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Stability Analysis of Two Maxwell Fluid Layers in Horizontal Magnetization

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Original Research Article

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Abstract: This paper investigates the linear stability of two superposed Maxwell viscoelastic fluid layers in the presence of a uniform horizontal magnetic field. A normal-mode analysis is applied to the governing MHD equations to derive the dispersion relation for interfacial disturbances. The results show that the viscosity and permeability of porous medium both have destabilizing influence on the growth rate of unstable mode of disturbance.

Keywords: Viscoelastic fluid, Maxwell fluid, Non-Newtonian fluid, Viscosity, Superposed fluid layers, Interfacial disturbances.

1. INTRODUCTION

Instability at the interface between two incompressible, inviscid, and perfectly conducting fluids of unequal densities, where the lighter accelerates into the heavier, has been widely investigated. A comprehensive treatment was given by Chandrasekhar [1]. Robert [2] later extended the analysis by incorporating finite kinematic viscosity and magnetic resistivity, assuming these parameters to be constant and identical in both fluids. Jukes [3] examined the Rayleigh–Taylor instability in magnetohydrodynamics (MHD) with finite conductivity and found that finite resistivity introduces additional modes of instability.

For more realistic astrophysical and geophysical applications, viscous effects have been considered by several researchers. Bhatia and Chhonkar [4] studied the instability of two superposed viscous conducting fluids, while Hooper and Grimshaw [5] investigated its nonlinear behavior. D'Anglo and Song [6-7] analyzed the Kelvin–Helmholtz instability in dusty plasma, and Gupta and Bhatia [8] examined partially ionized viscous plasma layers in a horizontal magnetic field. Srivastava and Khare [9], Daval Osorozco [10], and Elgowaing and Ashgriz [11] further explored related configurations involving vertical fields, rotation, and viscous layers. Bhatia and Sharma [12] extended the study to viscous fluids in a porous medium under a uniform vertical magnetic field.

Following Wooding [13], who first analyzed thermal flow instability through porous media, subsequent works by Rudraiah and Srimani [14], and Sharma and Rani [15] addressed the role of permeability in instability phenomena, motivated by applications in geophysics and petroleum recovery. The significance of porosity in astrophysical contexts has also been highlighted by McDonnel [16].

Further investigations include Kumar and Singh [17] on MHD Hele–Shaw flow of elastico-viscous fluids and Yadav and Ray [18] on unsteady motion of *n*-immiscible viscoelastic fluids through porous channels. Ali and Bhatia [19] examined Rayleigh–Taylor instability in a stratified conducting fluid through porous media under a two-dimensional magnetic field, while El Sayeed [20], analyzed related electrohydrodynamic and viscoelastic flow problems. The stability of two superposed Walters' B' viscoelastic fluids is considered in porous medium by S.K. Kango [21].

Given the practical importance of viscoelastic fluids, the present study focuses on the stability of two superposed viscoelastic Maxwellian fluids through a porous medium in a uniform two-dimensional horizontal magnetic field, extending earlier work by Sharma and Kumar [22] for a one-dimensional field.

2. Perturbation Equations:

We consider the motion of an incompressible, infinitely conducting viscoelastic fluid (of variable viscosity μ(z)) through a porous medium. The fluid is assumed to be immersed in a uniform two dimensional horizontal magnetic field $H = (H_x, H_y, 0)$

The constitutive equations for Oldroyd viscoelastic fluid are given by

$$T_{ij} = -p\delta_{ij} + \tau_{ij} \qquad \ldots(1)$$

and

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\tau_{ij} = 2\mu e_{ij} \qquad \ldots(2)$$

where

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad \ldots(3)$$

Here e_{ij} is the rate-of-strain tensor, τ_{ij} is the shear stress tensor, δ_{ij}. is Kronecker tensor, μ is the coefficient of viscosity, p is the pressure and λ is stress relaxation time respectively.

The equations of motion of an electrically conducting viscoelastic Maxwellian fluid through a porous medium in a uniform magnetic field are

$$\frac{\rho}{\varepsilon}\frac{Du_i}{Dt} = -g\rho\lambda_i + \frac{\partial T_{ij}}{\partial x_j}\mu_e \varepsilon_{ijk}\varepsilon_{jlm}\frac{\partial H_{l_n}}{x_j}H_k - \frac{\mu}{k_1}u_i, \qquad \ldots(4)$$

where $\frac{D}{dt} = \frac{\partial}{\partial t} + u_j\frac{\partial}{\partial x}$

is the mobile operator, H_1 is magnetic field, ρ is the density of the fluid, *g* is acceleration due to gravity, k_1 is the permeability of the porous medium, ε is the medium porosity, u_1. is the Darcian (filter) velocity of the fluid and $\lambda_i = (0, 0, 1)$ is a unit vector along the vertical.

The hydromagnetic equations of motion of an Maxwellian viscoelastic fluid through a porous medium become, on using the constitutive equation (1) for the viscoelastic fluid in conjunction with relations (2) and (3), become

$$\frac{\rho}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{Du}{Dt} = \left(1+\lambda\frac{\partial}{\partial t}\right)\left[-\nabla p + \mu_e(\nabla\times H)\times H + q\rho\right]$$
$$+\left[\frac{\mu}{\varepsilon}\nabla^2 u + \frac{1}{\varepsilon}\{(\nabla u).\nabla\mu + (\nabla u).\nabla\mu\} - \frac{\mu}{k_1}u\right] \qquad \ldots(5)$$

where $g = (0, 0, -g)$. The relevant equations governing the motion of an incompressible ideally conducting Maxwellian viscoelastic fluid through a porous medium in a uniform magnetic field are, therefore, eq. (5) and

$$\varepsilon\frac{\partial\rho}{\partial t} + (u.\nabla)\rho = 0 \qquad \ldots(6)$$

$$\varepsilon\frac{\partial H}{\partial t} = \nabla\times(u\times H) \qquad \ldots(7)$$

and

$$\nabla.u = 0, \nabla.H = 0 \qquad \ldots(8)$$

Let $\delta\rho$, δp and $h = (h_x, h_y, h_z)$ denote respectively the perturbation in density ρ, pressure p and the magnetic field H due to small disturbance given to the system which produces the velocity field $u = (u, v, w)$ in the system. Retaining only the linear terms in the perturbed quantities, we obtain the linearized perturbation equations

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\rho}{\varepsilon}\frac{\partial u}{\partial t}=\left(1+\lambda\frac{\partial}{\partial t}\right)\left[-\nabla\delta p+g\delta\rho+(\nabla\times h)\times H\right]$$

$$+\left[\frac{\mu}{\varepsilon}\nabla^2 u+\frac{1}{\varepsilon}\{(\nabla u.\nabla)u+\nabla u.\nabla\mu\}-\frac{\mu}{k_1}u\right] \qquad \ldots (9)$$

$$\varepsilon\frac{\partial h}{\partial t}=\nabla\times(u\times H) \qquad \ldots(10)$$

$$\varepsilon\frac{\partial}{\partial t}\delta\rho=-(u.\nabla)\rho \qquad \ldots(11)$$

and $\nabla.u=0,\ and\ \nabla.H=0$ …(12)

Analysing the disturbances in terms of normal modes we assume that the perturbed quantities vary in space *(x, y, z)* and time *(t)* as
F(z) exp [ik_x x + ik_y y + nt], ... (13)

where F(z) is some function of z, k_x and k_y are the horizontal wave numbers $\left(k^2=k_x^2+k_y^2\right)$ and n (may be complex) denotes the rate at which the system departs away from equilibrium.

On using expression (13) in eqs. (9) to (12) and eliminating some of the variables we finally obtain an equation in w, writing $D=\frac{d}{dz}$

$$\frac{n}{\varepsilon}\left[k^2\rho w-D\{\rho(Dw)\}\right]-\frac{gk^2(D\rho)q}{n\varepsilon}-\frac{1}{n\varepsilon}\left(k_xH_x+k_yH_y\right)^2\left(D^2-k^2\right)w$$

$$+\left(\frac{1}{1+\lambda n}\right)\left[\frac{\mu}{\varepsilon}\left(D^2-k^2\right)^2 w+\frac{2}{\varepsilon}\left(D^2-k^2\right)Dw(D\lambda)\right.$$

$$\left.+\frac{1}{\varepsilon}D^2\mu\left(D^2+k^2\right)w+\frac{1}{k_1}\{\mu wk^2-D(\mu Dw)\}\right]=0 \qquad \ldots (14)$$

3. TWO VISCOELASTIC SUPERPOSED FLUIDS OF UNIFORM DENSITIES

We now consider the case when two superposed conducting viscoelastic fluids of uniform densities ρ_j and ρ_2 and uniform viscosities μ_1 and μ_2 occupy the regions z < 0 and z > 0 and are separated by a horizontal boundary at z = 0. For both the fluids, eq. (14) reduces to
(D^2-k^2) (D^2-M^2) w = 0, …(15)

Where

$$M^2=k^2+(1+\lambda n)\left[\frac{\varepsilon}{k_1}+\frac{n}{v}\left\{1+\frac{1}{n^2\rho}\left(k_xH_x+k_yH_y\right)^2\right\}\right] \qquad \ldots (16)$$

and $v=\frac{\mu}{\rho}$ is the coefficient of kinematic viscosity.

Since w must vanish both when z→ -∞ (in the lower fluid) and z→ +∞ (in the upper fluid), the solutions of eq. (15) appropriate to the two regions are
$w_1 = P_1$ exp (+kz) +Q_1 exp (M_1z), (z < 0). …(17)

and $w_2 = P_2$ exp (-kz) + Q_2 exp (-M_2z), (z < 0). …(18)

where P_1, Q_1, P_2, Q_2 are constants and M_1, M_2 are the positive square roots of (16) for the two regions. In writing the solutions (17) and (18) it is assumed that M_1 and M_2 are so defined that either real parts are positive.

For determining the four constants P_1, P_2, Q_1, Q_2 we require the four boundary conditions. The three conditions require continuity of
W, Dw and $\mu(D^2 + k^2)$ w ... (19, 20 & 21)

across the interface z = 0. Integrating eq. (14), we obtain the fourth condition

$$\left[\left\{\rho_2 - \frac{\mu_2}{n(1+n\lambda)}\left(D^2-k^2\right) + \frac{\mu_2\varepsilon}{nk_1(1+n\lambda)} + \frac{1}{n^2}(k.H)^2\right\}Dw_2\right]_{z=0}$$
$$-\left[\left\{\rho_1 - \frac{\mu_1}{n(1+n\lambda)}\left(D^2-k^2\right) + \frac{\mu_1\varepsilon}{nk_1(1+n\lambda)} + \frac{1}{n^2}(k.H)^2\right\}Dw_1\right]_{z=0}$$
$$= -\frac{k^2}{n^2}g(\rho_2-\rho_1)w_0 - \frac{2k^2}{n}(\mu_2-\mu_1)(Dw)_0 \qquad \ldots(22)$$

where w_0 and $(Dw)_0$ are the unique values of these quantities at z = 0.

On applying conditions (19) to (22) to the solution (17)-(18), we get

$$P_1 + Q_1 = P_2 + Q_2 \qquad \ldots(23)$$
$$kP_1 + M_1 Q_1 = -kP_2 - M_2Q_2 \qquad \ldots(24)$$
$$\mu_1\left\{2k^2P_1 + \left(M_1^2+k^2\right)Q_1\right\} = \mu_2\left\{2k^2P_2 + \left(M_2^2+k^2\right)Q_2\right\} \qquad \ldots(25)$$
$$\left[\left\{\rho_2 + \frac{\mu_2\varepsilon}{nk_1(1+n\lambda)} + \frac{1}{n^2}(k.H)^2\right\}(P_2k + Q_2M_2) - \frac{\mu_2}{n(1+n\lambda)}Q_2M_2\left(M_2^2-k^2\right)\right] -$$
$$-\left[\left\{\rho_1 + \frac{\mu_1\varepsilon}{nk_1(1+n\lambda)} + \frac{1}{n^2}(k.H)^2\right\}(P_1k + Q_1M_1) - \frac{\mu_1}{n(1+n\lambda)}Q_1M_1\left(M_1^2-k^2\right)\right]$$
$$= \frac{gk^2}{2n^2}(\rho_2-\rho_1)(P_1+Q_1+P_2+Q_2) + \frac{k^2}{n}(\mu_1-\mu_2)(kp_1 + M_1Q_1 - kP_2 - M_2Q_2) \qquad \ldots(26)$$

Eliminating constants P_1, P_2, Q_1, Q_2 from eqs. (23)-(26), we obtain the characteristic equations

$$(M_1-k)\left[(R-1)\left\{\alpha_2 n + \frac{(k.V_A)^2}{n}\right\} + 2k^2(\alpha_1\nu_1 - \alpha_2\nu_2)\right.$$
$$\left.\left\{\alpha_2 + \frac{C}{k}(M_2-k) + \frac{M_2}{n^2k}(k.V_A)^2\right\}\right] - 2k\left[\left\{\alpha_1 n + \frac{(k.V_A)^2}{n}\right\}\right.$$
$$\left.\left\{\alpha_2 + \frac{C}{k}(M_2-k) + \frac{M_2}{n^2k}(k.V_A)^2\right\}\right] + \left\{\alpha_2 n + \frac{(k.V_A)^2}{n}\right\}$$
$$\left.\left\{\alpha_1 + \frac{C}{k}(M_1-k) + \frac{M_1}{n^2k}(k.V_A)^2\right\}\right]$$
$$+(M_2-k)\left[(R-1)\left\{\alpha_1 n + \frac{(k.V_A)^2}{n}\right\} - 2k^2(\alpha_1\nu_1 - \alpha_2\nu_2)\right.$$
$$\left.\left\{\alpha_1 + \frac{C}{k}(M_1-k) + \frac{M_1}{n^2k}(k.V_A)^2\right\}\right] \qquad \ldots(27)$$

where

$$\alpha_1 = \frac{\rho_1}{\rho_1+\rho_2}, \alpha_2 = \frac{\rho_2}{\rho_2+\rho_2}, (\alpha_1+\alpha_2=1), \qquad \ldots(28)$$

$$R = -\frac{gk}{n^2}(\alpha_1-\alpha_2), \qquad \ldots(29)$$

$$C = \frac{k^2}{n}\left(\frac{\mu_2-\mu_1}{\rho_1+\rho_1}\right) = -\frac{k^2}{n}(\alpha_1 v_1 - \alpha_2 v_2) \qquad \ldots(30)$$

and $$(k.V_A)^2 = \frac{(k.H)^2}{\rho_1+\rho_2} = \frac{(k_x H_x + k_y H_y)^2}{\rho_1+\rho_2} \qquad \ldots(31)$$

V_A is the Alfven velocity vector.

The dispersion relation (27) is quite complex, particularly as M_1 and M_2 involve square roots. We, therefore, carry out the stability analysis for highly viscous, conducting superposed fluids. Then we can write.

$$M_1 = k + (1+n\lambda)\left[\frac{\varepsilon}{2kk_1} + \frac{n}{2kv_1} + \frac{(k.V_A)^2}{2nkv_1\alpha_1}\right] \qquad \ldots(32)$$

$$M_2 = k + (1+n\lambda)\left[\frac{\varepsilon}{2kk_1} + \frac{n}{2kv_2} + \frac{(k.V_A)^2}{2nkv_2\alpha_2}\right] \qquad \ldots(33)$$

neglecting square and higher order terms in $\frac{1}{v_{1,2}}$

Substituting the values of M_1 and M_2 in eq. (27) and putting $\nu_1 = \nu_2 = \nu$ (the case of equal kinematic viscosities), we obtain the dispersion relation in the dimensionless form as

$$\sum_{i=0}^{7} B_i Y^i = 0 \qquad \ldots(34)$$

Here V_x and V_y are the Alfven velocities in x and y directions and θ is the angle between k and H_x. The coefficients B_i's are not given here because they are very lengthy expression.

4. CONCLUSION

The dispersion relation (34) is quite complex. In order to study the effects of various physical parameters, we have therefore performed the numerical calculations of eq. (34) to locate the roots of Y (positive real part) against wave number x, for several values of parameters. The numerical calculations are presented in Table I to II, where we have taken $V_1 = V_2 = 0.5$ and $\theta = 45°$.

From Fig. 1 we see that as N (viscosity) increases, growth rate Y increases showing destabilizing character of viscosity.

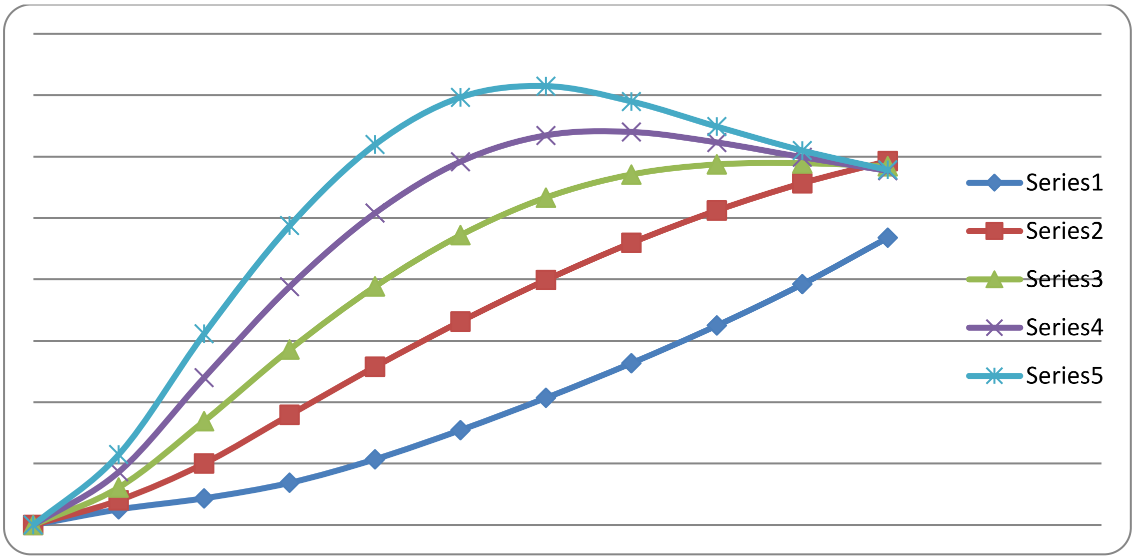

Fig. 1:
Values of growth rate (real positive value of Y) against wave number *x for N* = 1.0, 2.0, 3.0, 4.0 and 5.0 respectively for series 1,2,3,4,5 when $V_1 = V_2 = 0.5$, $P = 1.0$, $T = 1.0$ and □ = 45°

Fig. 2 shows that as *P* (permeability) increases, the growth rate *Y* increases showing thereby destabilizing influence of permeability of the porous medium on the unstable mode of disturbance.

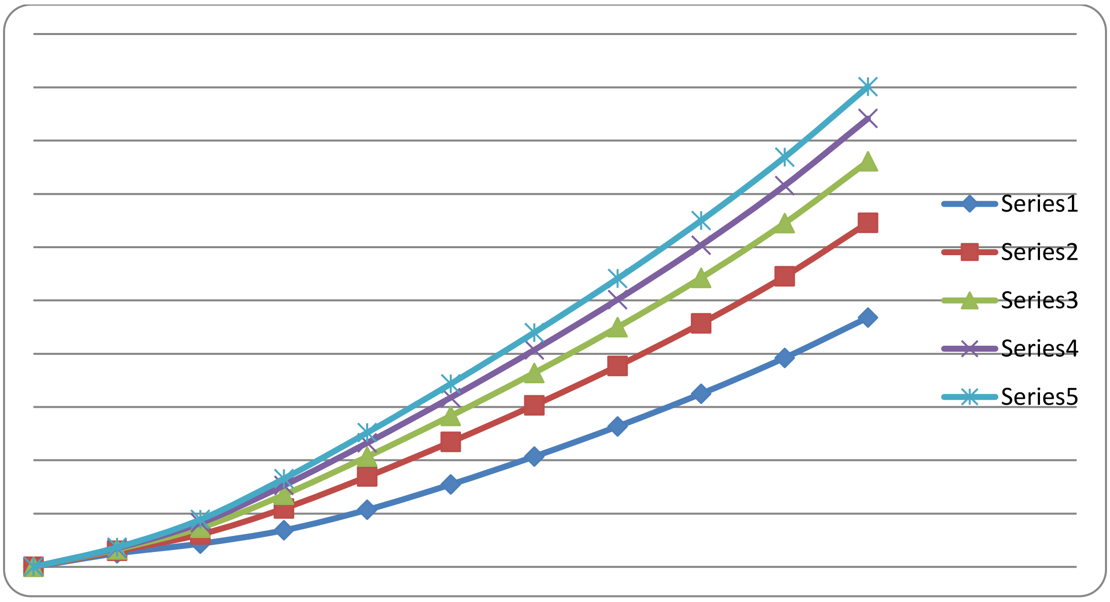

Fig. 2:
Values of growth rate (real positive value of Y) against wave number *x* for *p* = 1.0, 2.0, 3.0, 4.0 and 5.0 respectively for series 1,2,3,4,5 when $V_1 = V_2 = 0.5$, T = 1.0, $N = 1.0$ and □ = 45°

Thus we may conclude that permeability of porous medium and viscosity have destabilizing influence on the Rayleigh-Taylor instability of superposed viscoelastic fluids.

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