OPEN ACCESS

Saudi Journal of Civil Engineering
Abbreviated Key Title: Saudi J Civ Eng
ISSN 2523-2657 (Print) | ISSN 2523-2231 (Online)
Scholars Middle East Publishers, Dubai, United Arab Emirates
Journal homepage: https://saudijournals.com/journal/sjce/home

Original Research Article

Use of a Refined Theory for Three-Dimensional Bending Analysis of Isotropic Rectangular Thick Plates

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DOI: https://doi.org/10.36348/sjce.2024.v08i08.001 | **Received:** 05.09.2024 | **Accepted:** 14.10.2024 | **Published:** 16.10.2024

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Abstract

In this paper, a refined plate theory (Alternative II theory) is presented for the three-dimensional bending analysis of an Isotropic thick plate. The theory has similarity to the first order shear deformation theory but requires no shear correction factors. The kinematics equations were developed based on the Alternative II Refined plate theory. Thereafter, using a complete three-dimensional constitutive relation, the total potential energy was developed. A governing equation and two compatibility equations were obtained by the variation of the total potential energy with respect to displacement and rotations respectively. Solving the governing and compatibility equations, a polynomial displacement function was obtained. The stiffness coefficients were then obtained using the displacement function. Thereafter, the equations for the in-plane normal and shear stresses, transverse normal and shear stresses as well as the lateral displacement were developed using the stiffness coefficients and the displacement function. Numerical values of the lateral displacement parameters were determined for a rectangular plate of aspect ratio 2.0, 1.0 and 0.5 for span to thickness ratios of 20, 10 and 7.14286. Also, numerical values of the lateral displacement and stresses were determined for a square plate for span to thickness ratios of 4, 10, 100 and 1000. The results from this work were compared with the work of previous researchers using simple percentage difference. It was observed that refined plate theories overestimate the lateral displacement of a plate. Hence, three-dimensional analysis is recommended for thick plate analysis.

Keywords: Refined Theory, Isotropic, Three-Dimensional, Bending Analysis, Thick Plate.

1.0 INTRODUCTION

The analysis of plates has been in existence for decades. It remains an important subject of interest because of the various uses of plates in engineering. Such uses are seen in the construction of structures such as wall panels, bridges, building floors, roofs, offshore platforms, aircraft, vehicles, ships among others [1]. Researchers have carried out various analysis of plates using different theories in the past. One of the earliest theories used in plate analysis is the classical plate theory. This theory assumes transverse inextensibility of the plate, hence neglecting strain along the thickness of the plate. Also, the classical plate theory neglects transverse shear stresses in a plate. This is done by the assumption that a vertical section of the plate remains normal to the middle surface even after deformation [2, 3]. It is obvious that the classical plate theory idealizes a plate as a two-dimensional material. Such approximations are only introduced to facilitate analysis, but it is known that significant shear tresses occur when the span to thickness ratio of the plate becomes relatively low [4]. Such plates that have a low span to thickness ratio (less than 20) are known as thick plates. To take care of the shear deformation that occurs in thick plates, Reissner and Mindlin introduced the first order shear deformation theory. The first order shear deformation theory takes account shear deformation but assumes a constant shear stress across the thickness of the plate [5]. This poses another problem as it defies the shear free surface condition. Hence, shear correction factors are required to provide a correct relationship between shear strain and stress across the thickness of the plate [6, 7]. These shear correction factors also pose challenges as they are dependent on geometry, loading and support conditions. To solve the problems associated with the classical plate theory and the first order shear deformation theory, higher order theories are developed [8]. These higher order theories take into consideration the shear deformation by using a shear deformation profile that ensures distribution of shear stress across the thickness of the plate and ensuring zero traction at the surface [9]. It should be noted that even the first order and higher order shear deformation theories solve plate as partial three-dimensional material because of the assumption of transverse inextensibility. This results in the stress along the thickness of the plate being assumed to be zero. Therefore, only five stress components are solved. Even though results obtained from these approximations are acceptable for many practical purposes, they do not give the accurate values of the stresses acting on the plate as well as the displacements under applied loads. It is important to analyze a plate using three-dimensional analysis to get the full response of the plate under loads since it is a three-dimensional body. Most works on three-dimensional analysis are for functionally graded (or sandwich) plates [10-14]. However [15], carried out analytical three-dimensional bending analysis of simply supported isotropic rectangular plate using a third order shear deformation theory while [16], investigated the three-dimensional stability analysis of plate using a direct variational energy method. The present study therefore presents an analytical approach to the three-dimensional bending analysis of homogeneous, rectangular, isotropic thick plate using a modified first order shear deformation theory which requires no shear correction factor yet satisfies shear free surface condition.

2.0 Refined Plate Theory: The refined plate theory is formulated as shown below:

2.1 Basic Assumptions

Consider a rectangular plate of total thickness, h, as shown in Figure 1a. The deformed section of the plate is as shown in Figure 1b. The plate is made of isotropic material. The assumptions of the present theory are as follows:

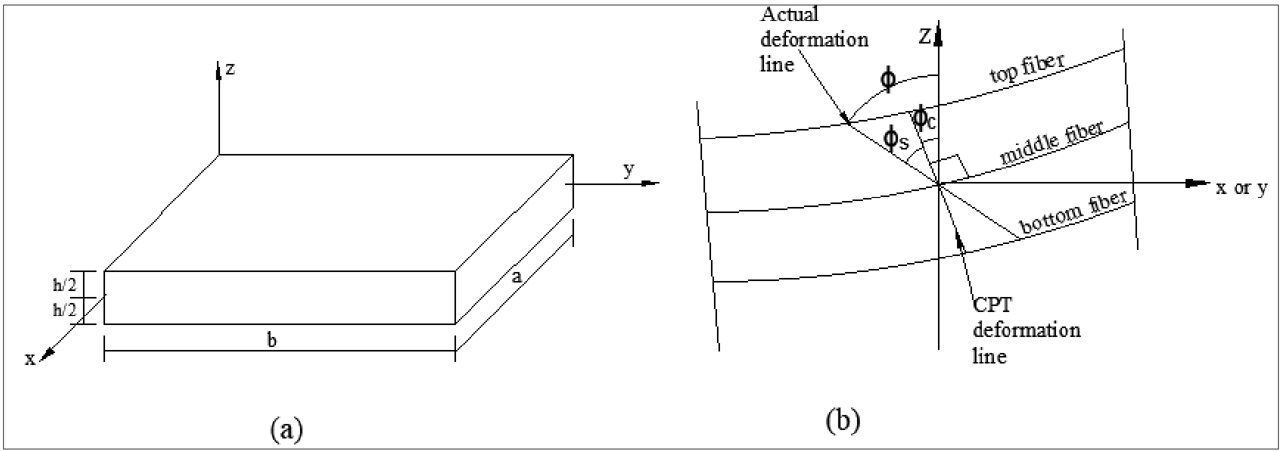

Figure 1: Geometry and deformed section of rectangular plate

i. The plate material is flat before loading.

ii. The deflection (w) of the middle in-plane surface of the plate is small when compared with the thickness of the plate. That is w/t < 0.3.

iii. The middle surface of the plate never stretches nor compresses before, during or after bending.

iv. A straight and flat x-z or y-z section, which is normal to middle x-y plane before bending shall remain straight and flat but not normal to the middle x-y surface after bending.

v. The actual transverse shear stresses, that is, *x-z* and *y-z* shear stresses distributed across the thickness of the plate are the product of nominal *x-z* and *y-z* shear stresses and shear stress shape profile, g(z). That is:

$$\tau_{xza} = \tau_{xz}\, g(z)$$
$$\tau_{yza} = \tau_{yz}\, g(z)$$

From assumption (iii), it follows that the in-plane displacements u and v consist of only bending, and shear components. This is as shown on Equations (1) and (2).

$$u = u_c + u_s \qquad (1)$$
$$v = v_c + v_s \qquad (2)$$

The bending components u_c and v_c are assumed to be similar to the displacements given by the classical plate theory, except for the removal of the negative sign. Therefore, the expression for u_c and v_c can be given as:

$$u_c = z\frac{\partial w}{\partial x},\ v_c = z\frac{\partial w}{\partial y} \qquad (3)$$

From assumption (iv), the shear components u_s and v_s are assumed to have a linear shear deformation profile in the displacement field but by multiplying the transverse shear stresses with a shear stress profile as given in assumption (v) ensures that shear stresses are zero at the top and bottom faces of the plate. Hence, no shear correction factor is required. Consequently, the expression for u_s and v_s can be given as:

$$u_s = z\phi_x,\ v_s = z\phi_y \qquad (4)$$

2.2 Kinematics

If Equations (3) and (4) are substituted into Equations (1) and (2), the displacements are given.
Hence, the kinematic relations are as shown on Equations (5) to (10).

$$\varepsilon_x = \frac{\partial u}{\partial x} = z\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial \phi_x}{\partial x}\right) \quad (5)$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = z\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial \phi_y}{\partial y}\right) \quad (6)$$

$$\varepsilon_z = \frac{\partial w}{\partial z} \quad (7)$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = z\left(2\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right) \quad (8)$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 2\frac{\partial w}{\partial x} + \phi_x \quad (9)$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = 2\frac{\partial w}{\partial y} + \phi_y \quad (10)$$

2.3 Constitutive Relations

The constitutive relation for a three-dimensional isotropic material is used in this work. This is as given in Equation (11).

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = E^* \begin{bmatrix} (1-\mu) & \mu & \mu & 0 & 0 & 0 \\ \mu & (1-\mu) & \mu & 0 & 0 & 0 \\ \mu & \mu & (1-\mu) & 0 & 0 & 0 \\ 0 & 0 & 0 & (0.5-\mu) & 0 & 0 \\ 0 & 0 & 0 & 0 & (0.5-\mu) & 0 \\ 0 & 0 & 0 & 0 & 0 & (0.5-\mu) \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} \quad (11)$$

Where:

$$E^* = \frac{E}{(1+\mu)(1-2\mu)} \quad (12)$$

E and μ are the young modulus of elasticity and the Poisson ratio respectively.
By substituting Equations (5) to (10) into Equation (11), The Equation of stresses can therefore be written as:

$$\sigma_x = E^*\left[z(1-\mu)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial \phi_x}{\partial x}\right) + z\mu\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial \phi_y}{\partial y}\right) + \mu\frac{\partial w}{\partial z}\right] \quad (13)$$

$$\sigma_y = E^*\left[z\mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial \phi_x}{\partial x}\right) + z(1-\mu)\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial \phi_y}{\partial y}\right) + \mu\frac{\partial w}{\partial z}\right] \quad (14)$$

$$\sigma_z = E^*\left[z\mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial \phi_x}{\partial x}\right) + z\mu\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial \phi_y}{\partial y}\right) + (1-\mu)\frac{\partial w}{\partial z}\right] \quad (15)$$

$$\tau_{xy} = E^*\left[z(0.5-\mu)\left(2\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right)\right] \quad (16)$$

$$\tau_{xz} = E^*\left[(0.5-\mu)\left(2\frac{\partial w}{\partial x} + \phi_x\right)\right] \quad (17)$$

$$\tau_{yz} = E_0\left[(0.5-\mu)\left(2\frac{\partial w}{\partial y} + \phi_y\right)\right] \quad (18)$$

2.4 Total Potential Energy

The internal work is as shown in Equation (19).

$$U = \frac{1}{2}\int_0^a \int_0^b \int_{-0.5z}^{0.5z} \left(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_z\varepsilon_z + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz}\right) \partial x \, \partial y \, \partial z \quad (19)$$

If Equations (5) to (10) and Equations (13) to (18) are substituted into Equation (19) and simplified, the internal work is given as:

$$U = \frac{E^*}{2}\int_0^a\int_0^b\int_{-0.5z}^{0.5z}\left(z^2(1-\mu)\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + 2z^2(1-\mu)\frac{\partial^2 w}{\partial x^2}\cdot\frac{\partial \phi_x}{\partial x} + 2z^2\mu\frac{\partial^2 w}{\partial x^2}\cdot\frac{\partial^2 w}{\partial y^2} + 2z^2\mu\frac{\partial^2 w}{\partial x^2}\cdot\frac{\partial \phi_y}{\partial y} + 2z\mu\frac{\partial^2 w}{\partial x^2}\cdot\frac{\partial w}{\partial z}\right.$$
$$+ z^2(1-\mu)\left(\frac{\partial \phi_x}{\partial x}\right)^2 + 2z^2\mu\frac{\partial^2 w}{\partial y^2}\cdot\frac{\partial \phi_x}{\partial x} + 2z^2\mu\frac{\partial \phi_x}{\partial x}\cdot\frac{\partial \phi_y}{\partial y} + 2z\mu\frac{\partial \phi_x}{\partial x}\cdot\frac{\partial w}{\partial z} + z^2(1-\mu)\left(\frac{\partial^2 w}{\partial y^2}\right)^2$$
$$+ 2z^2(1-\mu)\frac{\partial^2 w}{\partial y^2}\cdot\frac{\partial \phi_y}{\partial y} + 2z\mu\frac{\partial^2 w}{\partial y^2}\cdot\frac{\partial w}{\partial z} + z^2(1-\mu)\left(\frac{\partial \phi_y}{\partial y}\right)^2 + 2z\mu\frac{\partial \phi_y}{\partial y}\cdot\frac{\partial w}{\partial z} + (1-\mu)\left(\frac{\partial w}{\partial z}\right)^2$$
$$+ 4z^2(0.5-\mu)\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2 + 4z^2(0.5-\mu)\frac{\partial \phi_x}{\partial y}\cdot\frac{\partial^2 w}{\partial x\partial y} + 4z^2(0.5-\mu)\frac{\partial \phi_y}{\partial x}\cdot\frac{\partial^2 w}{\partial x\partial y} + z^2(0.5-\mu)\left(\frac{\partial \phi_x}{\partial y}\right)^2$$
$$+ 2z^2(0.5-\mu)\frac{\partial \phi_x}{\partial y}\cdot\frac{\partial \phi_y}{\partial x} + z^2(0.5-\mu)\left(\frac{\partial \phi_y}{\partial x}\right)^2 + 4(0.5-\mu)\left(\frac{\partial w}{\partial x}\right)^2 + 4(0.5-\mu)\frac{\partial w}{\partial x}\cdot\phi_x$$
$$\left. + (0.5-\mu){\phi_x}^2 + 4(0.5-\mu)\left(\frac{\partial w}{\partial y}\right)^2 + 4(0.5-\mu)\frac{\partial w}{\partial y}\cdot\phi_y + (0.5-\mu){\phi_y}^2\right)\partial x\,\partial y\,\partial z \qquad (20)$$

Substituting x = aR, y = bQ, z = tS and ∝ = b/a into Equation (20) gives the average strain energy as given in Equation (21).

$$U = \frac{abE^*t}{2}t^2\int_0^1\int_0^1\int_{-0.5}^{0.5}\left(S^2(1-\mu)\frac{1}{a^4}\left(\frac{\partial^2 w}{\partial R^2}\right)^2 + 2S^2(1-\mu)\frac{1}{a^3}\frac{\partial^2 w}{\partial R^2}\cdot\frac{\partial \phi_x}{\partial R} + 2S^2\mu\frac{1}{a^4\propto^2}\frac{\partial^2 w}{\partial R^2}\cdot\frac{\partial^2 w}{\partial Q^2}\right.$$
$$+ 2S^2\mu\frac{1}{a^3\propto}\frac{\partial^2 w}{\partial R^2}\cdot\frac{\partial \phi_y}{\partial Q} + 2\frac{S}{t^2}\mu\frac{1}{a^2}\frac{\partial^2 w}{\partial R^2}\cdot\frac{\partial w}{\partial S} + S^2(1-\mu)\frac{1}{a^2}\left(\frac{\partial \phi_x}{\partial R}\right)^2 + 2S^2\mu\frac{1}{a^3\propto^2}\frac{\partial^2 w}{\partial Q^2}\cdot\frac{\partial \phi_x}{\partial R}$$
$$+ 2S^2\mu\frac{1}{a^2\propto}\frac{\partial \phi_x}{\partial R}\cdot\frac{\partial \phi_y}{\partial Q} + 2\frac{S}{t^2}\mu\frac{1}{a}\frac{\partial \phi_x}{\partial R}\cdot\frac{\partial w}{\partial S} + S^2(1-\mu)\frac{1}{a^4\propto^4}\left(\frac{\partial^2 w}{\partial Q^2}\right)^2 + 2S^2(1-\mu)\frac{1}{a^3\propto^3}\frac{\partial^2 w}{\partial Q^2}\cdot\frac{\partial \phi_y}{\partial Q}$$
$$+ 2\frac{S}{t^2}\mu\frac{1}{a^2\propto^2}\frac{\partial^2 w}{\partial Q^2}\cdot\frac{\partial w}{\partial S} + S^2(1-\mu)\frac{1}{a^2\propto^2}\left(\frac{\partial \phi_y}{\partial Q}\right)^2 + 2\frac{S}{t^2}\mu\frac{1}{a\propto}\frac{\partial \phi_y}{\partial Q}\cdot\frac{\partial w}{\partial S} + \frac{(1-\mu)}{t^4}\left(\frac{\partial w}{\partial S}\right)^2$$
$$+ 4S^2(0.5-\mu)\frac{1}{a^4\propto^2}\left(\frac{\partial^2 w}{\partial R\partial Q}\right)^2 + 4S^2(0.5-\mu)\frac{1}{a^3\propto^2}\frac{\partial \phi_x}{\partial Q}\cdot\frac{\partial^2 w}{\partial R\partial Q} + 4S^2(0.5-\mu)\frac{1}{a^3\propto}\frac{\partial \phi_y}{\partial R}\cdot\frac{\partial^2 w}{\partial R\partial Q}$$
$$+ S^2(0.5-\mu)\frac{1}{a^2\propto^2}\left(\frac{\partial \phi_x}{\partial Q}\right)^2 + 2S^2(0.5-\mu)\frac{1}{a^2\propto}\frac{\partial \phi_x}{\partial Q}\cdot\frac{\partial \phi_y}{\partial R} + S^2(0.5-\mu)\frac{1}{a^2}\left(\frac{\partial \phi_y}{\partial R}\right)^2$$
$$+ 4(0.5-\mu)\frac{1}{a^2t^2}\left(\frac{\partial w}{\partial R}\right)^2 + 4(0.5-\mu)\frac{1}{at^2}\frac{\partial w}{\partial R}\cdot\phi_x + \frac{(0.5-\mu)}{t^2}{\phi_x}^2 + 4(0.5-\mu)\frac{1}{a^2\propto^2t^2}\left(\frac{\partial w}{\partial Q}\right)^2$$
$$\left. + 4(0.5-\mu)\frac{1}{a\propto t^2}\frac{\partial w}{\partial Q}\cdot\phi_y + \frac{(0.5-\mu)}{t^2}{\phi_y}^2\right)\partial R\,\partial Q\,\partial S \qquad (21)$$

Evaluating the integrals with respect to S in Equation (21), gives the internal work as:

$$U = \frac{abD_0}{2a^4}\int_0^1\int_0^1\left((1-\mu)\left(\frac{\partial^2 w}{\partial R^2}\right)^2 + 2(1-\mu)a\frac{\partial^2 w}{\partial R^2}\cdot\frac{\partial \phi_x}{\partial R} + 2\mu\frac{1}{\propto^2}\frac{\partial^2 w}{\partial R^2}\cdot\frac{\partial^2 w}{\partial Q^2} + 2\mu\frac{a}{\propto}\frac{\partial^2 w}{\partial R^2}\cdot\frac{\partial \phi_y}{\partial Q} + (1-\mu)a^2\left(\frac{\partial \phi_x}{\partial R}\right)^2\right.$$
$$+ 2\mu\frac{a}{\propto^2}\frac{\partial^2 w}{\partial Q^2}\cdot\frac{\partial \phi_x}{\partial R} + 2\mu\frac{a^2}{\propto}\frac{\partial \phi_x}{\partial R}\cdot\frac{\partial \phi_y}{\partial Q} + (1-\mu)\frac{1}{\propto^4}\left(\frac{\partial^2 w}{\partial Q^2}\right)^2 + 2(1-\mu)\frac{a}{\propto^3}\frac{\partial^2 w}{\partial Q^2}\cdot\frac{\partial \phi_y}{\partial Q}$$
$$+ (1-\mu)\frac{a^2}{\propto^2}\left(\frac{\partial \phi_y}{\partial Q}\right)^2 + 12(1-\mu)\frac{a^4}{t^4}\left(\frac{\partial w}{\partial S}\right)^2 + 4(0.5-\mu)\frac{1}{\propto^2}\left(\frac{\partial^2 w}{\partial R\partial Q}\right)^2 + 4(0.5-\mu)\frac{a}{\propto^2}\frac{\partial \phi_x}{\partial Q}\cdot\frac{\partial^2 w}{\partial R\partial Q}$$
$$+ 4(0.5-\mu)\frac{a}{\propto}\frac{\partial \phi_y}{\partial R}\cdot\frac{\partial^2 w}{\partial R\partial Q} + (0.5-\mu)\frac{a^2}{\propto^2}\left(\frac{\partial \phi_x}{\partial Q}\right)^2 + 2(0.5-\mu)\frac{a^2}{\propto}\frac{\partial \phi_x}{\partial Q}\cdot\frac{\partial \phi_y}{\partial R} + (0.5-\mu)a^2\left(\frac{\partial \phi_y}{\partial R}\right)^2$$
$$+ 48(0.5-\mu)\frac{a^2}{t^2}\left(\frac{\partial w}{\partial R}\right)^2 + 48(0.5-\mu)\frac{a^3}{t^2}\frac{\partial w}{\partial R}\cdot\phi_x + 12(0.5-\mu)\frac{a^4}{t^2}{\phi_x}^2 + 48(0.5-\mu)\frac{a^2}{\propto^2t^2}\left(\frac{\partial w}{\partial Q}\right)^2$$
$$\left. + 48(0.5-\mu)\frac{a^3}{\propto t^2}\frac{\partial w}{\partial Q}\cdot\phi_y + 12(0.5-\mu)\frac{a^4}{t^2}{\phi_y}^2\right)\partial R\,\partial Q \qquad (22)$$

where D_0 is given in Equations (23)

$$D_0 = \frac{Et^3}{12(1+\mu)(1-2\mu)} \qquad (23)$$

The external work is expressed in the non-dimensional form as:

$$V = ab\int_0^1\int_0^1 [qw]\,\partial R\,\partial Q \qquad (24)$$

Combining Equations (22) and (24) gives the total potential energy as:

$$\Pi = \frac{abD_0}{2a^4}\int_0^1\int_0^1\Bigg((1-\mu)\left(\frac{\partial^2 w}{\partial R^2}\right)^2 + 2(1-\mu)a\frac{\partial^2 w}{\partial R^2}.\frac{\partial\phi_x}{\partial R} + 2\mu\frac{1}{\propto^2}\frac{\partial^2 w}{\partial R^2}.\frac{\partial^2 w}{\partial Q^2} + 2\mu\frac{a}{\propto}\frac{\partial^2 w}{\partial R^2}.\frac{\partial\phi_y}{\partial Q} + (1-\mu)a^2\left(\frac{\partial\phi_x}{\partial R}\right)^2$$
$$+ 2\mu\frac{a}{\propto^2}\frac{\partial^2 w}{\partial Q^2}.\frac{\partial\phi_x}{\partial R} + 2\mu\frac{a^2}{\propto}\frac{\partial\phi_x}{\partial R}.\frac{\partial\phi_y}{\partial Q} + (1-\mu)\frac{1}{\propto^4}\left(\frac{\partial^2 w}{\partial Q^2}\right)^2 + 2(1-\mu)\frac{a}{\propto^3}\frac{\partial^2 w}{\partial Q^2}.\frac{\partial\phi_y}{\partial Q}$$
$$+ (1-\mu)\frac{a^2}{\propto^2}\left(\frac{\partial\phi_y}{\partial Q}\right)^2 + 12(1-\mu)\frac{a^4}{t^4}\left(\frac{\partial w}{\partial S}\right)^2 + 4(0.5-\mu)\frac{1}{\propto^2}\left(\frac{\partial^2 w}{\partial R\,\partial Q}\right)^2$$
$$+ 4(0.5-\mu)\frac{a}{\propto^2}\frac{\partial\phi_x}{\partial Q}.\frac{\partial^2 w}{\partial R\,\partial Q} + 4(0.5-\mu)\frac{a}{\propto}\frac{\partial\phi_y}{\partial R}.\frac{\partial^2 w}{\partial R\,\partial Q} + (0.5-\mu)\frac{a^2}{\propto^2}\left(\frac{\partial\phi_x}{\partial Q}\right)^2$$
$$+ 2(0.5-\mu)\frac{a^2}{\propto}\frac{\partial\phi_x}{\partial Q}.\frac{\partial\phi_y}{\partial R} + (0.5-\mu)a^2\left(\frac{\partial\phi_y}{\partial R}\right)^2 + 48(0.5-\mu)\frac{a^2}{t^2}\left(\frac{\partial w}{\partial R}\right)^2 + 48(0.5-\mu)\frac{a^3}{t^2}\frac{\partial w}{\partial R}.\phi_x$$
$$+ 12(0.5-\mu)\frac{a^4}{t^2}{\phi_x}^2 + 48(0.5-\mu)\frac{a^2}{\propto^2 t^2}\left(\frac{\partial w}{\partial Q}\right)^2 + 48(0.5-\mu)\frac{a^3}{\propto t^2}\frac{\partial w}{\partial Q}.\phi_y$$
$$+ 12(0.5-\mu)\frac{a^4}{t^2}{\phi_y}^2 - \frac{2qa^4w}{D_0}\Bigg)\partial R\,\partial Q \qquad (25)$$

2.5 Compatibility Equations and the Governing Equation

Carrying out minimization of the total potential energy functional with respect to the rotational displacements gives compatibility equations. Equation (26) and (27) are obtained when Equation (25) is minimized with respect to Φ_x and Φ_y respectively.

$$2(1-\mu)a\frac{\partial^3 w}{\partial R^3} + 2(1-\mu)a^2\frac{\partial^2\phi_x}{\partial R^2} + 2\mu\frac{a}{\propto^2}\frac{\partial^3 w}{\partial R\,\partial Q^2} + 2\mu\frac{a^2}{\propto}\frac{\partial^2\phi_y}{\partial R\,\partial Q} + 4(0.5-\mu)\frac{a}{\propto^2}\frac{\partial^3 w}{\partial R\,\partial Q^2} + 2(0.5-\mu)\frac{a^2}{\propto^2}\frac{\partial^2\phi_x}{\partial Q^2}$$
$$+ 2(0.5-\mu)\frac{a^2}{\propto}\frac{\partial^2\phi_y}{\partial R\,\partial Q} + 48(0.5-\mu)\frac{a^3}{t^2}\frac{\partial w}{\partial R} + 24(0.5-\mu)\frac{a^4}{t^2}\phi_x = 0 \qquad (26)$$

$$2\mu\frac{a}{\propto}\frac{\partial^3 w}{\partial R^2\,\partial Q} + 2\mu\frac{a^2}{\propto}\frac{\partial^2\phi_x}{\partial R\,\partial Q} + 2(1-\mu)\frac{a}{\propto^3}\frac{\partial^3 w}{\partial Q^3} + 2(1-\mu)\frac{a^2}{\propto^2}\frac{\partial^2\phi_y}{\partial Q^2} + 4(0.5-\mu)\frac{a}{\propto}\frac{\partial^3 w}{\partial R^2\,\partial Q} + 2(0.5-\mu)\frac{a^2}{\propto}\frac{\partial^2\phi_x}{\partial R\,\partial Q}$$
$$+ 2(0.5-\mu)a^2\frac{\partial^2\phi_y}{\partial R^2} + 48(0.5-\mu)\frac{a^3}{\propto t^2}\frac{\partial w}{\partial Q} + 24(0.5-\mu)\frac{a^4}{t^2}\phi_y = 0 \qquad (27)$$

Solving Equations (26) and (27) gives:

$$\Phi_x = \frac{C_R}{a}\frac{\partial w}{\partial R} \qquad (28)$$

$$\Phi_y = \frac{C_Q}{a\propto}.\frac{\partial w}{\partial Q} \qquad (29)$$

Where C_R and C_Q are constants.

Minimizing Equation (25) with respect to deflection, w, gives the governing equation as follows.

$$(1-\mu)\frac{\partial^4 w}{\partial R^4} + (1-\mu)a\frac{\partial^3\phi_x}{\partial R^3} + 2\mu\frac{1}{\propto^2}\frac{\partial^4 w}{\partial R^2\,\partial Q^2} + \mu\frac{a}{\propto}\frac{\partial^3\phi_y}{\partial R^2\,\partial Q} + \mu\frac{a}{\propto^2}\frac{\partial^3\phi_x}{\partial R\,\partial Q^2} + (1-\mu)\frac{1}{\propto^4}\frac{\partial^4 w}{\partial Q^4} + (1-\mu)\frac{a}{\propto^3}\frac{\partial^3\phi_y}{\partial Q^3}$$
$$+ 12(1-\mu)\frac{a^4}{t^4}\frac{\partial^2 w}{\partial S^2} + 4(0.5-\mu)\frac{1}{\propto^2}\frac{\partial^4 w}{\partial R^2\,\partial Q^2} + 2(0.5-\mu)\frac{a}{\propto^2}\frac{\partial^3\phi_x}{\partial R\,\partial Q^2} + 2(0.5-\mu)\frac{a}{\propto}\frac{\partial^3\phi_y}{\partial R^2\,\partial Q}$$
$$+ 48(0.5-\mu)\frac{a^2}{t^2}\frac{\partial^2 w}{\partial R^2} + 24(0.5-\mu)\frac{a^3}{t^2}\frac{\partial\phi_x}{\partial R} + 48(0.5-\mu)\frac{a^2}{\propto^2 t^2}\frac{\partial^2 w}{\partial Q^2} + 24(0.5-\mu)\frac{a^3}{\propto t^2}\frac{\partial\phi_y}{\partial Q}$$
$$- \frac{qa^4}{D_0} = 0 \qquad (30)$$

If Equations (28) and (29) are substituted into Equation (30) and simplified, it gives:

$$(1-\mu)(1+C_R)\frac{\partial^4 w}{\partial R^4}+\left(\frac{2\mu+\mu C_Q+\mu C_R+4(0.5-\mu)+2(0.5-\mu)C_R+\ 2(0.5-\mu)C_Q}{\propto^2}\right)\frac{\partial^4 w}{\partial R^2\,\partial Q^2}+\frac{(1-\mu)\left(1+C_Q\right)}{\propto^4}\frac{\partial^4 w}{\partial Q^4}+24(0.5-\mu)(2+C_R)\frac{a^2}{t^2}\frac{\partial^2 w}{\partial R^2}+\frac{24(0.5-\mu)\left(2+C_Q\right)}{\propto^2}\frac{a^2}{t^2}\frac{\partial^2 w}{\partial Q^2}+12(1-\mu)\frac{a^4}{t^4}\frac{\partial^2 w}{\partial S^2}-\frac{qa^4}{D_0}=0 \quad (31)$$

Equation (31) can be arranged as shown on Equation (32),

$$\left(\frac{\partial^4 w}{\partial R^4}+\frac{\nabla_1}{\propto^2}\frac{\partial^4 w}{\partial R^2\,\partial Q^2}+\frac{\nabla_2}{\propto^4}\frac{\partial^4 w}{\partial Q^4}-\frac{qa^4}{D_0(1-\mu)(1+C_R)}\right)+\left(\nabla_3\frac{\partial^2 w}{\partial R^2}+\frac{\nabla_4}{\propto^2}\frac{\partial^2 w}{\partial Q^2}\right)+\left(\nabla_5\frac{\partial^2 w}{\partial S^2}\right)=0 \quad (32)$$

Where: $\nabla_1, \nabla_2, \nabla_3,$ and ∇_5 are constants.

It can be seen from Equation (32) that one of the valid solutions is that each term in the brackets must be equal to zero. That is:

$$\frac{\partial^4 w}{\partial R^4}+\frac{\nabla_1}{\propto^2}\frac{\partial^4 w}{\partial R^2\,\partial Q^2}+\frac{\nabla_2}{\propto^4}\frac{\partial^4 w}{\partial Q^4}-\frac{qa^4}{D_0(1-\mu)(1+C_R)}=0 \quad (33)$$

$$\nabla_3\frac{\partial^2 w}{\partial R^2}+\frac{\nabla_4}{\propto^2}\frac{\partial^2 w}{\partial Q^2}=0 \quad (34)$$

$$\nabla_5\frac{\partial^2 w}{\partial S^2}=0 \quad (35)$$

Equation (35) can be written as

$$w_0.\nabla_5\frac{\partial^2 w_s}{\partial S^2}=0 \quad (36)$$

Where $w_0 =\ w_x w_y$ represents the in-plane component of the deflection and w_s represents the out-plane component of the deflection. For non-trivial solution of Equation (36), then:

$$\frac{\partial^2 w_s}{\partial S^2}=0 \quad (37)$$

From Equation (37),

$$\frac{\partial w_s}{\partial S}=k_1 \quad (38)$$

Therefore:

$$w_S=k_0+k_1 S \quad (39)$$

Since at the middle surface there is no strain, then, it can be deduced from Equation (38) that $k_1 = 0$ and substituting same in Equation (39) gives:

$$w_S=k_0 \quad (40)$$

Equation (40) suggests that z component of deflection, w_S of the middle surface of the plate is a constant and not a variable. Therefore, the deflection, w is a function of only x and y (or R and Q).

Solving Equation (33) gives the deflection of the plate, w in the form shown in Equation (41).

$$w=[1\ R\ R^2\ R^3\ R^4]\begin{bmatrix}a_0\\a_1\\a_2/2\\a_3/6\\a_4/24\end{bmatrix}\times[1\ Q\ \ Q^2\ Q^3\ Q^4]\begin{bmatrix}b_0\\b_1\\b_2/2\\b_3/6\\b_4/24\end{bmatrix} \quad (41)$$

Equation (41) can be written in a terse as:

$$w = A\,h \quad (42)$$

Where A and h are the coefficient of deflection and the shape function of the deflected curve respectively.

If Equation (42) is substituted into Equations (28) and (29), they give:

$$\Phi_x=\frac{AC_R}{a}\frac{dh}{dR}=\frac{B_R}{a}\frac{dh}{dR} \quad (43)$$

$$\Phi_y=\frac{AC_Q}{a\propto}.\frac{dh}{dQ}=\frac{B_Q}{a\propto}.\frac{dh}{dQ} \quad (44)$$

2.6 Bending Analysis

Bearing in mind that deflection, w is not a function of S, substituting Equations (42), (43) and (44) into Equation (25) and simplifying gives Equation (45).

$$\Pi = \frac{abD_0}{2a^4}\Bigg((1-\mu)[A^2 + 2AB_R + B_R{}^2]K_{RR} + \frac{1}{\alpha^2}[2\mu A^2 + 2\mu AB_Q + 2\mu AB_R + 2\mu B_R B_Q + 4(0.5-\mu)A^2 + 4(0.5-\mu)AB_R + 4(0.5-\mu)AB_Q + (0.5-\mu)B_R{}^2 + 2(0.5-\mu)B_R B_Q + (0.5-\mu)B_Q{}^2]K_{RQ} + \frac{(1-\mu)}{\alpha^4}[A^2 + 2AB_Q + B_Q{}^2]K_{QQ} + 12(0.5-\mu)\frac{a^2}{t^2}[4A^2 + 4AB_R + B_R{}^2]K_R + \frac{12(0.5-\mu)}{\alpha^2}\frac{a^2}{t^2}[4A^2 + 4AB_Q + B_Q{}^2]K_Q - \frac{2qa^4A}{D_0}K_q\Bigg) \quad (45)$$

Where:

$$K_{RR} = \int_0^1\int_0^1\left(\frac{\partial^2 h}{\partial R^2}\right)^2 dR\,dQ,\ K_{RQ} = \int_0^1\int_0^1\left(\frac{\partial^2 h}{\partial R\,\partial Q}\right)^2 dR\,dQ,\ K_{QQ} = \int_0^1\int_0^1\left(\frac{\partial^2 h}{\partial Q^2}\right)^2 dR\,dQ,$$

$$K_R = \int_0^1\int_0^1\left(\frac{\partial h}{\partial R}\right)^2 dR\,dQ,\ K_Q = \int_0^1\int_0^1\left(\frac{\partial h}{\partial Q}\right)^2 dR\,dQ\ K_q = \int_0^1\int_0^1 h\,dR\,dQ,$$

Minimizing Equation (45) with respect to B_R and B_Q produces Equations (46) and (47) respectively.

$$\left[2(1-\mu)K_{RR} + \frac{2(0.5-\mu)}{\alpha^2}K_{RQ} + 24(0.5-\mu)\left(\frac{a}{t}\right)^2 K_R\right]B_R + \left[\frac{1}{\alpha^2}K_{RQ}\right]B_Q + \left[2(1-\mu)K_{RR} + \frac{2(1-\mu)}{\alpha^2}K_{RQ} + 48(0.5-\mu)\left(\frac{a}{t}\right)^2 K_R\right]A = 0 \quad (46)$$

$$\left[\frac{1}{\alpha^2}K_{RQ}\right]B_R + \left[\frac{2(0.5-\mu)}{\alpha^2}K_{RQ} + \frac{2(1-\mu)}{\alpha^4}K_{QQ} + \frac{24(0.5-\mu)}{\alpha^2}\left(\frac{a}{t}\right)^2 K_Q\right]B_Q + \left[\frac{2(1-\mu)}{\alpha^2}K_{RQ} + \frac{2(1-\mu)}{\alpha^4}K_{QQ} + \frac{48(0.5-\mu)}{\alpha^2}\left(\frac{a}{t}\right)^2 K_Q\right]A = 0 \quad (47)$$

Solving Equations (46) and (47) simultaneously would give:

$$B_R = T_R.A \quad (48)$$

$$B_Q = T_Q.A \quad (49)$$

Where:

$$T_R = \frac{(m_{12}m_{23} - m_{13}m_{22})}{(m_{11}m_{22} - m_{12}m_{21})} \quad (50)$$

$$T_Q = \frac{(m_{21}m_{13} - m_{11}m_{23})}{(m_{11}m_{22} - m_{12}m_{21})} \quad (51)$$

$$m_{11} = 2(1-\mu)K_{RR} + \frac{2(0.5-\mu)}{\alpha^2}K_{RQ} + 24(0.5-\mu)\left(\frac{a}{t}\right)^2 K_R \quad (52)$$

$$m_{12} = m_{21} = \frac{1}{\alpha^2}K_{RQ} \quad (53)$$

$$m_{13} = 2(1-\mu)K_{RR} + \frac{2(1-\mu)}{\alpha^2}K_{RQ} + 48(0.5-\mu)\left(\frac{a}{t}\right)^2 K_R \quad (54)$$

$$m_{22} = \frac{2(0.5-\mu)}{\alpha^2}K_{RQ} + \frac{2(1-\mu)}{\alpha^4}K_{QQ} + \frac{24(0.5-\mu)}{\alpha^2}\left(\frac{a}{t}\right)^2 K_Q \quad (55)$$

$$m_{23} = \frac{2(1-\mu)}{\alpha^2}K_{RQ} + \frac{2(1-\mu)}{\alpha^4}K_{QQ} + \frac{48(0.5-\mu)}{\alpha^2}\left(\frac{a}{t}\right)^2 K_Q \quad (56)$$

If Equation (45) is minimized with respect to A, it gives:

$$2(1-\mu)[A + B_R]K_{RR} + \frac{2}{\alpha^2}[2\mu A + \mu B_Q + \mu B_R + 4(0.5-\mu)A + 2(0.5-\mu)B_R + 2(0.5-\mu)B_Q]K_{RQ} + \frac{2(1-\mu)}{\alpha^4}[A + B_Q]K_{QQ} + 48(0.5-\mu)\left(\frac{a}{t}\right)^2[2A + B_R]K_R + \frac{48(0.5-\mu)}{\alpha^2}\left(\frac{a}{t}\right)^2[2A + B_Q]K_Q - \frac{2qa^4}{D_0}K_q = 0 \quad (57)$$

If Equations (48) and (49) are substituted into Equation (57) and simplified, it gives:

$$2(1-\mu)A[1+T_R]K_{RR}+\frac{2}{\propto^2}A[2\mu+\mu T_Q+\mu T_R+4(0.5-\mu)+2(0.5-\mu)T_R+2(0.5-\mu)T_Q]K_{RQ}+\frac{2(1-\mu)A}{\propto^4}[1+T_Q]K_{QQ}+48(0.5-\mu)A\left(\frac{a}{t}\right)^2[2+T_R]K_R+\frac{48(0.5-\mu)}{\propto^2}A\left(\frac{a}{t}\right)^2[2+T_Q]K_Q=\frac{2qa^4}{D_0}K_q \quad (58)$$

Equation (58) can be written in the form shown in Equation (59)

$$AK_T=\frac{qa^4}{D_0}K_q \quad (59)$$

Where:

$$K_T=(1-\mu)[1+T_R]K_{RR}+\frac{1}{\propto^2}[2\mu+\mu T_Q+\mu T_R+4(0.5-\mu)+2(0.5-\mu)T_R+2(0.5-\mu)T_Q]K_{RQ}+\frac{(1-\mu)}{\propto^4}[1+T_Q]K_{QQ}+24(0.5-\mu)\left(\frac{a}{t}\right)^2[2+T_R]K_R+\frac{24(0.5-\mu)}{\propto^2}\left(\frac{a}{t}\right)^2[2+T_Q]K_Q \quad (60)$$

From Equation (59), it follows that:

$$A=\frac{K_q qa^4}{K_T D_0} \quad (61)$$

Substituting Equation (61) into Equations (48) and (49) respectively gives:

$$B_R=T_R.\frac{K_q}{K_T}\frac{qa^4}{D_0} \quad (62)$$

$$B_Q=T_Q.\frac{K_q}{K_T}\frac{qa^4}{D_0} \quad (63)$$

If Equations (61), (62) and (63) are substituted into Equations (42), (43) and (44) respectively, the following equations are obtained.

$$w=\frac{K_q}{K_T}\frac{qa^4}{D_0}h \quad (64)$$

$$\Phi_x=T_R.\frac{K_q}{K_T}\frac{qa^3}{D_0}\frac{dh}{dR} \quad (65)$$

$$\Phi_y=\frac{T_Q}{\propto}.\frac{K_q}{K_T}\frac{qa^3}{D_0}\frac{dh}{dQ} \quad (66)$$

Substituting $x=aR$, $y=bQ$, $z=tS$ and $\propto=b/a$ into Equations (13) to (18) bearing in mind that the deflection, w, is not a function of z or s gives the following.

$$\sigma_x=E^*\left[\frac{tS(1-\mu)}{a^2}\left(\frac{\partial^2 w}{\partial R^2}+a\frac{\partial\phi_x}{\partial R}\right)+\frac{tS\mu}{a^2\propto^2}\left(\frac{\partial^2 w}{\partial Q^2}+a\propto\frac{\partial\phi_y}{\partial Q}\right)\right] \quad (67)$$

$$\sigma_y=E^*\left[\frac{tS\mu}{a^2}\left(\frac{\partial^2 w}{\partial R^2}+a\frac{\partial\phi_x}{\partial R}\right)+\frac{tS(1-\mu)}{a^2\propto^2}\left(\frac{\partial^2 w}{\partial Q^2}+a\propto\frac{\partial\phi_y}{\partial Q}\right)\right] \quad (68)$$

$$\sigma_z=E^*\left[\frac{tS\mu}{a^2}\left(\frac{\partial^2 w}{\partial R^2}+a\frac{\partial\phi_x}{\partial R}\right)+\frac{tS\mu}{a^2\propto^2}\left(\frac{\partial^2 w}{\partial Q^2}+a\propto\frac{\partial\phi_y}{\partial Q}\right)\right] \quad (69)$$

$$\tau_{xy}=E^*\left[\frac{tS(0.5-\mu)}{a^2}\left(\frac{2}{\propto}\frac{\partial^2 w}{\partial R\,\partial Q}+\frac{a}{\propto}\frac{\partial\phi_x}{\partial Q}+a\frac{\partial\phi_y}{\partial R}\right)\right] \quad (70)$$

$$\tau_{xz}=E^*\left[\frac{(0.5-\mu)}{a}\left(2\frac{\partial w}{\partial R}+a\phi_x\right)\right] \quad (71)$$

$$\tau_{yz}=E^*\left[\frac{(0.5-\mu)}{a}\left(\frac{2}{\propto}\frac{\partial w}{\partial Q}+a\phi_y\right)\right] \quad (72)$$

Consequently, Equations (64), (65) and (66) are substituted into Equations (67) to (72), then Equations (12) and (23) substituted into the resultant Equations to obtain the equations for the stresses in non-dimensional form as:

$$\bar{\sigma}_x=\sigma_x.\frac{t^2}{qa^2}=12S\left[(1-\mu)(1+T_R)\frac{\partial^2 h}{\partial R^2}+\frac{\mu}{\propto^2}(1+T_Q)\frac{\partial^2 h}{\partial Q^2}\right]\frac{K_q}{K_T} \quad (73)$$

$$\bar{\sigma}_y=\sigma_y.\frac{t^2}{qa^2}=12S\left[\mu(1+T_R)\frac{\partial^2 h}{\partial R^2}+\frac{(1-\mu)}{\propto^2}(1+T_Q)\frac{\partial^2 h}{\partial Q^2}\right]\frac{K_q}{K_T} \quad (74)$$

$$\bar{\sigma}_z = \sigma_{z.}\frac{t^2}{qa^2} = 12S\mu\left[(1+T_R)\frac{\partial^2 h}{\partial R^2} + \frac{(1+T_Q)}{\propto^2}\frac{\partial^2 h}{\partial Q^2}\right]\frac{K_q}{K_T} \qquad (75)$$

$$\bar{\tau}_{xy} = \tau_{xy}.\frac{t^2}{qa^2} = 12\frac{S}{\propto}\left[(0.5-\mu)(2+T_R+T_Q)\frac{\partial^2 h}{\partial R\,\partial Q}\right]\frac{K_q}{K_T} \qquad (76)$$

$$\bar{\tau}_{xz} = \frac{\tau_{xz}}{q}\left(\frac{t}{a}\right) = 12\left[(0.5-\mu)(2+T_R)\frac{\partial h}{\partial R}\right]\frac{K_q}{K_T}\left(\frac{a}{t}\right)^2 \qquad (77)$$

$$\bar{\tau}_{yz} = \frac{\tau_{yz}}{q}\left(\frac{t}{a}\right) = 12\left[\frac{(0.5-\mu)}{\propto}(2+T_Q)\frac{\partial h}{\partial Q}\right]\frac{K_q}{K_T}\left(\frac{a}{t}\right)^2 \qquad (78)$$

For the out-of-plane displacement w, if Equation (23) is substituted into Equation (64) and simplified, it gives the non-dimensional form of the deflection, $\bar{w}$ as:

$$\bar{w} = w\frac{Et^3}{qa^4} = 12(1-2\mu)(1+\mu)\frac{K_q}{K_T}h \qquad (79)$$

3.0 Numerical Example

It is required to analyze a thick rectangular SSSS isotropic plate whose Poisson's ratio is 0.3. The in-plane normal stresses (σ_x) and (σ_y) are to be obtained at coordinate (0.5, 0.5, 0.5). The in-plane shear stress (τ_{xy}) is to be obtained at (0, 0, 0.5). The out-plane shear stress (τ_{xz}) is to be obtained (0, 0.5, 0), while the out-plane shear stress (τ_{yz}) is to be obtained at (0.5, 0, 0). Also, the transverse displacement (w) is to be obtained at (0.5, 0.5, 0). The shape function for the given plate is h = $(R - 2R^3 + R^4)(Q - 2Q^3 + Q^4)$.

The stiffness coefficients are: $K_{RR} = K_{QQ} = 0.23619$, $K_{RQ} = 0.23592$, $K_R = K_Q = 0.02390$, Kq = 0.04.

3.1 RESULT AND DISCUSSION

The results of the numerical examples are presented in Table 1 and Table 2 in comparison with the works of previous scholars.

Table 1 shows the values of the non-dimensional displacement, $\bar{w}$ of simply supported isotropic rectangular plate under uniformly distributed transverse load with those obtained by previous scholars. It is observed from Table 1 that the results of the lateral displacement (w) obtained from three-dimensional analysis vary with the results of refined plate theories. It is therefore seen that refined plate theories overestimate the lateral displacement of a plate. The variation of the lateral displacement obtained from this present study and that of Shimpi and Patel (2006) has a maximum percentage difference of 19.94% at a span to thickness $\left(\frac{a}{t}\right)$ value of 20 and aspect ratio $\left(\frac{b}{a}\right)$ of 1.0 while its minimum percentage difference is 11.65% occurring at a span to thickness $\left(\frac{a}{t}\right)$ value of 7.14286 and aspect ratio $\left(\frac{b}{a}\right)$ of 0.5. With the works of Reissner, the maximum percentage difference is 19.89% at a span to thickness $\left(\frac{a}{t}\right)$ value of 20 and aspect ratio $\left(\frac{b}{a}\right)$ of 1.0 while its minimum percentage difference is 10.99% occurring at a span to thickness $\left(\frac{a}{t}\right)$ value of 7.14286 and aspect ratio $\left(\frac{b}{a}\right)$ value of 0.5.

Table 4.1: Results of non-dimensional displacement, ($\bar{w}$) for a rectangular plate under uniformly distributed transverse load

Plate dimensional parameters		$\bar{w} = \frac{wE}{tq} x \frac{1}{2(1+\mu)}$ at x = a/2, y = b/2				
$\frac{b}{a}$	$\frac{a}{t}$	Present Study	Shimpi and Patel (2006), (**SP**)	Reissner (**R**). Taken from reference of Srinivas, (1970)	Percentage difference between Present study and previous works. (%) **SP**	**RO**
2.0	**20**	5850.92	6855.0	6852.9	17.16	17.12
	10	376.12	437.52	437.02	16.32	16.19
	7.14286	101.17	116.91	116.66	15.56	15.31
1.0	**20**	2302.15	2761.3	2760.0	19.94	19.89
	10	150.02	178.45	178.13	18.95	18.74
	7.14286	41.09	48.40	48.247	17.79	17.42
0.5	**20**	376.12	437.52	437.02	16.32	16.19
	10	25.96	29.604	29.492	14.04	13.61
	7.14286	7.57	8.452	8.4025	11.65	10.99

It is seen from Table 1, that the variation in the lateral displacement increases with an increase in the span to thickness $\left(\frac{a}{t}\right)$ value. All these suggest that the refined plate theory is not very suitable for thick plate analysis, hence a three-dimensional analysis is recommended for thick plate analysis.

Table 2 shows the results of the non-dimensional lateral displacement and stresses obtained from this study and that obtained by Ibearugbulem *et al*., (2021) for a square isotropic plate. It can be seen from Table 2 that the values of the non-dimensional lateral displacement, $\overline{w}$, obtained in this study follow similar trend with the values obtained by Ibearugbulem *et al*., (2021) as the values decrease with an increase in the span to thickness, $\frac{a}{t}$ value.

Table 4.2: Results of non-dimensional displacement and stresses for simply supported isotropic rectangular square plate under uniformly distributed transverse load

	$\overline{w} = \frac{100E_0t^3w}{qa^4}$ (0.5, 0.5, 0)	$\overline{\sigma}_x = \frac{\sigma_x t^2}{qa^2}$ (0.5, 0.5, 0.5)	$\overline{\sigma}_y = \frac{\sigma_y t^2}{qa^2}$ (0.5, 0.5, 0.5)	$\overline{\tau}_{xy} = \frac{\tau_{xy} t^2}{qa^2}$ (0, 0, 0.5)	$\overline{\tau}_{xz} = \frac{\tau_{xz} t}{qa}$ (0, 0.5, 0)
	$\frac{a}{t} = 4$				
Present study (**P**)	5.0159	0.34042	0.34042	-0.14524	0.26151
Ibearugbulem *et al*., (2021), (**I**)	4.8207	0.3832	0.3832	-0.1635	0.1976
% Diff. b/w **P** and **I**	3.89	12.57	12.57	12.57	24.44
	$\frac{a}{t} = 10$				
Present study (**P**)	3.9004	0.34048	0.34048	-0.14526	0.26154
Ibearugbulem *et al*., (2021), (**I**)	4.4039	0.3944	0.3944	-0.1683	0.2425
% Diff. b/w **P** and **I**	12.91	15.86	15.86	15.88	7.27
	$\frac{a}{t} = 100$				
Present study (**P**)	3.6900	0.34049	0.34049	-0.14527	0.26155
Ibearugbulem *et al*., (2021), (**I**)	4.3032	0.3971	0.3971	-0.1694	0.2533
% Diff. b/w **P** and **I**	16.62	16.65	16.65	16.63	3.14
	$\frac{a}{t} = 1000$				
Present study (**P**)	3.6879	0.34049	0.34049	-0.14527	0.26155
Ibearugbulem *et al*., (2021), (**I**)	4.3021	0.3971	0.3971	-0.1694	0.2534
% Diff. b/w **P** and **I**	16.65	16.65	16.65	16.63	3.10

The variation in the values of the non-dimensional lateral displacement, $\overline{w}$ obtained in this present study and that obtained by Ibearugbulem *et al*., (2021) has a maximum percentage difference of 16.65% and a minimum value of percentage difference of 3.89% at span to thickness, $\frac{a}{t}$ values of 1000 and 4 respectively.

This shows that as span to thickness, $\frac{a}{t}$ values get smaller, the results obtained by the Alternative II refined plate theory approaches the value obtained using a third order shear deformation theory by Ibearugbulem *et al*., (2021). For the non-dimensional parameters of the in-plane normal stresses, $\overline{\sigma}_x$ and $\overline{\sigma}_y$, the maximum percentage difference is obtained as 16.65% at a span to thickness, $\frac{a}{t}$ value of 100 and above while the least percentage difference is obtained as 12.57% at a span to thickness, $\frac{a}{t}$ value of 4. The variation in the values of the non-dimensional parameters of the in-plane shear stress, $\overline{\tau}_{xy}$ follows the same trend as that of the in-plane normal stresses. It has a maximum percentage difference of 16.63% at a span to thickness, $\frac{a}{t}$ value of 100 and above while the least percentage difference is obtained as 12.57% at a span to thickness, $\frac{a}{t}$ value of 4. For the transverse shear stress, $\overline{\tau}_{xz}$ the variation has a maximum percentage difference of 24.44% at a span to thickness, $\frac{a}{t}$ value of 4 while the least percentage difference is obtained as 3.10% at a span to thickness, $\frac{a}{t}$ value of 1000. This indicates that the values of the transverse shear stress, $\overline{\tau}_{xz}$ obtained using the Alternative II theory varies more significantly with those obtained using third order shear deformation theory for thick plates than in thin plates.

4.0 CONCLUSION

From the results obtained from this study, it can be concluded that refined plate theories overestimate the lateral displacements in thick plates. Hence, three-dimensional analysis is recommended. Also, it is seen from the comparison done with the works of previous authors that the Alternative II plate theory produced reasonably results, hence can be used in thick plate analysis.

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