

Impact of Variable Thermal Conductivity on Unsteady Flow of a Magnetized Exothermic Fluid Across a Porous Microchannel

Godwin Ojmeri^{1*}, Mohammed Maigemu Dago¹, Abdulsalam Shuaibu¹

¹Department of Mathematics, College of Science, Federal University of Agriculture, Zuru, P. M. B. 28, Kebbi State, Nigeria

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*Corresponding author: Godwin Ojmeri

Department of Mathematics, College of Science, Federal University of Agriculture, Zuru, P. M. B. 28, Kebbi State, Nigeria

Abstract

This work presents an unsteady analysis of the hydromagnetic flow of an exothermic fluid with thermal characteristics through a microchannel. Following the discretization of the partial differential equations, the modelled time-dependent governing equations are solved using the implicit finite-difference technique (IFDM). Graphs depicting the influence of key parameters are created, and the findings are thoroughly described. As the temperature rises due to the exothermic process, the buoyant force frequently overcomes the Darcy resistance. This creates a rise in fluid velocity, especially if the medium is highly permeable. The interconnecting ligaments (pores) significantly improve the fluid-solid contact area over a simple microchannel. Furthermore, it is discovered that varying thermal conductivity has a substantial impact on temperature and hydromagnetic fluid in the microchannel. This study's findings will benefit applications in biomedical and chemical engineering, including catalytic packed-bed reactors, bioreactors, and waste treatment. In these systems, the porous structure functions as both a flow regulator and a thermal stabilizer. Bio-microfluidic systems and heat management in microelectronics, where properties vary fast with temperature, can all benefit from the findings of this study.

Keywords: Unsteady Flow, Exothermic Fluid, Variable Thermal Conductivity, Darcy Porous Medium, Superhydrophobic Surface (SHS), Implicit Finite Difference Method (IFDM).

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1.0 INTRODUCTION

Porous material is a solid substance that exists in the form of a dense bed of separate solids or foam and has extremely permeable interconnecting pores. Interconnected solid pores are typically filled with a fluid in either a liquid or gaseous phase. Natural porous media include biological stuff, rocks, and sand. Other examples include stone and sand layers in oil and gas reservoirs, as well as sand in groundwater (Tripathy *et al.*, 2015). Because of these numerous applications, many researchers have been inspired to investigate the properties of porous media in response to a variety of events. Recently, Li *et al.*, (2026) investigated the unsteady magnetohydrodynamic (MHD) flow and heat transfer of fractional viscoelastic nanofluids across an infinite vertical plate in a porous medium. Both ramping and isothermal wall temperatures were considered, as well as the effects of heat injection and suction. Numerical studies reveal that increased porous medium permeability promotes fluid flow, whereas a greater magnetic field inhibits it. Bordoloi *et al.*, (2023) offered

an analytical solution for two-dimensional steady, viscous, and incompressible hydromagnetic free convective flow through a symmetrically extended upright porous plate immersed in a porous medium, where thermo-diffusion and chemical reaction coexist. Kadeethum *et al.*, (2022) demonstrated natural convection in porous media for highly nonlinear metaphysical engineering applications. Gireesha and Sindhu (2020) investigated the MHD convection flow of Casson fluid in microchannels with porous materials. Alam *et al.*, (2021) used network simulation procedures to quantitatively assess MHD radiative fluid flow in an inclined porous plate with heat and mass permeability. They noticed that fluid momentum tended to rise or decrease slowly away from the plate after progressively rising or dropping toward it. There have been reports that the fluid's temperature follows the same trend. Abbas *et al.*, (2022) used porous media to explore several industrial operations, including heating and cooling electrical systems. Ajibade *et al.*, (2023) investigated the effect of viscosity and Darcy dissipation on a steady,

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fully developed free convection flow in an interconnected channel partially saturated with porous media due to heat generation using a homotopy perturbation approach. Hamza *et al.*, (2022) studied the unsteady hydromagnetic buoyancy-induced flow of a chemically reactive fluid with convective boundary heating in a vertical tube coated with porous material. The fluid rate increased when the porous medium, thermal Biot number, slip parameter, and viscous reactive fluid parameter all increased. Allan and Dardery (2018) investigated the effects of convective heat and mass transport on unsteady coupled convection through an upward porous material. Bordoloi *et al.*, (2021) studied the effect of diffusion and temperature on three-dimensional flow across a porous upright plate in a porous medium with alternating suction and porosity. Ahmed and Bordoloi (2021) investigated three-dimensional flow through a porous vertical plate in a porous medium with sinusoidal suction and permeability influenced by thermal diffusion. According to the preceding discussion, the impact of mass flow under symmetric wall concentration conditions has not been described using a superhydrophobic microchannel.

Variable physical properties, such as viscosity and heat conductivity, have been assumed constant. However, the temperature dependency of viscosity and thermal conductivity must be considered in order to avoid errors in fluid device construction and sophisticated fluid flow modeling. Meanwhile, significant industrial applications that require fluid flow, such as geothermal systems, crude oil exploitation, and equipment lubrication, necessitate temperature-dependent fluid properties. As a result, understanding how thermal conductivity varies with temperature is critical for accurately predicting flow dynamics and associated heat transfer properties (Ajibade and Kabir 2020, Hamza *et al.*, 2015). There are numerous published studies on unsteady and steady flows with varied thermal conductivity. Hamza *et al.*, (2025a) investigated how temperature-dependent variable thermal conductivity influences natural convective flow when suction/injection and porosity effects are present. Ojmeri *et al.*, (2025a) studied the effects of Casson fluid flow and varied thermal conductivity on a magnetized oscillating plate submerged in porous material during asymmetric thermal conditions. Salawu *et al.*, (2021) investigated the free convective flow of a moving vertical plate by modeling buoyancy force as a quadratic Boussinesq approximation and changing thermal conductivity with a suitable shooting-numeration technique. They concluded that fluid flow and heat distribution along a channel vary with temperature. Pradip *et al.*, (2021) evaluated the effect of viscosity and thermal conductivity changes on natural hydromagnetic flow across an isothermal upwelling plate submerged in a thermally conductive liquid. Reddy (2014) investigated natural hydromagnetic flux across a curved isothermal porous wall with varied levels of thermal conductivity. Animasaun and Sandeep (2016) studied the impact of

nanofluid concentration on variable viscosity and thermal conductivity. Salahuddin and Awais (2022) resolved the Carreau cross fluids generated by different thermal radiation and thermal conductivity factors. Megahed *et al.*, (2019) proposed a Carreau fluid with variable thermal conductivity and heat flux, which is affected by thermal radiation. Irfan *et al.*, (2018) studied the effects of varying thermal conductivity on heat source/sink and generalized Fourier's law in Carreau fluid flow. Hamza *et al.*, (2022) studied fully developed mixed convection caused by varying thermal conductivity over a microchannel whose plates were subjected to velocity slip and temperature jump effects. References for other studies demonstrating the significance and exciting applications of variable thermal include Haque *et al.*, (2015), Malik *et al.*, (2016), and Uwanta and Hamza (2014). In all of these studies, the effect of exothermic fluid in a superhydrophobic microchannel was overlooked.

For many years, the composition of flow models over viscous or chemically reactive (exothermic) fluids with MHD-free and mixed convections across various geometric shapes has piqued researchers' interest due to its intriguing and promising applications in a variety of chemical engineering and petroleum-related disciplines. These applications include chemical vapor deposition systems, tubular laboratory reactors, oxidation of solid materials in large containers, cooling of rocket boosters and re-entry vehicles, film vacuum in combustion chambers, cross-hatching on ablative surfaces, self-propagating reactions, combustion in subterranean tanks, and so on (Muthuraj and Srinivas 2010). Frank-Kamenetskii (1969) was the first to introduce the modeling of reactive viscous fluid. According to Makinde (2008), the majority of lubricants used in engineering and industrial processes, including polyglycols, hydrocarbon oils, synthetic esters, and others, are reactive. In response to the numerous benefits of magnetized chemical reactive fluid, Omokhuale *et al.*, (2025) recently explored the impact of superhydrophobicity and wall concentration dynamics on magnetized mixed convection in an exothermic fluid via a microchannel. Hamza *et al.*, (2026) investigated the mixed flow of a chemically reactive fluid with electrokinetic and magnetic effects through a superhydrophobic microchannel. Ojmeri and Onwubuya (2023) conducted a theoretical investigation of a chemically reactive free convection flow in a microchannel, which was influenced by magnetic fields and superhydrophobicity. They concluded from their findings that greater values of the viscous heating parameter cause an increase in fluid temperature and momentum, whereas the presence of a superhydrophobic surface causes a decrease in shear stress in the microchannel. Obalulu *et al.*, (2021) statistically demonstrated the significance of Arrhenius-controlled heat transfer flow on a non-Newtonian fluid undergoing chemical reaction in an upstanding channel. Ojmeri and Hamza (2022) proposed a computer model for an

Arrhenius-kinetically propelled heat emission and absorption fluid in a microchannel. Ahmad and Jha (2015) conducted a numerical and analytical investigation on the effect of heat transfer flow on the reaction liquid of an exothermic chemical in a porous tube. Their findings indicate that the quantity of heat transmission on the surface of pipes rises over time, both under acceptable boundary conditions. Ahmad *et al.*, (2017) investigated a fully developed, continuous flow of viscous reactive fluid generated by the Soret effect in a vertical porous pipe, combining convective heat and mass transfer. The nonlinear ordinary differential equations of the flow are solved using a semi-analytical technique. The Frank-Kamenetskii, Soret, and mixed convective flow control parameters have been shown to have a considerable impact on the conduit's flow pattern, heat gradient, and mass gradient. Jha *et al.*, (2011a, 2011b) studied an exothermic reaction fluid's unsteady free-convection flow in a tube and an upright channel restricted between two measured upright parallel plates. Their findings reveal that increasing the chemical reactant parameter increases the magnitudes of shear stress and heat transmission on both sides.

Despite these intriguing contributions, the influence of varying thermal conductivity was not discussed.

The absence of a Darcy porous medium in an exothermic fluid microchannel causes the system to transition from a high-surface-area, drag-dominated regime to a clear-flow mode. While this reduces flow resistance, it also raises the risk of thermal instability and decreases heat dissipation efficiency. Exothermic fluids are more prone to thermal runaway, a phenomenon in which internal heat generation from chemical reactions accelerates uncontrollably. As a result, the model fails to adequately describe the stability of internal temperature and transport mechanisms in chemically reactive or multicomponent flows that are commonly encountered in medical imaging and cancer therapy, polymer processing, chemical vapor deposition, heating, ventilation, and air conditioning (HVAC) processes, and nanofluids for tunable, enhanced thermal conductivity for superior cooling. Physically, the Ojmeri and Onwubuya (2023) model is unsuitable for microchannel flows with rapidly changing internal temperature

gradients, as well as for precise temperature management in advanced systems such as electronic cooling, aerospace, medical imaging, and renewable energy. To address these limitations, the current study aims to improve the Ojmeri and Onwubuya (2023) model by including the time component, variable thermal conductivity, and Darcy porous medium effects into the governing equations. This expansion seeks to create a more complete and physically realistic model that describes a broader range of microscale transport mechanisms in SHO slit microchannels. Finally, it is believed that this new model would aid in the management of active thermal systems, real-time heat flow control for complicated applications, smart insulator design that adapts to temperature fluctuations, and improved heat transfer, all of which are critical for industrial operations.

2.0 Problem Description and Illustration

Consider an electrically conducting fluid traveling upward via a vertical parallel-plate microchannel in an unstable, fully developed free-convection flow with Arrhenius reaction kinetics submerged in a porous medium. Due to a unique microfabrication technology, one wall possesses superhydrophobic capabilities and is exceptionally resistant to wetting, but the other wall remains unmodified and retains its slip-resistant qualities. As shown in Figure 1, the SHS is maintained at $y_0 = 0$, while the nonslip surface is maintained at $y_0 = L$. Assuming that the thermal conductivity (kf) of the fluid varies as a linear function of temperature in the form, $kf = km [1 + \delta (T' - T'_m)]$, where km is the fluid free stream thermal conductivity and δ is a constant depending on the nature of the fluid, where $\delta > 0$ for fluids such as water and air, while $\delta < 0$ for fluids such as lubrication oils (Elbarbary and Elgazery 2004). It is assumed that a transverse magnetic strength $(0, B_0, 0)$ is applied normally to the channel. Using the standard Boussinesq buoyancy approximation with beginning and boundary conditions, and following Ojmeri and Onwubuya (2023), assuming that the fluid is impacted by thermal properties, the current problem can be described in dimensional partial differential form as follows:

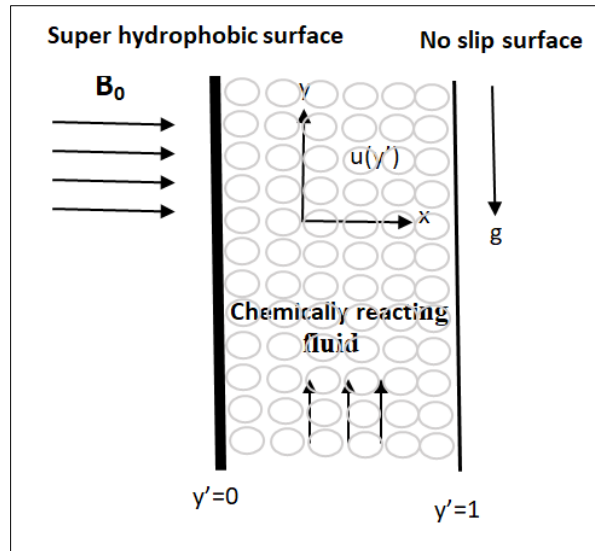


Figure 1: Physical Configuration of the Flow Dynamics

Continuity Equation

$$\frac{\partial u'}{\partial x'} = 0 \tag{1}$$

Velocity Equation

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0' u'}{\rho} - \frac{u'}{K} + g\beta(T' - T_0) \tag{2}$$

Temperature Equation

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{QC_0^* A}{\rho C_p} e^{\left(\frac{-E}{RT'}\right)} + \frac{1}{\rho C_p} \frac{\partial}{\partial y'} \left[K_f \frac{\partial T'}{\partial y'} \right] \tag{3}$$

With the relevant initial and boundary conditions as:

$$t' \leq 0: u' = 0, T' = T'_m, \text{ for } 0 \leq y' \leq L, \\ t' > 0:$$

$$\left. \begin{aligned} u(y') = \lambda' \frac{\partial u'}{\partial y'} \\ T(y') = T_0 + \gamma' \frac{\partial T'}{\partial y'} \end{aligned} \right\} \text{ at } y' = 0 \tag{4}$$

$$\left. \begin{aligned} u(y') = 0 \\ T(y') = T_0 \end{aligned} \right\} \text{ at } y' = L$$

Where the dimensionless quantities and variables used are given below:

$$t = \frac{t'v'}{h^2}, u = \frac{u'}{U}, y = \frac{y'}{h}, T = \frac{T' - T_0}{T_w - T_0}, x = \frac{x'v'}{Uh^2}, \\ \alpha = \delta^* (T'_h - T'_m), M^2 = \frac{\sigma B_0'^2 h^2}{\rho \nu}, Da = \frac{K}{h^2} \tag{5}$$

$$\varepsilon = \frac{RT_0}{E}, \lambda = \frac{QC_0^* AEH^2}{RT_0^2} e^{\left(\frac{-E}{RT_0}\right)}, (Y, \gamma, \Gamma) = (Y', \gamma', \Gamma')/h$$

Substituting eqn (5) into eqns (1) – (4), we have:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - M_1^2 u + \theta \tag{6}$$

$$Pr \frac{\partial \theta}{\partial t} = (1 + \beta\theta) \frac{\partial^2 \theta}{\partial y^2} + \lambda e^{\frac{\theta}{1+\varepsilon\theta}} + \beta \left[\frac{\partial \theta}{\partial y} \right]^2 \tag{7}$$

Subject to the initial and boundary conditions:

$$t \leq 0: u = 0, \theta = 0, 0 \leq y \leq 1 \tag{8}$$

$$t > 0: u = \Gamma \frac{\partial u}{\partial y}, \theta = 1 + \gamma \frac{\partial \theta}{\partial y} \text{ at } y = 0 \\ u = 0, \theta = 0, \text{ at } y = 1$$

Where β is the variable thermal conductivity parameter, λ is the exothermic heat source, ε is the

activation energy, γ is the temperature jump parameter, Γ is the velocity slip parameter ε is the velocity slip parameter, M is the magnetic parameter, and $M_1^2 = M^2 + \frac{1}{Da}$.

3.0 METHODOLOGY

3.1 Numerical Solution

Equations (6) and (7), subject to equation (8), are computationally solved using a semi-implicit finite difference method. The forward difference methods calculate all time derivatives, while the central difference formula approximates spatial derivatives. The system is unconditionally stable and convergent, which allows for bigger time steps (Chinyoka *et al.*, 2005). In both space and time, a uniform discretization is utilized, with some time-dependent factors treated implicitly in the formulation and others addressed explicitly.

The following is the semi-implicit finite difference scheme for equations. (6) and (7), subject to eq. (8):

$$\frac{U_i^{j+1} - U_i^j}{\Delta t} = \frac{U_{i+1}^{j+1} - 2U_i^{j+1} + U_{i-1}^{j+1}}{(\Delta Y)^2} + \theta_i^j - M_1^2 U_i^j \tag{9}$$

$$Pr \left[\frac{\theta_i^{j+1} - \theta_i^j}{\Delta t} \right] = (1 + \alpha\theta_i^j) \left[\frac{\theta_{i+1}^{j+1} - 2\theta_i^{j+1} + \theta_{i-1}^{j+1}}{(\Delta Y)^2} \right] + \\ \lambda e^{\left(\frac{\theta_i^j}{1+\varepsilon\theta_i^j}\right)} + \alpha \left[\frac{\theta_{i+1}^{j+1} - \theta_{i-1}^{j+1}}{2\Delta y} \right]^2 \tag{10}$$

While the dimensional boundary conditions are:

$$U_i^{j+1} = \Gamma \frac{U_{i+1}^{j+1} - U_{i-1}^{j+1}}{2\Delta Y}, \theta_i^j = 1 + \gamma \frac{U_{i+1}^{j+1} - U_{i-1}^{j+1}}{2\Delta Y} \tag{11}$$

$$U_i^j = 0, \theta_i^j = 0$$

4.0 RESULTS AND DISCUSSION

This section shows the findings of numerically solving the time-dependent free convection flow of a chemically reactive heat transfer with changing thermal conductivity through a microchannel in the presence of a

Darcy porous material. The implicit finite difference method is used to solve extremely nonlinear partial differential equations. Illustrative graphs are created to demonstrate the influence of key parameters in the flow setup. For this analysis, $\gamma = \Gamma = 1$, $M=0.5$, $Da=0.1$, $\beta=0.5$, $\lambda = 0.1$ are the default values considered.

Figures 2 and 3 depict the influence of time (t) on the velocity and temperature gradients, confirming the accuracy of the numerical approach. The graphs show that the influence of time increases temperature and velocity, respectively, until a steady state ($SS=1$) time is reached, which occurs over long periods of time. As a result, the numerical solution closely approximates the steady-state scenario, providing strong evidence of the numerical scheme's reliability and validity in this study.

Figure 4 shows the fluid motion pattern for different values of magnetic parameter M and time t . For each value of M examined, the flow pattern obviously diminishes; nonetheless, the graph shows that it grows with t . The magnetic component creates a resistive force (Lorentz force) that inhibits fluid flow independent of t value.

Figure 5 illustrates the effect of the Darcy porous medium on the velocity profile. The presence of a porous material (as measured by the Darcy porous number) often increases fluid velocity. This effect is especially noticeable at hot superhydrophobic surfaces (SHS), where the combination of interfacial slip and porous permeability lowers flow resistance. Furthermore, porous structures improve the fluid-solid contact area but can cause considerable pressure losses. However, using porous fins rather than solid fins can lower the pressure-drop penalty by 43-67% while maintaining excellent convective efficiency.

Figures 6 and 7 show how α and non-dimensional time (t) affect temperature and momentum curves, respectively. Varying α with t strengthens temperature and velocity profiles until they reach a steady-state time. Increasing α values improves the energy difference between the plate's outer region and the surrounding boundary layer, as shown in the relationship $\beta = \delta(T'_h - T'_m)$. According to Elbarbary and Elgazery (2004), this allows heat to move quickly from a plate to a liquid inside the boundary layer. As a result, the temperature and velocity gradients are strengthened. It suggests both the fluid velocity and the thickness of the thermal boundary layer increase as α

increases. The findings from this study agree with those reported by Hamza *et al.*, (2024a). Outcomes from this research are essential for engineers to predict how materials will behave under extreme conditions, such as the high temperatures in furnaces or the cryogenic environments of aerospace technology.

Figures 8 and 9 show how temperature and velocity gradients affect the chemical reaction parameters λ and t . As the value of λ grows, the temperature gradually improves until it reaches a steady state. Similarly, λ increases fluid velocity within the microchannel. As the velocity slip at the SHO walls grows, the fluid wall effect decreases. As a result, the fluid accelerates its approach to the wall. Elevated λ causes a large temperature increase, leading to increased fluid velocity and decreased viscosity. In the industrial environment, this phenomenon is best shown by the automated transport of lubricants or polymers through micro-scale manufacturing channels.

Figure 10 depicts the effect of altering thermal conductivity against time on heat transport across the boundary wall. The heat transfer rate increases as thermal conductivity varies with temperature. Physically, this means that as thermal conductivity becomes more sensitive to temperature, the rate of heat transfer from the surface to the fluid accelerates.

Figure 11 plots the impact of Darcy porous media (Da) and magnetic parameter on the skin friction coefficient. This graphic shows that increasing levels of Da increase the frictional force between the fluid and the microchannel surfaces. Lower skin friction is observed for an increasing number of magnetic field effects, as expected.

Figure 12 illustrates the effect of the exothermic source term (λ) on the Nusselt number. Higher λ values reduce the rate of heat in the microchannel.

The variable thermal conductivity parameter α has a dual role in microchannel flow. Increasing α improves the fluid's thermal conductivity, resulting in greater Nusselt numbers and better heat dissipation efficiency. Mechanically, this heat enhancement reduces fluid viscosity, accelerating the flow while increasing skin friction due to amplified velocity gradients at the SHO walls. On the other hand, high-conductivity porous materials (such as metal foams) effectively "bridge" heat from the fluid's center to the channel walls, bypassing the fluid's own insulating effect.

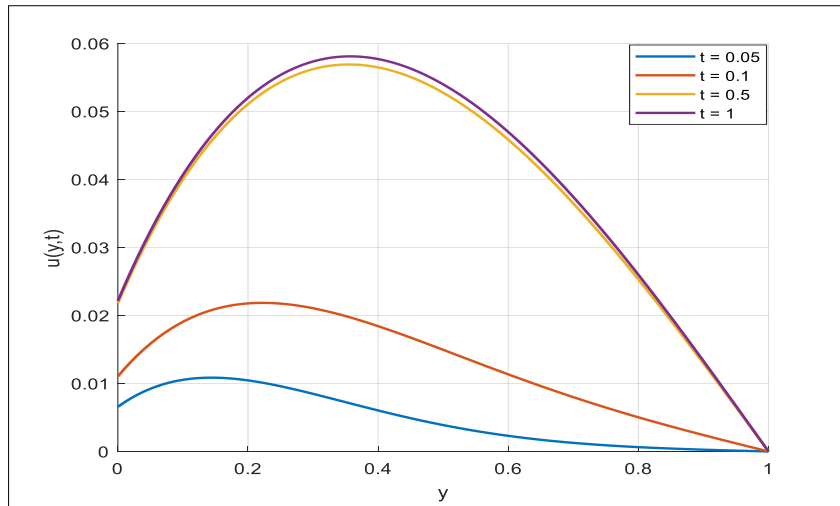


Figure 2: Effect of time (t) on the velocity profile

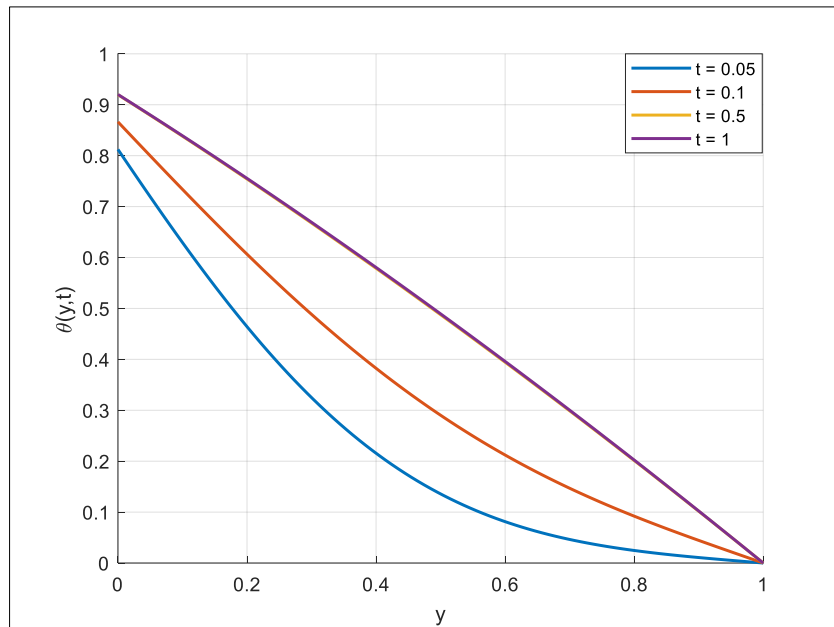


Figure 3: Effect of time (t) on the temperature profile

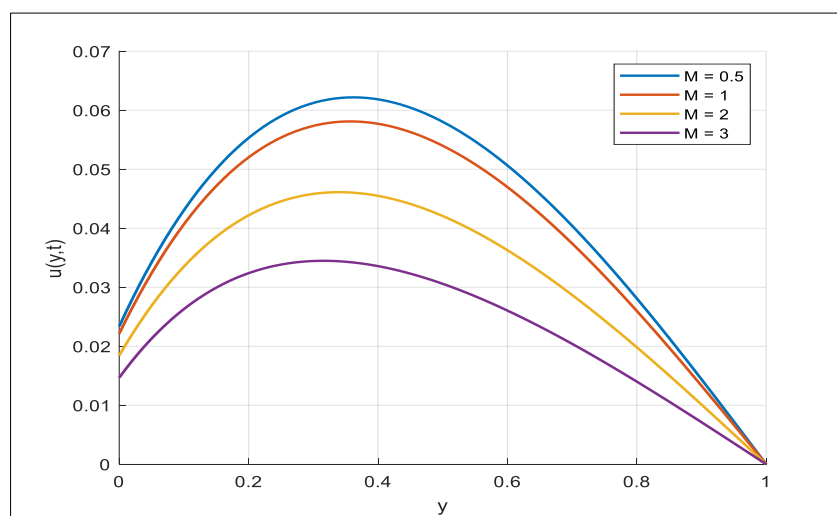


Figure 4: Effect of magnetic parameter (M) on the velocity profile

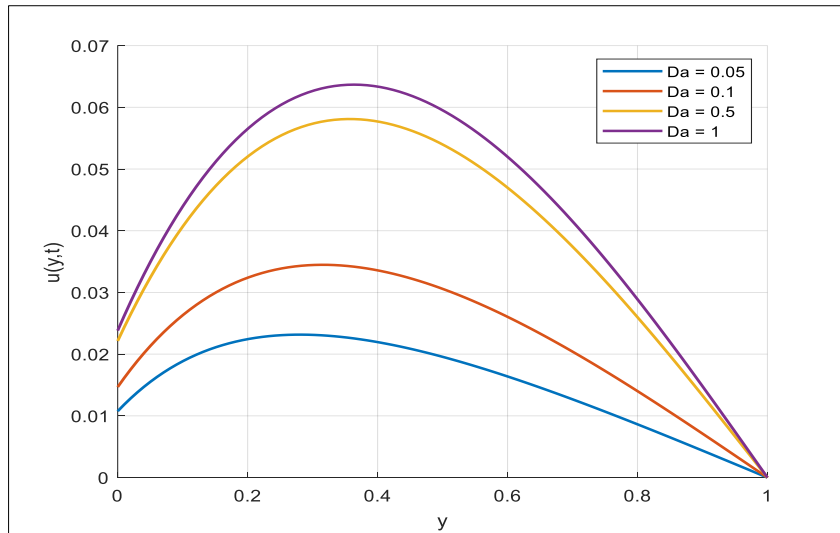


Figure 5: Effect of Darcy porous medium (Da) on the velocity profile

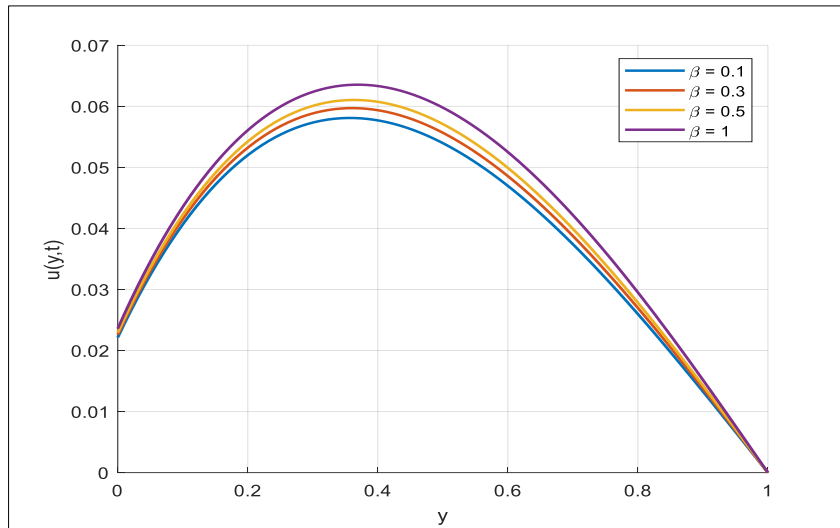


Figure 6: Effect of variable thermal conductivity (β) on the velocity profile

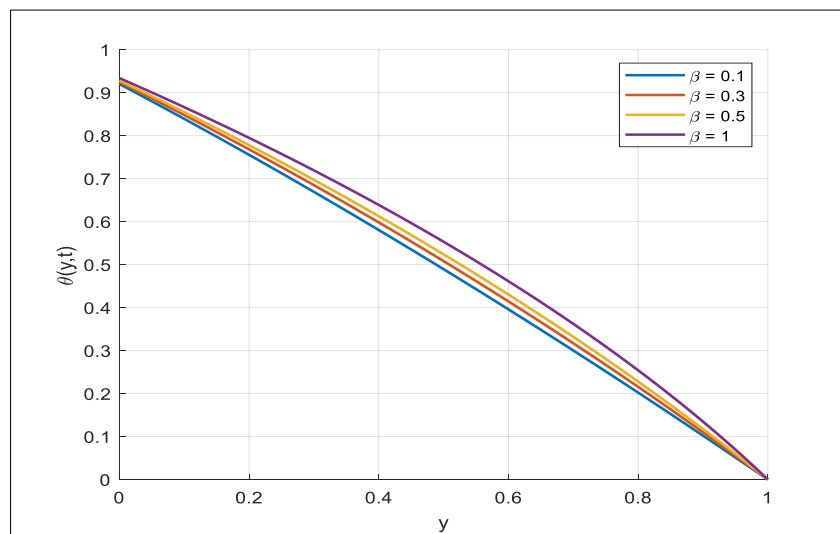


Figure 7: Effect of variable thermal conductivity (β) on the temperature profile

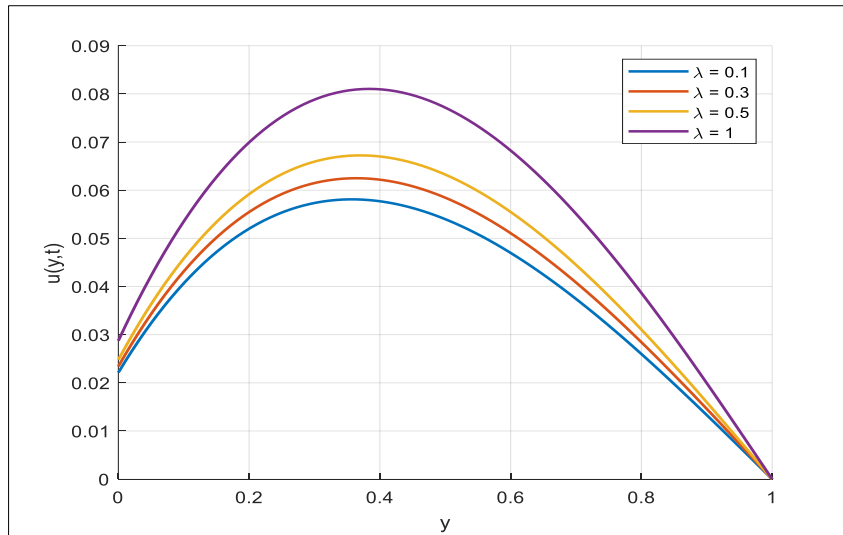


Figure 8: Effect of exothermic source term (λ) on the velocity profile

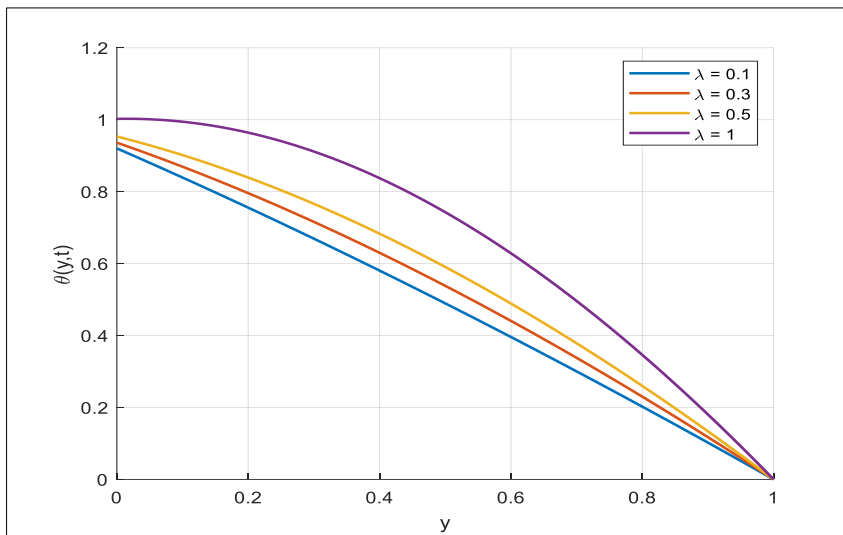


Figure 9: Effect of exothermic source term (λ) on the temperature profile

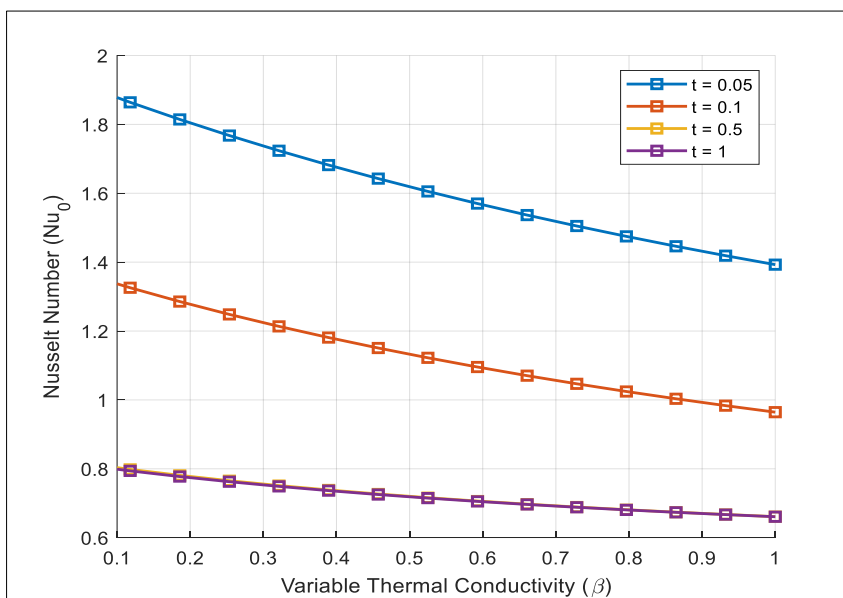


Figure 10: Effect of variable thermal conductivity (β) and time (t) on the Nusselt number

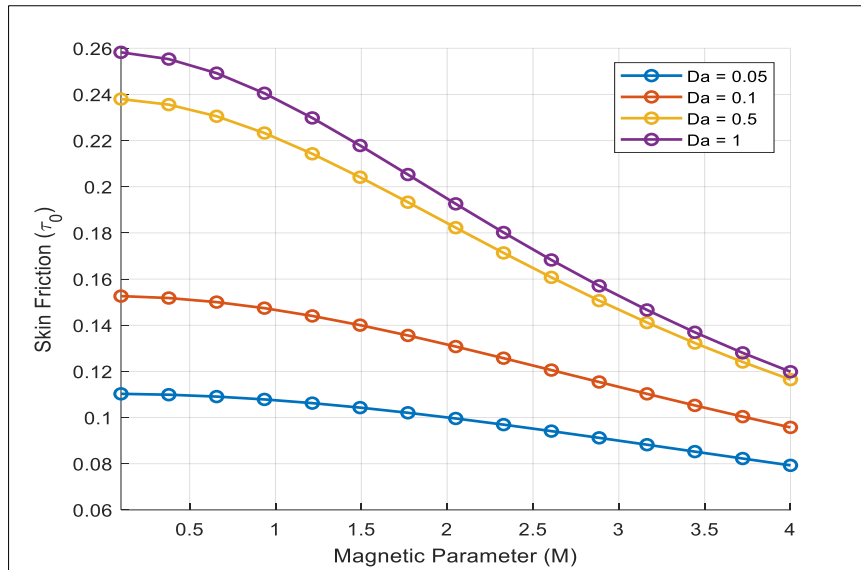


Figure 11: Effect of Darcy porous medium (*Da*) and Magnetic parameter (*M*) on the skin friction

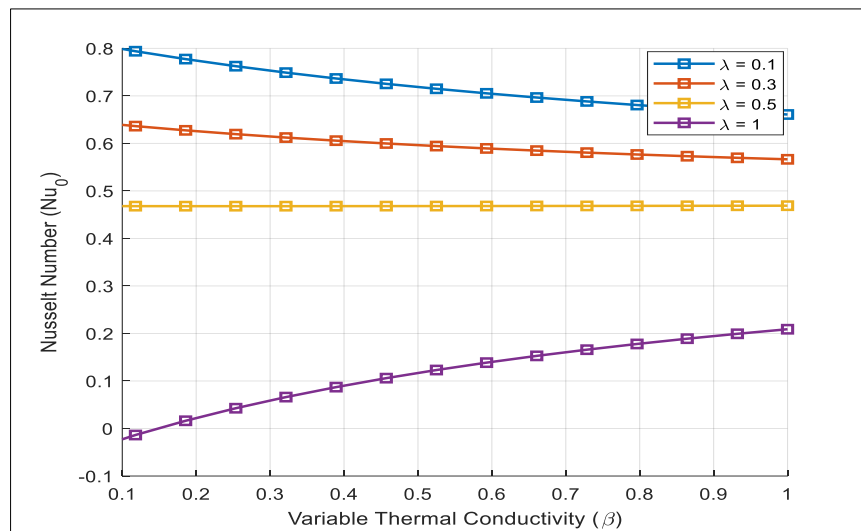


Figure 12: Effect of variable thermal conductivity (β) and exothermic source term (λ) on the Nusselt number

Nomenclature

- B_0 =magnetic flux density [$\text{kg/s}^2.\text{m}^2$]
- g =acceleration due to gravity [m/s^2]
- h = channel width [m]
- M = magnetic field [-]
- β = variable thermal conductivity
- ε = activation energy [J/mol]
- Nu =rate of heat transfer [-]
- T =temperature of the fluid [K]
- u =fluid’s velocity [ms^{-1}]
- y = length between plates
- t = time variable

Greek Letters

- Γ = velocity slip parameter (-)
- γ = temperature jump parameter (-)
- λ =viscous heating parameter (-)
- β^* =coefficient of thermal expansion [K^{-1}]
- μ =variable fluid’s viscosity [$\text{kgm}^{-1}\text{s}^{-1}$]
- k =thermal conductivity [$\text{m.kg/s}^3\text{K}$]

- α^* =thermal diffusivity [m^2s^{-1}]
- σ = electric fluid’s conductivity [$\text{s}^3\text{m}^2/\text{kg}$]
- ρ = density of the fluid [kgm^{-3}]
- ν =fluid’s viscosity [m^2s^{-1}]

5.0 CONCLUSION

The synergy between these features—drag reduction from SHS and high convective intensity from the porous medium—often provides superior overall efficiency (Performance Evaluation Criterion) compared to using either feature alone. Specifically, increasing the "porous media presence" is a documented method to mitigate the thermal runaway phenomena, as it breaks the synchronous surge of velocity and temperature that typically occurs in uncontrolled exothermic flows.

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