\left(\frac{d^2h}{dQ^2}\right)^2 + \left(2{J_1}^2H_1 - 2J_1J_2H_2 - 2J_1J_3H_2\right).\frac{1}{P^2}\left(\frac{d^2h}{dR^2}.\frac{d^2h}{dQ^2}\right) + (1+\mu)J_2J_3H_3.\frac{1}{P^2}\left(\frac{d^2h}{dR^2}.\frac{d^2h}{dQ^2}\right) + \left[\left(\frac{1-\mu}{2}\right)\left({J_2}^2H_3 + {J_3}^2H_3\right)\right].\frac{1}{P^2}\left(\frac{d^2h}{dR^2}.\frac{d^2h}{dQ^2}\right) + \propto^2\left(\frac{1-\mu}{2}\right).\left({J_2}^2H_4\right)\left(\frac{dh}{dR}\right)^2 + \frac{\propto^2}{P^2}\left(\frac{1-\mu}{2}\right)\left({J_3}^2H_4\right)\left(\frac{dh}{dQ}\right)^2\right]dRdQ - \frac{ab{J_1}^2}{2}\int_0^1\int_0^1[m\lambda^2(h^2)]\partial R\partial Q \ldots\ldots\ldots\ldots\ldots\ldots (2.25)$$

Where the aspect ratio P, is given as; P = b/a.

General Governing Equation

Adopting Ritz method by minimizing the total energy equation, we obtain;

$$\frac{d\mathcal{G}}{dJ_1} = 0, \frac{d\mathcal{G}}{dJ_2} = 0, \frac{d\mathcal{G}}{dJ_3} = 0 \ldots\ldots\ldots\ldots\ldots (2.26)$$

Evaluating Eq. (2.26), Eq. (2.27a) - (2.27c) are obtained.

$$\frac{D}{a^4}\left[\left(T_1H_1+\frac{T_3}{P^4}H_1+\frac{2T_2}{P^2}H_1\right)J_1+\left(-T_1H_2+\frac{T_2}{P^2}H_2\right)J_2+\left(-\frac{T_3}{P^4}H_2-\frac{T_2}{P^2}H_2\right)J_3\right]=(m\lambda^2T_6)J_1 \text{ ……….. (2.27a)}$$

$$\frac{D}{a^4}\left[\left(-T_1H_2-\frac{T_2}{P^2}H_2\right)J_1+\left(T_1H_3+\left(\frac{1-\mu}{2P^2}\right)T_2H_3+\left(\frac{1-\mu}{2}\right)\propto^2 T_4H_4\right)J_2+\left(\left(\frac{1+\mu}{2P^2}\right)T_2H_3\right)J_3\right]=0 \text{ ……… (2.27b)}$$

$$\frac{D}{a^4}\left[\left(-\frac{T_3}{P^4}H_2-\frac{T_2}{P^2}H_2\right)J_1+\left(\left(\frac{1+\mu}{2P^2}\right)T_2H_3\right)J_2+\left(\frac{T_3}{P^4}H_3+\left(\frac{1-\mu}{2P^2}\right)T_2H_3+\left(\frac{1-\mu}{2P^2}\right)\propto^2 T_5H_4\right)J_3\right]=0 \text{ …….. (2.27c)}$$

Where;

$$T_1=\int_0^1\int_0^1\left(\frac{\partial^2h}{\partial R^2}\right)^2\partial R\partial Q\,,T_2=\int_0^1\int_0^1\left(\frac{\partial^2h}{\partial R^2}.\frac{\partial^2h}{\partial Q^2}\right)\partial R\partial Q\,,T_3=\int_0^1\int_0^1\left(\frac{\partial^2h}{\partial Q^2}\right)^2\partial R\partial Q\,,T_4=\int_0^1\int_0^1\left(\frac{\partial h}{\partial R}\right)^2\partial R\partial Q\,,T_5=\int_0^1\int_0^1\left(\frac{\partial h}{\partial Q}\right)^2\partial R\partial Q\,,T_6=\int_0^1\int_0^1(h)^2\partial R\partial Q \text{ ………….. (2.28)}$$

Re-writing Eq. (2.27a) - (2.27c) in matrix form, yields Eq. (2.29)

$$\begin{bmatrix}L_{11} & L_{12} & L_{13}\\ L_{21} & L_{22} & L_{23}\\ L_{31} & L_{32} & L_{33}\end{bmatrix}\begin{Bmatrix}J_1\\ J_2\\ J_3\end{Bmatrix}=\frac{a^4}{D}\begin{bmatrix}m\lambda^2T_6\\ 0\\ 0\end{bmatrix}\begin{Bmatrix}J_1\\ J_2\\ J_3\end{Bmatrix} \text{ ………………… (2.29)}$$

Eq. (2.29) is the general governing Equation for thick rectangular plates of arbitrary boundary conditions.

$$L_{11}=H_1\left(T_1+\frac{2T_2}{P^2}+\frac{T_3}{P^4}\right),L_{12}=-H_2\left(T_1+\frac{T_2}{P^2}\right),L_{13}=-H_2\left(\frac{T_2}{P^2}+\frac{T_3}{P^4}\right),L_{21}=L_{12},L_{22}=T_1H_3+\left(\frac{1-\mu}{2P^2}\right)T_2H_3+\left(\frac{1-\mu}{2}\right)\propto^2 K_4H_4,L_{23}=\left(\frac{1+\mu}{2P^2}\right)K_2H_3,\ L_{31}=L_{13},\ L_{32}=L_{23},L_{33}=\left(\frac{1-\mu}{2P^2}\right)T_2H_3+\frac{T_3}{P^4}H_3+\left(\frac{1-\mu}{2P^2}\right)\propto^2 T_5H_4 \text{ ………………. (2.30)}$$

Analytical Equation for Free-Vibration Analysis

The general governing equation derived and presented in Eq. (2.29) can be written as;

$$\begin{bmatrix}S_{11} & S_{12} & S_{13}\\ S_{21} & S_{22} & S_{23}\\ S_{31} & S_{32} & S_{33}\end{bmatrix}\begin{Bmatrix}J_1\\ J_2\\ J_3\end{Bmatrix}=\frac{ma^4\lambda^2}{D}\begin{bmatrix}1\\ 0\\ 0\end{bmatrix}\begin{Bmatrix}J_1\\ J_2\\ J_3\end{Bmatrix} \text{ ……………….. (3.1)}$$

Where;

$$S_{ij}=L_{ij}*{}^{1}\!/_{T_6} \text{ …………….. (3.2)}$$

Solving Eq. (3.1) using substitution method yields;

$$J_2=\left[\frac{-S_{23}.S_{31}+S_{33}.S_{21}}{{S_{32}}^2-S_{33}.S_{22}}\right]J_1,J_3=\left[\frac{-S_{23}.S_{21}+S_{22}.S_{31}}{{S_{32}}^2-S_{33}.S_{22}}\right]J_1 \text{ ……………………………………. (3.3)}$$

Substituting Eq. (3.3) into the first equation line in the matrix given in Eq. (3.1) gives;

$$S_{11}+S_{12}.\left[\frac{-S_{23}.S_{31}+S_{33}.S_{21}}{{S_{32}}^2-S_{33}.S_{22}}\right]+S_{13}.\left[\frac{-S_{23}.S_{21}+S_{22}.S_{31}}{{S_{32}}^2-S_{33}.S_{22}}\right]=\frac{ma^4\lambda^2}{D}=\Delta^2 \text{ …………………. (3.4)}$$

Where Δ^2 is a non – dimensional natural frequency parameter given as;

$$\Delta^2=\frac{ma^4\lambda^2}{D} \text{ …………. (3.5)}$$

Boundary Conditions

The general Polynomial Deflection Function '*w*' for thick plates is given in Eq. (2.3a). A close look at Eq. (2.3a) shows that it is a product of two orthogonal beams; one in x(R)-axis and the other in the y(Q)-axis. The deflection Equations for the two beams are given as Eqs. (3.6a) and (3.6b) respectively.

$$w_x=(a_0+a_1R+a_2R^2+a_3R^3+a_4R^4)\text{………….. (3.6a)}$$

$$w_y=(b_0+b_1Q+b_2Q^2+b_3Q^3+b_4Q^4) \text{ ……………. (3.6b)}$$

For the beam simply supported at both edges, the boundary conditions are as;

$$\text{(i)} At\ R\ =\ 0;\ w_x=0,\frac{\partial^2w_x}{\partial R^2}=0 \text{ (ii) } At\ R\ =\ 1;\ w_x=0,\frac{\partial^2w_x}{\partial R^2}=0 \text{ ………….. (3.7)}$$

Upon satisfaction of the conditions in Eq. (3.7), the following equation is obtained;

$$w=w_x.w_y=a_4(R-2R^3+R^4).b_4(Q-2Q^3+Q^4) \text{ ………………… (3.8)}$$

$$w=J_1h=J_1(R-2R^3+R^4).(Q-2Q^3+Q^4) \text{ ……………………….. (3.9)}$$

Where;

$$J_1 = a_4 b_4 \,, h = (R - 2R^3 + R^4).(Q - 2Q^3 + Q^4) \text{ .... } (3.10)$$

H is the shape function for the SSSS thick plate, J_1 is the amplitude.
Substituting Eq. (3.10) into Eq. (2.28) gives;

$$T_1 = 0.23619; \; T_2 = 0.23592 \,; \; T_3 = 0.23619; \; T_4 = 0.0239; \; T_5 = 0.0239 \text{ .... } (3.11)$$

NUMERICAL RESULTS AND DISCUSSION

Eq. (3.4) is a simple linear equation used to generate the fundamental natural frequency parameters for the plate at different aspect ratios, P= b/a and different span-depth ratios (a/t). Eq. (3.11) provides the stiffness 'T_i' for SSSS thick plate. The values of the non-dimensional fundamental natural frequency parameter Δ at different values of span-depth ratios and aspect ratios (b/a) generated from Eq. (3.4) are presented on Table-1. Tables 2 and 3 show the comparison of the results of the present study with the results of previous scholars. The graphs of the non-dimensional fundamental natural frequency parameter Δ against the span-depth ratio of SSSS thick rectangular plate for the present study and previous researchers are presented in Figs 1 and 2.

Table-1: Non-dimensional fundamental natural frequency parameter, Δ

$\propto = {}^a/_t$	$^b/_a = 1.0$	$^b/_a = 1.2$	$^b/_a = 1.4$	$^b/_a = 1.6$	$^b/_a = 1.8$	$^b/_a = 2.0$	$^b/_a = 2.2$	$^b/_a = 2.4$
	$\lambda = \frac{\Delta}{a^2}\sqrt{\frac{D}{m}}$							
	Δ							
5	17.8439	15.3339	13.7871	12.7692	12.0648	11.5576	11.1806	10.8928
10	19.2148	16.3460	14.6052	13.4708	12.6911	12.1323	11.7184	11.4032
15	19.5059	16.5569	14.7736	13.6142	12.8184	12.2487	11.8269	11.5060
20	19.6110	16.6327	14.8340	13.6655	12.8638	12.2902	11.8656	11.5427
25	19.6602	16.6681	14.8622	13.6894	12.8851	12.3096	11.8837	11.5597
30	19.6872	16.6875	14.8776	13.7025	12.8966	12.3201	11.8935	11.5691
40	19.7140	16.7068	14.8930	13.7155	12.9082	12.3307	11.9033	11.5783
50	19.7265	16.7158	14.9001	13.7215	12.9135	12.3355	11.9079	11.5826
60	19.7333	16.7206	14.9039	13.7248	12.9164	12.3382	11.9104	11.5850
70	19.7374	16.7236	14.9063	13.7268	12.9182	12.3398	11.9119	11.5864
80	19.7400	16.7255	14.9078	13.7281	12.9193	12.3408	11.9128	11.5873
90	19.7419	16.7268	14.9088	13.7290	12.9201	12.3415	11.9135	11.5879
100	19.7432	16.7277	14.9096	13.7296	12.9207	12.3421	11.9140	11.5884

Table-2: Present Study Compared with Hashemi and Arsanjani, (2004) [8] for SSSS thick Plates

$P = {}^b/_a$	$\propto = {}^a/_t$	Present Study, (P.S).	Hashemi and Arsanjani, (2004) [8]	% Difference.= $\frac{(P.S - H.A) * 100}{P.S}$
		$\lambda = \frac{\Delta}{a^2}\sqrt{\frac{D}{m}}$		
		Δ		
1	100.00	19.7432	19.7322	0.06
	20.00	19.6110	19.5676	0.22
	10.00	19.2148	19.084	0.68
	6.67	18.6065	18.3661	1.29
	5.00	17.8439	17.5055	1.90
1.5	100.00	14.2607	14.2525	0.06
	20.00	14.1915	14.1662	0.18
	10.00	13.9818	13.9085	0.52
	6.67	13.653	13.5147	1.01
	5.00	13.2287	13.025	1.54
2	100.00	12.3421	12.3343	0.06
	20.00	12.2902	12.2696	0.17
	10.00	12.1323	12.0752	0.47
	6.67	11.8829	11.7747	0.91
	5.00	11.5576	11.3961	1.40
2.5	100.00	11.4541	11.4464	0.07
	20.00	11.4094	11.3906	0.16
	10.00	11.2731	11.226	0.42
	6.67	11.057	10.9617	0.86
	5.00	10.7738	10.6307	1.33

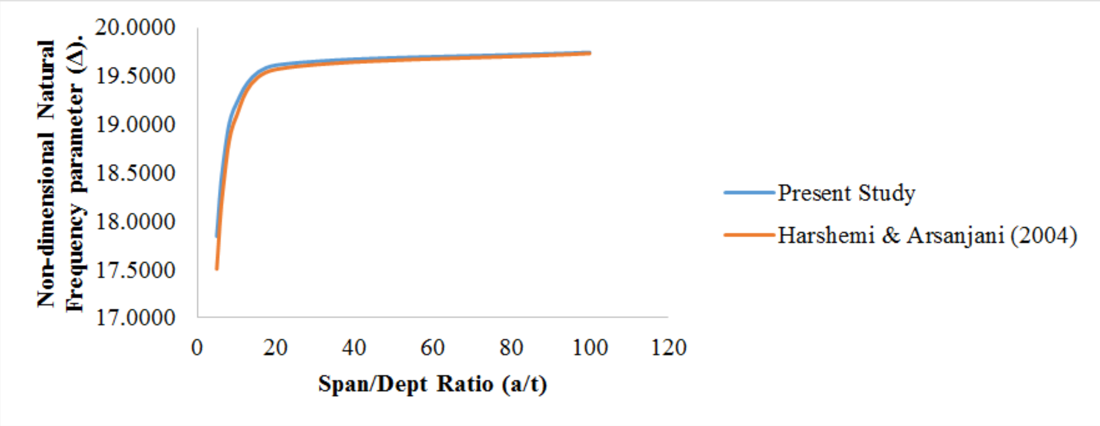

Fig-2: Present Study result compared with the Results of Harshemi & Arsanjani (2004) [8]

Table-3: Present Study Compared with Rajesh and Meera (2016) [9] for SSSS thick Plates, at P=b/a = 2

$\propto = {}^{a}/_{t}$	Present Study (P.S)	Rajesh & Meera (2016). (R.M)	% Difference. = $\frac{(P.S - R.M) * 100}{P.S}$
	$\lambda = \frac{\Delta}{a^2}\sqrt{\frac{D}{m}}$		
	Δ		
5	11.5576	11.3716	1.61
7	11.9232	11.8087	0.96
9	12.0842	12.0073	0.64
10	12.1323	12.0675	0.53
11	12.1683	12.1127	0.46
13	12.2175	12.1749	0.35
15	12.2487	12.2145	0.28
20	12.2902	12.2675	0.18
25	12.3096	12.2924	0.14
30	12.3201	12.306	0.11
40	12.3307	12.3195	0.09
50	12.3355	12.3258	0.08
60	12.3382	12.3292	0.07
70	12.3398	12.3313	0.07
80	12.3408	12.3326	0.07
90	12.3415	12.3335	0.06
100	12.3421	12.3342	0.06

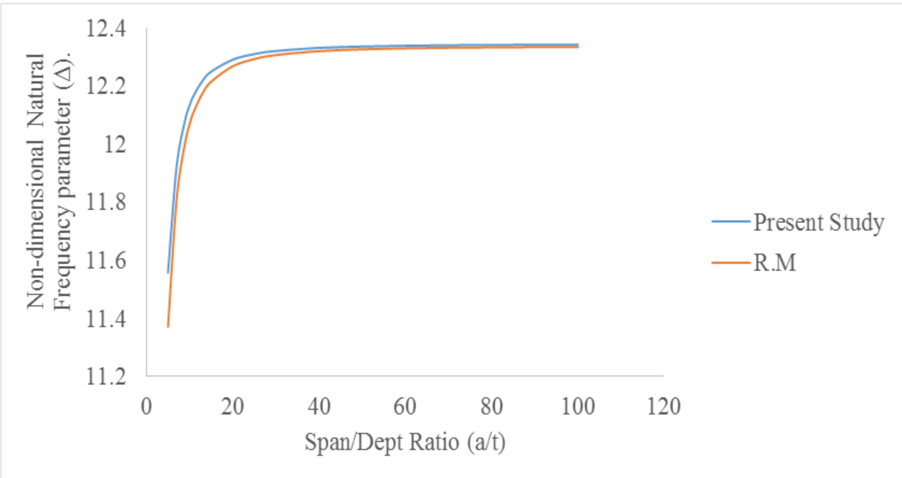

Fig-3: Present Study result compared with the Results of Rajesh and Meera (2016) [9] for aspect ratio, P = 2.0

Looking at Table-2, at the same value of '*P*', there is a decrease in the value of the non-dimensional natural frequency parameter as 'a/t' decreases. Also at any value of 'a/t', there is a decrease in the value of the non-dimensional natural frequency parameter **Δ** as '*P*' increases, having the highest value at *P* = 1 (square plate). Comparison between present study and the results of Hashemi and Arsanjani [8], shows a percentage difference range of (0.06 to 1.90). Figure-1 showed that the two results are close to each other. This confirms the efficacy of the present study.

It was observed from Table-3 at the same value of '*P*', there is an increase in the value of the non-dimensional natural frequency parameter **Δ** as 'a/t' increases. Comparing the present study with the results of Rajesh *et al*., [9], shows a percentage difference range of (0.06 to 1.90). These differences are quite negligible and acceptable in statistics as being close. Also, Figure-2 showed that the two results are close to each other which attest to the efficacy of the present study.

CONCLUSIONS

The governing equation and the simple linear equation derived in this work produces speedy and acceptable results for free-vibration analysis of isotropic thick rectangular plates with simple supports at the four edges. The non-dimensional natural frequency parameters obtained in this work are close to the results of previous researchers and therefore are very reliable.

REFERENCES

1. Gorman, D. J. (1982). Free Vibration Analysis of Rectangular Plates. Elsevier North Holland Inc, USA.
2. Ezeh, J. C., Ibearugbulem, O. M., & Onyechere I. C (2013). Pure bending analysis of thin rectangular flat plates using ordinary finite difference method. *International Journal of Emerging Technology and Advanced Engineering*. 3(3), 20-23.
3. Ventsel, E., & Krauthammer K. (2001). Thin Plates and Shells. New York, Marcel Decker Inc.
4. Sayyad, A. S., & Ghugal Y. M. (2012). Bending and free vibration analysis of thick isotropic plates by using exponential shear deformation theory. *Journal of Applied and Computational Mechanics*. 6**,** 65–82.
5. Sayyad, I. I., Chikalthankar, S. B., & Nandedkar, V. M. (2013). Bending and free vibration analysis of isotropic thick plate using refined plate theory. *Bonfring International Journal of Industrial Engineering and Management Science*. 3(2), 40-46.
6. Sayyad, A. S. (2011). Comparison of various shear deformation theories for the free-vibration of thick isotropic beam. International *Journal of Civil and Structural Engineering*. 2(1), 85-97.
7. Sadrnejad, S. A., Daryan, A. S., & Ziaei, M (2009). Vibration Equations of Thick Rectangular Plates Using Mindlin Plate Theory. *Journal of Computer Science*, 5(11), 838-842.
8. Hashemi, S. H., & Arsanjani, M. (2004). Exact characteristic equations for some of classical boundary conditions of vibrating moderately thick rectangular plates. *International Journal of Solid and Structures*. 42, 819-853.
9. Rajesh, K., & Meera, S. K. (2016). Linear free vibration analysis of rectangular Mindlin plates using coupled displacement field method. *Mathematical Models in Engineering*. 1(1), 41-47.
10. Onyechere, I. C. (2019). Stability and Vibration Analysis of Thick Plates using Orthogonal Polynomial Displacement Functions. PhD Thesis. Department of Civil Engineering, Federal University of Technology, Owerri (FUTO), Nigeria.