

Investigation on Magnetohydrodynamic (MHD) Outer Boundary Layer Flow with Defect Layer Past a Flat Plate

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Abstract

Magnetohydrodynamic (MHD) flow past a flat plate over an outer flow turbulence subjected to a defect-layer has been examined. It finds a novel approach to a Blasius equation with a view to analyze outer flow turbulence by analytical method, neglecting numerical method to describe the physical situation on outer flow which does not seem to have appeared in the literature. In a defect-layer it is rigorously stated that outer flow in a defect layer is independent of Reynolds number. To solve Blasius equation subject to boundary conditions it is stated that numerical results are obtained by analytical method. A graphical representation shows that the velocity distribution is merged with different values of Hartmann number (magnetic pressure) so that velocity increases with indefinite period. In this situation, outer flow turbulence in a defect layer holds stress free so that the existence of a magnetic field dominates the entire outer flow situation. In relating to the physical situation of interest, the universe is expanding slowly and slowly subject to a Hot Big Bang with a decisive importance to a microwave background of radiation.

Keywords: MHD, Defect Layer, Magnetic Pressure, Reynolds Number, Microwave Radiation.

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INTRODUCTION

Over the past decades, the study of hydrodynamic flow has been carried out by several authors. The motivation of studies corresponds to the fluid behavior to transform into fluid flow with a decisive importance to Blasius solution. A literature survey reveals to the study of Blasius [1], Wang [2], Martin and Boyd [3], Aziz [4], Fang and Lee [5], and Cai [6]. An excellence of a literature on similarity solution to a magnetohydrodynamic (MHD) flow has become relevant to compare with analytical solution by avoiding numerical method. Mention may be made of their works of Ghosh [7-9], Ghosh *et al.*, [10], Sharma *et al.*, [11], and Nikodijevic *et al.*, [12]. Nevertheless, an orientation of a fluid flow behavior is to describe magnetohydrodynamic (MHD) flow past a flat plate to explore the situation with different conditions and configurations. Such type of study has received wide attention to many researchers to their works of Tel-bert *et al.*, [13], Ghosh [14, 15], Helmi [16], Fujii and Imura [17], Mabood and Ibrahim [18], and Ghosh [19].

However, the study of a flat plate over an outer flow turbulence has not received attention in literature. It finds a remarkable attention to readers with a decisive importance to the study of a pioneer book that has been written by Schlichting and Gersten [20]. It contains a perception of a defect layer formation on an outer flow turbulence which is independent of Reynolds number. We obtain the defect layer of the velocity compared to frictional velocity on the center line. It is independent of wall layer and therefore, also independent of Reynolds number. Hence, a defect layer on frictional velocity is subjected to an outer flow turbulence which is independent of Reynolds number.

In relating to the physical situation of interest to explore the situation of a strong magnetic field inside the solar corona takes place of solar light which is directly falls on the earth's surface so that a microwave background of radiation becomes relevant to the earth's atmosphere with reference to a controlled thermonuclear fusion reaction of the Sun. In turn, plasma fusion exerted by the supercritical state as stated by Ghosh [21], in taking into account of the hottest electron gyrating in a

strong magnetic field that leads to a strong ionizing radiation in the solar atmosphere in a Controllable region so that a microwave background of radiation in the earth's atmosphere becomes relevant to the study of a controlled thermonuclear fusion reaction of the Sun. In this situation, ionization takes place of an electromagnetic field to produce electrical energy from the solar corona which is propagated by the Sun so that microwave background of radiation becomes relevant to the earth's atmosphere which can be used for solar energy collector to preserve a huge amount of electricity by solar panel.

The motivation of present investigation is to study of a magnetohydrodynamic (MHD) of a flow of a defect layer past a flat plate over an outer flow turbulence. It finds wide applications of geophysics, astrophysics and fluid engineering to deal with MHD power generator, nuclear reactor, construction of turbine and other centrifugal machines and atmospheric re-entry vehicles. A defect layer emerges to a frictional velocity on the center line so that the frictional velocity on the outer flow turbulence becomes independent of Reynolds number. This problem is solved by analytical method in accordance with Blasius equation where numerical method becomes ignored. This type of problem does not seem to appear in the literature. In this problem, the effect of induced magnetic field is merged with the outer flow velocity field for any value of a Hartmann number. Although the outer flow velocity is closely resemblance to a magnetic force, it is rigorously stated that ionization takes place in the earth's upper atmosphere. Therefore, it appears to be real significance of a microwave background of radiation. To examine the frictional shear stress, it comes to a justification of stress free over the outer flow practically; there is no such boundary over the outer flow. Thus, the expansion of universe leads to a stress free situation. This implicates the situation of a Hot Big Bang of the universe.

Formulation of the Problem and Its Solution

Consider the steady hydromagnetic flow of a viscous incompressible electrically conducting fluid past an infinitely flat plate immersed in a fluid under the influence of a transverse magnetic field. We choose cartesian co-ordinate system in such a way that x' - axis is along the plate and the y' - axis is normal to the plate. The Maxwell's equations comprise five vector equations like $\nabla \times B = \mu_e J$ (Ampere's law), $\nabla \times E = -\frac{\partial B}{\partial t}$ (Faraday's law of induction), $\nabla \cdot B = 0$ (magnetic field continuity equation), $\nabla \cdot J = 0$ (the conservation of electric charge and $J = \sigma[E + u' \times B]$ (the Ohm's law for a moving Conductor).

u' , B , E , J , t , μ_e , σ are, respectively, velocity vector, magnetic field vector, electric field vector, current density vector, time, magnetic permeability and electrical conductivity.

The law of conservation of mass leads to the equation of continuity which is an agreement with fundamental equation of magnetohydrodynamics such as $\nabla \cdot u' = 0$

The MHD momentum equation becomes

$$u' \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial y'} = U \frac{\partial U}{\partial x'} + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} u' \quad (1)$$

The boundary conditions become

$$u' = U, v' = 0 \text{ at } y = 0 \quad (2)$$

$$u' = 0 \text{ as } y \rightarrow \infty$$

where ν is the Kinematic coefficient of viscosity, ρ is the fluid density, U is free stream velocity and B_0 is the magnetic field.

Introducing a dimensionless stream function with reference to Cauchy Riemann equation read

$$u' = \frac{\partial \Psi}{\partial y}, v' = -\frac{\partial \Psi}{\partial x} \quad (3)$$

Eq. (1) and Eq. (3), we arrive the form such as

$$\frac{\partial \Psi}{\partial y} \frac{\partial^2 \Psi}{\partial x \partial y} - \frac{\partial \Psi}{\partial x} \frac{\partial^2 \Psi}{\partial y^2} = U \frac{\partial U}{\partial x} + \nu \frac{\partial^3 \Psi}{\partial y^3} - \frac{\sigma B_0^2}{\rho} \frac{\partial \Psi}{\partial y} \quad (4)$$

Introducing dimensionless variables such as

$$\varepsilon = \frac{x}{\delta}, \eta = \frac{y}{\delta}, p = \frac{\Psi}{U\delta} \quad (5)$$

The boundary layer approximation for free stream velocity turns into

$$U \sim (R_e^{-1/2}) p \quad (6)$$

where R_e is the Reynolds number.

Eq. (4) with the help of dimensionless quantities (5) takes the form

$$U \frac{dU}{dx} + \frac{\nu U}{\delta^2} \frac{\partial^3 p}{\partial \eta^3} - \frac{\sigma B_0^2 U}{\rho} \frac{\partial p}{\partial \eta} = 0 \quad (7)$$

With the help of boundary layer approximation (6), we rewrite the free stream spatial velocity gradient in dimensionless form as

$$U \frac{dU}{dx} = \frac{p}{R_e \delta} \frac{dp}{d\varepsilon} \quad (8)$$

Combining Eqs. (7) and (8), the momentum equation in dimensionless form reduces to

$$\frac{1}{R_e} \frac{\delta p}{\nu U} \frac{dp}{d\varepsilon} + \frac{\partial^3 p}{\partial \eta^3} - M \frac{\partial p}{\partial \eta} = 0 \quad (9)$$

Where $M = \frac{\sigma B_0^2 U}{\rho}$ is the Hartmann number

Representing the free stream velocity the displacement velocity on the outer flow from the leading edge of the plate:

$$\frac{d\eta}{d\varepsilon} = U^*(\varepsilon) \quad (10)$$

In a defect layer formulation, the velocity is independent of Reynolds number. Combining Eqs. (9) and (10) subject to the outer part of a boundary layer becomes analyzed with the help of $\phi = \frac{U^*(\varepsilon)}{U}$ and finally, the effective equation becomes

$$\frac{\partial^3 p}{\partial \eta^3} + p \frac{\partial p}{\partial \eta} - M \frac{\partial p}{\partial \eta} = 0 \quad (11)$$

We denote F as p and $(\cdot)'$ as $\frac{\partial}{\partial \eta}$, Eq (11) takes the form

$$F''' + FF' - MF' = 0 \quad (12)$$

The corresponding boundary conditions are

$$F = 0; F' = 1 \text{ at } \eta = 0, \\ F' = 0 \text{ at } \eta \rightarrow \infty \quad (13)$$

where F is the dimensionless stream function and F' is the dimensionless velocity.

Eq. (12) together with the boundary conditions (13) can be solved analytically and the solution for the velocity distribution takes the form.

$$F'(\eta) = \frac{\sqrt{3}(M^4 + 2c_1)}{2} \text{Sec h}^2 \frac{\sqrt{3}}{2} (\eta - c_2) \sqrt{M^4 + 2c_1} \quad (14)$$

Differentiating with respect to η , the following frictional shear stress turns into

$$F''(\eta) = 0 \quad (15)$$

$$\text{where } c_1 = \frac{1}{0.0074 \times \sqrt{3}} - \frac{M^4}{2}$$

$$\text{and } c_2 = \frac{\sqrt{2}\pi}{\sqrt{3}} \sqrt{0.0074 \times \sqrt{3}}$$

RESULTS AND DISCUSSIONS

Numerical results for velocity distribution (F') are depicted graphically in Fig.1 for various values of Hartmann number M^2 (magnetic pressure). It shows that the profile is skewed in nature and the skewness is characterized by the magnetic pressure. Fig.1 demonstrates that the velocity profile is merged with any

value of M^2 if the value of M^2 is increased. Also, the profile goes on increasing with an indefinite period. As a result, the magnetic field dominates the MHD flow past a flat plate over an outer flow turbulence in a defect layer. Fig.1 reveals that the velocity increases with increase in M^2 for any value of Hartmann number (M^2). The tendency of an increased velocity experiences over an MHD flow situation past a flat plate immersed in a fluid so that the defect layer on outer flow turbulence is independent of Reynolds number. It is noticed that the expansion of universe leads to a magnetic pumping with an excellence of increased velocity that goes extensively until the effect of velocity (F') becomes zero at the point at infinity.

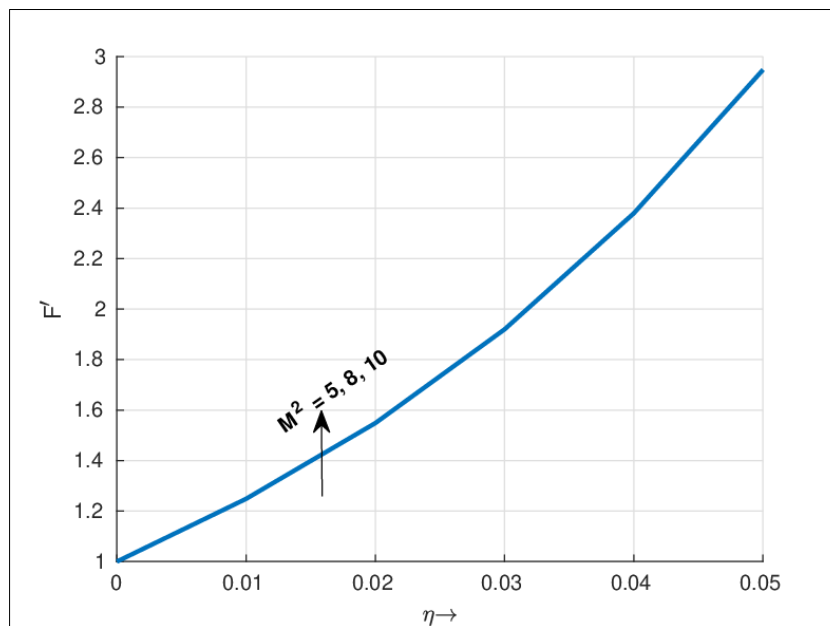


Fig. 1: Velocity distribution for varying M^2

Here, the outer flow velocity is associated with magnetic pressure it is noticed that ionization takes place in the earth's upper atmosphere. Therefore, it becomes a significant effects of microwave background of radiation to the earth's atmosphere which can be preserved in a solar energy collector from the Sun so that a huge amount of electricity is preserved via solar panel. Due to the existence of a magnetic field in the solar corona, electric current generates in an electromagnetic field in the interior state of the Sun. It proves that Maxwell equation of the Sun determines a huge amount of electricity which can be collected from the Sun via solar panel. Indeed, a controlled thermonuclear fusion reaction of the Sun leads to the expansion of universe to deal with microwave background of radiation with a decisive importance of a Hot Big Bang of the universe. Nevertheless, the outer flow is stress free with no specific boundary which communicates inviscid flow with no source or sink.

CONCLUSION

Magnetohydrodynamic (MHD) flow past a flat plate over a defect layer on outer part of boundary layer

has been studied. In a defect layer it is rigorously stated that the velocity is independent of Reynolds number. Blasius equation has been solved analytically by avoiding numerical method. To examine the outer part of the boundary layer subject to a defect layer the velocity distribution is merged with magnetic field. In this situation, velocity increases with indefinite period until the effect of velocity becomes zero at the point at infinity. Physical speaking of the universe is expanding slowly and slowly with reference to a Hot Big Bang to exert its influence on microwave background of radiation.

Conflict of Interest: Author claims that he has no conflict of interest.

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