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Original Research Article

Relationship between the Intrinsic Properties of Sands and the Parameters of Mathematical Particle Size Distribution Models for Predicting Geotechnical Quantities

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Abstract

This work characterizes the relationships between the intrinsic properties of sands and the parameters of four mathematical models that best simulate the experimental curves and geotechnical properties of sands used in construction. Origin.Pro.2019" software was used to smooth the grading curves, define the parameters of the mathematical models and link them to the geotechnical data. To achieve this objective, the correlations between the intrinsic properties of the sands are developed using mathematical models with the highest coefficient of determination (R^2) and the lowest statistical coefficient (χ^2). The correlations used are those with a coefficient of determination greater than or equal to 0.9. The results obtained show that the models used provide a good description of the experimental curves. The model parameters are correlated with the granulometric fractions and the geotechnical parameters. The evolution of the points expressing the parameters of the Gaussian and exponential models (A1, Xc, A, W, Yo) and the parameter (t1) as a function of seven randomly chosen geotechnical quantities, are polylinear and linear fits, respectively. This study is important for predicting a geotechnical quantity from a modelled grading curve, by solving the mathematical expressions of the models used.

Keywords: Intrinsic properties, correlation, experimental curve, predict, mathematical models, Particle size.

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1. INTRODUCTION

The construction of basic infrastructure in cities leads to considerable consumption of cement and sand as building materials. Existing resources such as sand are sometimes depleted and, in some cases, the expansion of cities is pushing people to settle on sand deposits, prompting researchers to look for alternative materials. The consumption of construction materials is 5% per year [1] for cement and 5.5% [2] for sand, which means that quality tests must be carried out before materials are used to guarantee the safety of the works. The city of Brazzaville, plagued by erosion, is the victim of an environmental problem: the silting up of certain districts and waterways. The depletion of natural sandpits can be compensated for by sand from erosion, watercourses and the crushing of rock materials. Sustainable urban development does not only mean prioritizing the future of the construction industry, but also solving all the problems linked to the environment [3]. To solve the problem of silting up of certain districts, watercourses and crusher sand, more precautions need to be taken. To test cement, some African countries import standardized sand, specified for example by European standard EN 196-1 [4], American standard ASTM C778 [5] and Indian standard ISO 650 [6]. The use of foreign standards requires laboratory testing equipment, chemicals and materials, such as the standardized sand used to test cement. Cement quality depends on standard sand, which in many African countries is an imported material. However, to reduce the environmental impact of the silting up of watercourses and certain districts, the use of erosion sand and rock crushing, non-conventional materials (recycled or natural), can be an alternative solution if preceded by the necessary geotechnical studies [7, 8, 9]. These sands could be used to formulate standard sands as part of the local materials development process, reducing the cost of laboratory testing and creating jobs by producing standard sands in situ. The

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perfect reproduction of standard sand would make it easier to compare results from one laboratory to another. However, a correlation between the mechanical properties of a mortar based on standard local sand and those of a mortar based on standard sand is sufficient to assess the quality of cements [3]. Nine sands commonly used in construction in Brazzaville were characterized in order to find a substitute for standard imported sand among these sands and their mixtures. The properties of the sands, the strength class of the cement and mortars and the relationships between the intrinsic properties of the mortars will be determined. As the percentage of fines in Brazzaville sands is less than 20%, the arrangement of the grains is influenced by the proportions of large and small spheres in relation to their diameter [10]. The particle size distribution is commonly used to classify sands and define their standards of use in geotechnical engineering. The particle size distribution makes it possible to predict the behavior of sands and their quantities according to certain points on the particle size curve (D10, D30, D60, etc.) [11]. The hydraulic, mechanical, chemical and thermal properties of soils depend on their particle size distribution [12-14]. Several mathematical models have been defined to better describe grain size distribution in soil [15]. Evaluation of these models shows that the performance of a model depends on the texture, the number of parameters and the number of grain distribution modes, which may be unimodal or multimodal. If mathematical models are unable to describe a given type of soil, it will not be possible to relate the model parameters to soil behavior. Bayat et al. [15] studied the effect of model parameters on the shape of the particle size curve, which made it easier for them to find possible models and potential parameter values. The relationships between the model parameters and the granulometric fractions of the soil remain undetermined and depend on the geology of the soil, which can change from one point to another. Several studies have established relationships between these quantities and behavior for certain soils and these relationships are not valid for all soils [16, 17]. The link between soil behavior and granulometry is established by mathematical models, which are commonly used to simulate the results of a scientific experiment and predict certain aspects of it [18-20]. The mathematical representation of the particle size distribution can be used to classify a soil using the most appropriate parameters. The relationships between the parameters of the particle size distribution models of reformulated local sands and their geotechnical properties have not yet been reported. The aim of this work is to study the existing mathematical models of particle size distribution that best apply to the particle size curves of river sands, erosional sands or reformulated sands. The links between the intrinsic properties of the sands and the parameters of the models are determined.

2. MATERIALS AND METHODS

2.1. Materials

The local sands used were collected from various locations, as shown in Table 1 below. In what follows, the sand collection sites will be designated by the letter S representing the sand, followed by a number, as shown in Table 1. They are among the sands most commonly used for construction in the city of Brazzaville, capital of the Republic of Congo. Three sands (S2, S3, S7) were extracted from the Congo River and (S2, S3) the Djoué quarry (S7) respectively. Three sands (S5, S6, S8) came from erosion in three districts of the city of Brazzaville and two sands (S1, S4) were taken from natural sandpits. The sand (S9) is obtained after crushing the Inkissi sandstone, taken from the south of the city of Brazzaville.

Table 1: Geographical location of sampling sites

Site	Sample	Localization
Matéssama	S1	15°16.05'E ; 04°06.04'S
Fleuve	S2	15°22.56'E ; 04°17.35'S
Yoro	S3	15°22.56'E ; 04°17.24'S
Académie	S4	15°18.24'E ; 04°08.14'S
Nkombo	S5	15°20.49'E ; 04°17.38'S
Mfilou	S6	15°19.74'E ; 04°20.15'S
Djoué	S7	15°17.35'E ; 04°19.21'S
Mboualé	S8	15°17.46'E ; 04°12.21'S
Concassé	S9	15°30.43'E ; 04°29.36'S

2.2. Methods

The particle size analysis of the various sands was carried out in accordance with standard NE 933-1 [21]. For each sample, 200 g, previously steamed for 24 h at 105°C, were passed through a series of mechanized 0.08, 0.125, 0.16, 0.25, 0.5, 1-, 1.6- and 2-mm sieves. Sand equivalent is an indicator used in geotechnical engineering to characterize the cleanliness of sand. It indicates the content of fines (elements with a diameter of less than 0.5 mm), essentially of clay, vegetable or organic origin on the surface of the grains, defined in accordance with standard NF EN 933-8 [22]. The granulometric fraction is deduced from the recommendations of the granularity nomograms, which consider coarse sand = Csa (0.63 – 2 mm); medium sand = Msa (0.2 - 0.63 mm); fine sand = Fsa (0.063 - 0.2 mm).

Density is used to match masses and volumes. It can also be used to determine the weight of the material in accordance with ISO/TS 17892-4 [23]. Absolute density is the ratio between the mass of the material and its actual volume, minus the volume of the pores (open and closed). It is equal to the real density in the case of non-porous materials and is defined in accordance with standard P18-558 [24].

The absorption coefficient, defined as the ratio between the mass of water absorbed by the sample after soaking in water and the dry mass of the sample. This soaking is obtained by immersing the sample in water for 24 hours at 20°C, measured in accordance with standard NF P 18-555 [25]. The modulus of fineness and their permissible ranges, uniformity coefficients and curvature coefficients are defined in accordance with the respective standards EN 196-1 [4] and NF P18-540 [26]. The uniformity coefficients UC and curvature coefficients CC were used to characterize the grain size of the sands, according to the following formulae:

$$UC = {}^{D_{60}}/_{D_{10}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

$$CC = {}^{D_{30}}/_{D_{10} * D_{60}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

Where, Dx is the particle size corresponding to x% by weight of the sieve.

The sands used in this study were selected in accordance with standards EN 196-1 [4], ASTM C778 [5], ISO 650 [6] used to select local sands that correspond to the range of standard sands; and standards EN 196-1 [4], SABS 1090 [27], BS 1200 [28] to select local sands corresponding to the range of sands used in the manufacture of cement mortars.

2.3. Modelling particle size curves

Four (4) empirical mathematical models were used to model the particle size distribution curves. These were the four-parameter Gauss model, the five-parameter exponential model, the three-parameter Li Yong model and the two-parameter Weibull model, the mathematical expressions of which are presented in Table 2. The quality of the smoothing of the experimental curves, based on the coefficient of determination, made it possible to determine the model that best describes the experimental curves. The models were evaluated on the basis of the coefficient of determination (R^2) and the Akaike information criterion (AIC) test, which take into account the number of variables in the model. The best fit of the theoretical curve to the experimental curve was judged by the model with the highest coefficient of determination, taken as the first basic criterion, the value of the chi-square (χ^2) and the value of the standard deviation. The Chi - sqr (χ^2) - is used to test the independence between two random variables.

The principle is to insert the mathematical model (Table 2) into the Origin Pro 2019b software, specifying the model parameters. After processing, the Origin Pro 2019b software presents the smoothing or theoretical particle size curves with the new values of the model parameters and the values of the statistical parameters. Once the particle size curves have been modelled, the parameters of the two models are compared with each other. The model with the highest coefficient of determination (R^2) and the lowest possible RSS and AIC is the one that provides the best response. In Table 2 are represented the expressions of the mathematical models used for the smoothing of the particle size curves.

Table 2: Mathematical expressions for the models used for smoothing the experimental sieve size

Models	**Mathematical expressions**	**Parameters**
Exponential	$p(d) = A1 * exp\left(-\frac{d}{t1}\right) + A2 * exp\left(-\frac{d}{t2}\right) + \ y0$ (3)	A1, t1, A2, t2, Y0
Gauss	$p(d) = y0 + \frac{A}{w\sqrt{\frac{\pi}{2}}} exp(-2(\frac{(d-xc)^2}{w^2}))$ (4)	Y0, A, w, xc
Li Yong	$p(d) = 1 - cd^{-\mu} exp(-\frac{d}{d_c})$ (5)	C, µ , d
Weibull	$\mathrm{p}(d) = 1 - exp(-(\frac{d}{b})^c)$ (6)	b, c

$p(d)$ is the fractionof soil with a grain diameter smaller than *d* and d_{max} and d_{min} are the maximum and minimum diameters respectively the experimental curves.

The Origin Pro 2019b software was used in the process of implementing the relationships between the experimental grain distribution curve obtained in the laboratory and the theoretical curve obtained from the mathematical models. The statistical parameters used to assess the correlation between the experimental curves and the theoretical curves corresponding to the different equations of the mathematical models are the R^2, the AIC and the RSS. These parameters are calculated automatically using Origin Pro 2019b software. The expressions for these parameters are as follows:

The coefficient of determination (R^2) is one of the first criteria for predicting the best equation for describing particle size curves.

$$R^2 = \frac{\sum_{i=1}^{N}\left(W*_{pre,i} - W*\bar{}_{pre,}\right)^2}{\sum_{i=1}^{N}\left(Nr*_{exp,i} - W*\bar{}_{exp,i}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(7)}$$

$W *_{pre,i}$: average reduced water content;
$W *\bar{}_{pre,}$: represents the i th predictive reduced mass;
$W *\bar{}_{exp,i}$: represents the i th reduced mass;
N : Number of observations.

RSS: the sum of residual squares. It is used to estimate the differences between the theoretical data points and the experimental data. The residual sum of squares is calculated using the following formula:

$$RSS = \sum_{i=1}^{n} e_i = \sum_{i=1}^{n} (y - y *_i)^2 = \sum_{i=1}^{n} w_i [y_i - (\beta_o + \beta_1 x_i)]^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

AIC: Akaike Information Criterion is a model comparison parameter, used to find the model that most closely matches the reality presented by the experimental data.

$$AIC = \begin{cases} Nln\left(\frac{RSS}{N}\right) + 2K, si\ \frac{N}{K} \geq 40 \\ Nln\left(\frac{RSS}{N}\right) + 2K + \frac{2K(K+1)}{N-K-1}, si\ \frac{N}{K} < 40 \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Where N is the number of points on the curve, K is the number of model parameters plus 1 and RSS is the sum of residual squares.

Choosing the best-performing model

A model simulates a particle size curve better when its coefficient of determination is close to unity, its RSS close to zero and its AIC low.

3. RESULTS AND DISCUSSION

3.1 RESULTS

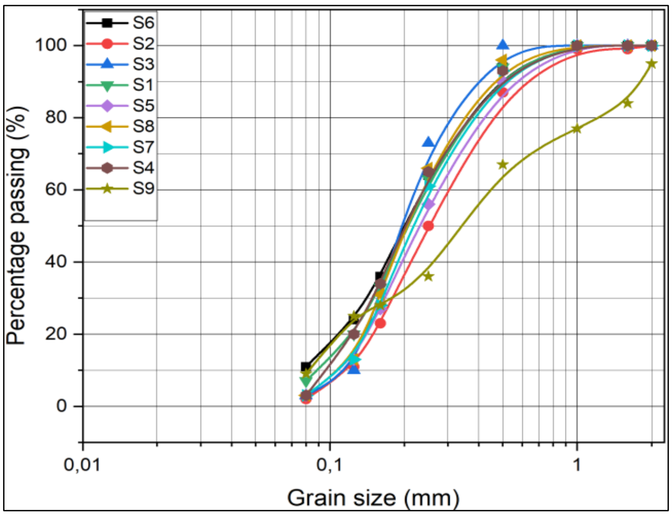

Figure 1: Sieve size curves for nine natural sands

The grain size curves in Figure 1 show concavities pointing downwards, suggesting that these sands are very fine. With the exception of sand S9, which is made up of 5 mm grains, the maximum grain diameter of the other eight sands is 1 mm and they all have more or less the same appearance. In other words, all eight sands (S1-S8) have the same eolian origin [30, 31].

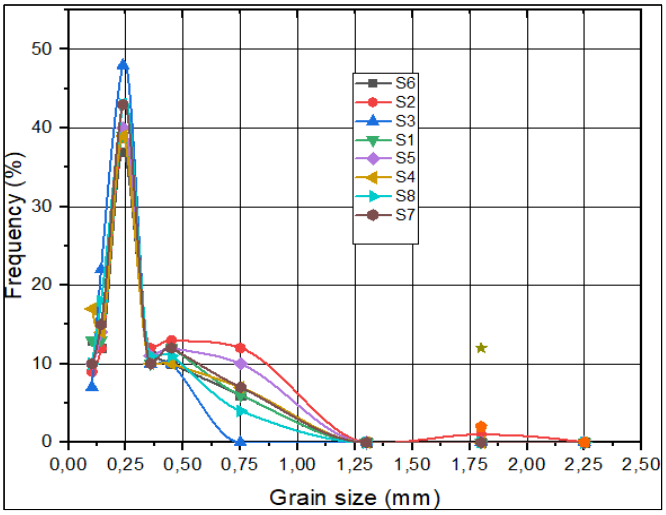

Figure 2: Evolution of the frequency of grain size for natural sands (S1-S8)

The Figure 2 shows the frequency curves for natural sands. It can be seen that the frequency curves for local natural sands have more or less the same shape and show two modes, one between 0.125 and 0.25 mm and the other between 0.375 and 1 mm. The presence of these two modes indicates granular differentiation in these sands. Of these two modes, the highest is found in the 0.125 - 0.25 mm ranges, where fine sands are found. These sands are mainly made up of fine particles. Standard sand with the highest mode between 0.625 and 0.75 mm is close to coarse sand with a grain diameter between 0.5 and 1 mm.

Table 3: Physical characteristics of natural sands

Sand	WA	BA	AD	FM	UC	CC
S1	0.19	1.77	2.60	1.23	2.67	1.04
S2	0.33	1.57	2.63	1.53	2.5	0.9
S3	0.16	1.5	2.62	1.17	1.69	0.8
S4	0.22	1.78	2.61	1.22	2.3	0.98
S5	0.42	1.76	2.61	1.43	2.45	0.97
S6	0.38	1.48	2.61	1.18	3.29	1.28
S7	0.27	1.58	2.58	1.33	2.27	0.93
S8	0.56	1.65	2.61	1.25	2 .09	1.01
S9	3.42	1.48	2.61	1.8	6.0	0.6

WA - water absorption (%); BA - bulk density (T/m^3); AD - absolute density (T/m^3); FM - fineness module; UC - uniformity coefficient; CC - curvature coefficient.

Table 3 shows that crusher sand S9 has the highest water absorption rate for a fines content of 11%. The bulk densities of the sands (S1-S9) are not within the permitted range of (2.63-267 T/m^3) [30]. Their absolute densities are close, which means that these local sands may have similar mineralogy and chemical composition. The bulk densities of sands S3, S6 and S9 are similar to those of dune sands [30].

Modelling of particle size distribution curves for natural and improved sands with S9 sand

The results obtained from the modelling of sands particle size curves show that the curves of the selected models follow the experimental curves well. The model selected is the best of the four models for natural sand (group 1) and sand enhanced with S9 crusher sand (group 2). In other words, the Gauss model has the highest coefficient of determination of the four models and is the best for improved natural sands. The exponential model is the best for natural sands. The models selected are those with a coefficient of determination greater than 0.9 and the lowest Chi-sqr. The exponential model better simulates the experimental curves of natural sands, and the Gaussian model better smoothes the experimental curves of natural sands improved with S9 sand. The statistical parameter values of the smoothed Experimental sieve size curves for natural sands are shown in Tables 4 & 5.

Table 4: Statistical parameters of the Gauss model after smoothing the particle size curves of natural sands

Model	**GAUSS**							
Equation	$p(d) = y0 + \frac{A}{w\sqrt{\frac{\pi}{2}}} exp(-2(\frac{d-xc}{w})$ (5)							
Sands	S1	S2	S3	S4	S5	S6	S7	S8
y0	-3986.01	-3981.29	99.94	-4085.77	-4182.27	-5728.27	-4211.78	-4322.22
xc	1.34	1.37	0.072	1.34	1.36	1.34	1.344	1.33
w	11.67	11.49	0.22	11.74	11.71	14.29	11.66	11.74
A	60024.28	59021.1	-26.89	61813.9	63114.5	104717.5	63243.77	65326.55
Chi-Sqr	332.88	252.42	4.49	375.16	296.20	294.01	369.05	451.3
R^2	0.88	0.92	0.998	0.87	0.90	0.88	0.88	0.85

With (y0, xc, w, A) - parameters of the GAUSS mathematical model, Chi - sqr (χ^2) - is used to test the independence between two random variables, R^2 - coefficient of determination

Table 5: Statistical parameters of the Exponential model after smoothing the particle size curves of natural sands

Model	**Exponential**							
Equation	$p(d) = A1 * exp\left(-\frac{d}{t1}\right) + A2 * exp\left(-\frac{d}{t2}\right) + y0$							
Sands	S1	S2	S3	S4	S5	S6	S7	S8
y0	101.12	100.84	102.01	100.7	101.22	101.06	101.27	101.40
A1	-75.84	-73.17	-87.37	-81.11	-75.57	-72.01	-79.31	-83.61
t1	0.19	0.24	0.16	0.18	0.22	0.192	0.196	0.18
A2	-73.32	-71.31	-90.18	-76.82	-73.39	-69.64	-76.73	-80.72
t2	0.19	0.24	0.16	0.18	0.22	0.192	0.196	0.18
Chi-Sqr	21.04	24.05	107.82	11.46	23.81	16.45	37.17	50.35
R^2	0.99	0.994	0.974	0.996	0.99	0.995	0.99	0.99

With (y0, A1, t1, A2, t2) - parameters of the Exponential mathematical model, Chi - sqr (χ^2) - is used to test the independence between two random variables, R^2 - coefficient of determination

Table 6: Classification of models according to statistical quantities for natural sands

Model	Group 1	Group 2
	Exponential	Gauss
S6	1	2
S2	1	2
S3	2	1
S1	1	2
S5	1	2
S8	1	2
S7	1	2
S4	1	2

Tables 4 and 5 show that the Exponential model has the highest coefficient of determination ($R^2 > 0.9$) and the lowest possible Chi-sqr (χ^2) compared with the Gaussian mathematical model (Table 6).

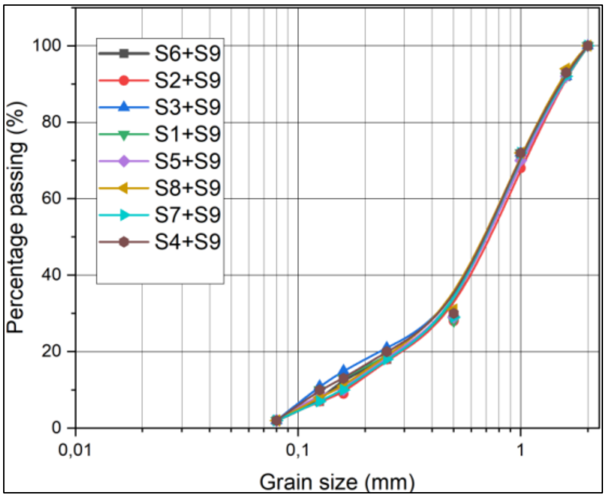

Figure 3: Sieve size curves for eight natural sands improved with S9 sand

Figure 3 shows that local sands improve with the addition of crushed sand (S9), thus entering the range of standardized sands, in accordance with standard EN 196-1 [4]. The grading curves show an ascending concavity, reflecting the presence of coarse elements in the natural sands (S1-S8) improved by the addition of crushed sand S9.

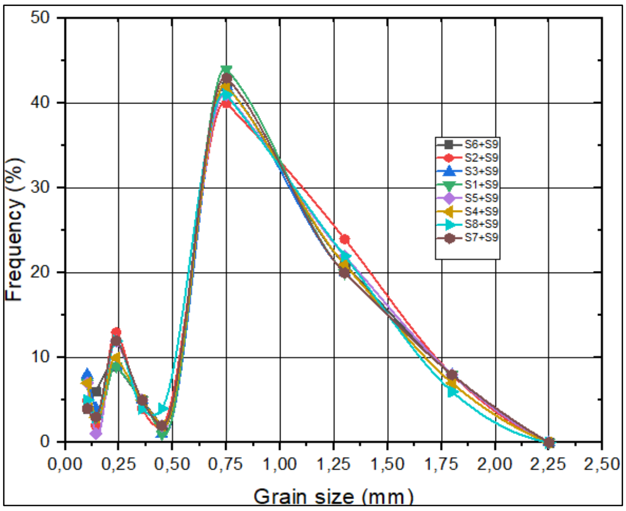

Figure 4: Evolution of the frequency of grain size for natural sands improved with S9 sand

When, natural sands are improved by adding S9 sand, the frequency curves follow the same pattern as those for standard sand limits [4] and show two modes, the highest of which is between 0.625 and 0.75 mm.

Table 7: Statistical parameters of the Gauss model after smoothing the particle size curves of sands improved by adding S9

Model	**GAUSS**							
Equation	$p(d) = y0 + \frac{A}{w\sqrt{\frac{\pi}{2}}} exp(-2(\frac{d-xc}{w})$ (3)							
Sands	S1+S9	S2+S9	S3+S9	S4+S9	S5+S9	S6 +S9	S7+S9	S8+S9
y0	-8.506	-24.625	-17.766	-19.7	-21.392	-18.15	-17.698	-30.949
xc	1.51	1.651	1.566	1.576	1.597	1.561	1.529	1.616
w	1.475	1.882	1.733	1.75	1.785	1.686	1.624	1.944
A	187.419	275.851	238.956	247.693	253.997	233.363	224.455	304.778
Chi-Sqr	19.505	9.934	24.839	17.673	12.86	14.276	10.239	9.834
R^2	0.991	0.995	0.989	0.992	0.994	0.994	0.995	0.996

With, (y0, xc, w, A) - parameters of the GAUSS mathematical model, Chi - sqr (χ^2) - is used to test the independence between two random variables, R^2 - coefficient of determination.

Table 8: Statistical parameters of the Exponential model after smoothing the particle size curves of sands improved by adding S9

Model	**Exponential**							
Equation	$p(d) = A1 * exp\left(-\frac{d}{t1}\right) + A2 * exp\left(-\frac{d}{t2}\right) + y0$							
Sands	S1+S9	S2+S9	S3+S9	S4+S9	S5+S9	S6 +S9	S7+S9	S8+S9
y0	176.882	196.569	186.766	168.879	186.778	170.491	162.953	166.443
A1	-58.441	-95.803	-1.51	-71.108	-5.913	-67.043	-69.631	-80.941
t1	2.075	2.414	0.005	1.913	0.003	1.955	1.802	1.825
A2	-121.1	-104.76	-185.90	-100.51	-189.32	-107.3	-98.44	-89.86
t2	2.074	2.414	2.309	1.913	2.26	1.955	1.801	1.825
Chi-Sqr	58.17	26.973	45.353	43.502	33.568	41.084	40.39	28.379
R^2	0.983	0.992	0.986	0.987	0.99	0.988	0.989	0.992

With (y0, A1, t1, A2, t2) - parameters of the Exponential mathematical model, Chi - sqr (χ^2) - is used to test the independence between two random variables, R^2 - coefficient of determination

Table 9: Classification of models according to statistical quantities for improved sands

Mixe	**Group 1**	**Group 2**
	Gauss	**Exponential**
S6 +S9	1	2
S2 + S9	1	2
S3 + S9	1	2
S1 + S9	1	2
S5 + S9	1	2
S8 + S9	1	2
S7 + S9	1	2
S4 + S9	1	2

Tables 7 and 8 show that the Gaussian model has the highest coefficient of determination (R^2) and the lowest Chi-sqr of the two mathematical models. Table 9 shows that the Gaussian model performs better than the exponential model. The Gaussian model simulates well the shape of the grading curves of natural sands improved with S9 crusher sand.

Table 10: Properties of local sands improved with S9 crusher sand

Sand	**WA**	**BA**	**AD**	**FM**	**UC**	**CC**
S1+S9	3.42	1.5	2.59	2.71	6.38	2.5
S2+S9	3.98	1.5	2.63	2.75	5.12	1.43
S3+S9	3.33	1.49	2.61	2.66	6.83	2.54
S4+S9	3.38	1.49	2.63	2.68	6.31	2.35
S5+S9	3.45	1.5	2.63	2.73	5.31	1.91
S6+S9	3.43	1.51	2.59	2.68	6.38	2.41
S7+S9	3.38	1.46	2.63	2.70	5.19	1.96
S8+S9	3.78	1.5	2.61	2.69	5.47	1.8

WA - water absorption (%); BA - bulk density (T/m^3); AD - absolute density (T/m^3); FM - fineness module; UC - uniformity coefficient; CC - curvature coefficient.

Table 11: Values of the correlation coefficients obtained between the parameters of the models used and the geotechnical quantities as well as the different granulometric fractions of natural sands.

Exponential					
Parameters	y0	A1	t1	A2	t2
FM	-0.369	0.468	0.941	0.436	0.941
UC	-0.6	0.892	0.372	0.883	0.372
CC	-0.413	0.605	-0.002	0.653	-0.002
Ad	-0.421	0.035	0.11	0.209	0.11
AD	-0.231	0.202	0.328	0.124	0.328
n	0.402	-0.021	-0.086	-0.199	-0.086
c	-0.402	0.021	0.086	0.199	0.086
e	0.398	-0.012	-0.102	-0.194	-0.102
Fsa	0.869	-0.585	-0.504	-0.751	-0.504
Msa	-0.349	0.496	0.766	0.459	0.75
Csa	0.602	0.793	0.994	0.772	0.994

With, AD - absolute density (T/m^3); Ad - Apparent density (T/m^3); FM - fineness module; UC - uniformity coefficient; CC - curvature coefficient; n - porosity; c - compactness (T/m^3); e - voids index; Fsa - fine sand (%); Msa - medium sand (%); Csa - coarse sand; Parameters of the Exponential model (y0, A1, t1, A2, t2).

The geotechnical quantities (FS, MS, CS) shown in table 11 are taken from figure 1, deduced from the recommendations of the granularity nomograms, which consider sands (fine, medium, coarse) in the range 0.06-2 mm.

Table 12: Values of the correlation coefficients obtained between the parameters of the models used and the geotechnical quantities as well as the different grain size fractions of sands improved with crushed sand S9

GAUSS				
Parameters	y_0	X_c	w	A
FM	0.622	0.254	-0.564	-0.619
UC	-0.031	-0.05	0.148	0.101
CC	-0.353	-0.584	0.096	0.25
Ad	0.218	-0.033	-0.127	-0.179
AD	-0.063	-0.224	-0.052	0.02
n	-0.18	-0.08	0.064	0.132
c	0.18	0.08	-0.064	-0.132
e	-0.178	-0.079	0.063	0.131
Fsa	-0.386	0.421	0.591	0.493
Msa	-0.515	-0.491	0.232	0.385
Csa	0.813	0.035	-0.791	-0.818

AD - absolute density (T/m^3); Ad - Apparent density (T/m^3); FM - fineness module; UC - uniformity coefficient; CC - curvature coefficient; n - porosity; c - compactness (T/m^3); e - voids index; Fsa - fine sand (%); Msa - medium sand (%); Csa - coarse sand; Parameters of the Gauss model (y0, xc, w, A).

The geotechnical quantities (FS, MS, CS) shown in table 12 are taken from figure 3, deduced from the recommendations of the granularity nomograms, which consider sands (fine, medium, coarse) in the range 0.06-2 mm.

The coefficients of determination for the mathematical Gauss model are of the order of R^2 (0.989-0.996) and for the exponential model of the order of R^2 (0.983 - 0.992).

Correlations between exponential model parameters (A1, t1, Yo) and geotechnical parameters (UC, Fsa, Csa) of natural sands.

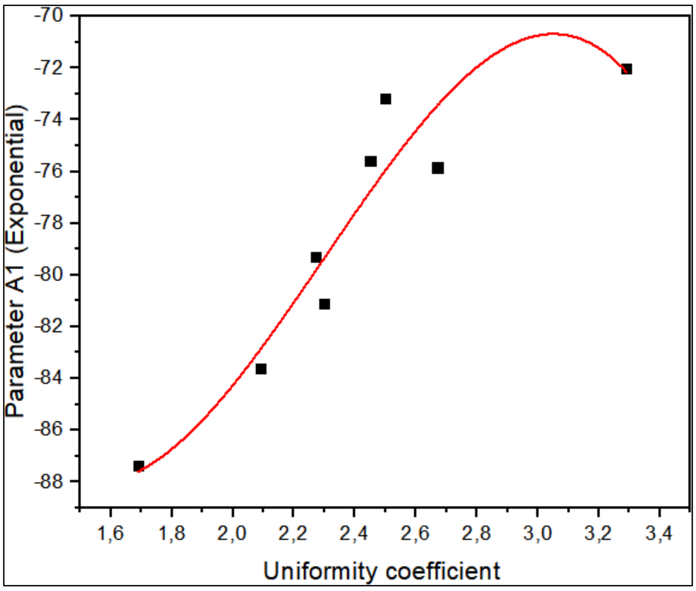

Figure 5: Evolution of parameter A1 of the exponential model as a function of the uniformity coefficient of natural sands

Figure 5 shows the evolution of the parameter A1 of the exponential model as a function of the uniformity coefficient, which is an increasing polylinear fit:

$Y = -5.384X^2 + 37.253X - 135.9$; …….. (10)

$A1 = -5.384(UC)^2 + 37.253(UC) - 135.9$ …………… (11)

$R^2 = 0.876$

Where: A1 - exponential model parameter, UC - uniformity coefficient, R^2 - coefficient of determination

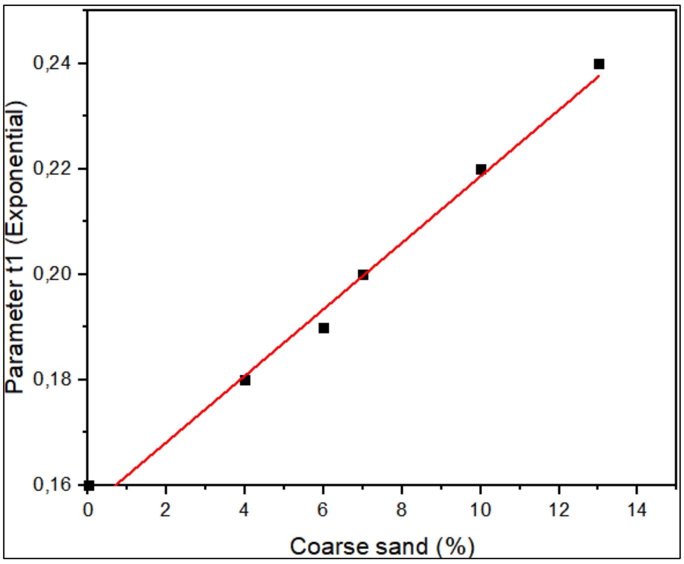

Figure 6: Evolution of the parameter t1 of the exponential model as a function of the fraction of coarse sands in natural sands

Figure 6 shows the evolution of the parameter t1 of the exponential model as a function of the coarse sand fraction, which is an increasing linear fit:

$Y = -0.0063X + 0.1556$ ……………. (12)

$t1 = -0.0063(CS) + 0.1556$ ………… (13)

$R^2 = 0.988$

With, t1 -parameter of the exponential model, CS -coarse sand fraction, -coefficient of determination

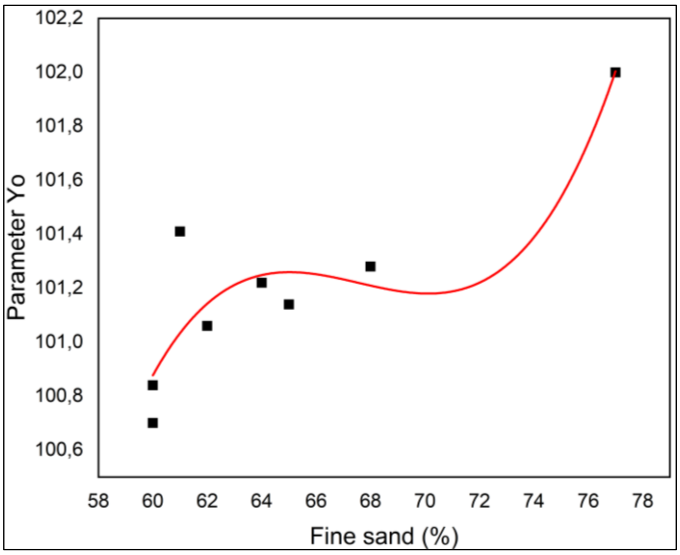

Figure 7: Evolution of the Yo parameter of the exponential model as a function of the fraction of fine sand in natural sands

Figure 7 shows the evolution of the parameter Yo of the exponential model as a function of the fine sand fraction, which is an increasing linear fit:

$Y = 0.0012X^3 - 0.243X^2 + 16.382X - 266.7$ ……………….. (14)

$Yo = 0.0012(Fsa)^3 - 0.243(Fsa)^2 + 16.382(Fsa) - 266.7$ ………….. (15)

$R^2 = 0.816$

With, Yo - parameter of the exponential model, Fsa - fraction of fine sand, R^2- coefficient of determination

Correlations between the parameters of the Gaussian mathematical model (A, W, Xc) and the geotechnical quantities (UC, Fsa, Csa) of natural sands improved by crushed sand S9.

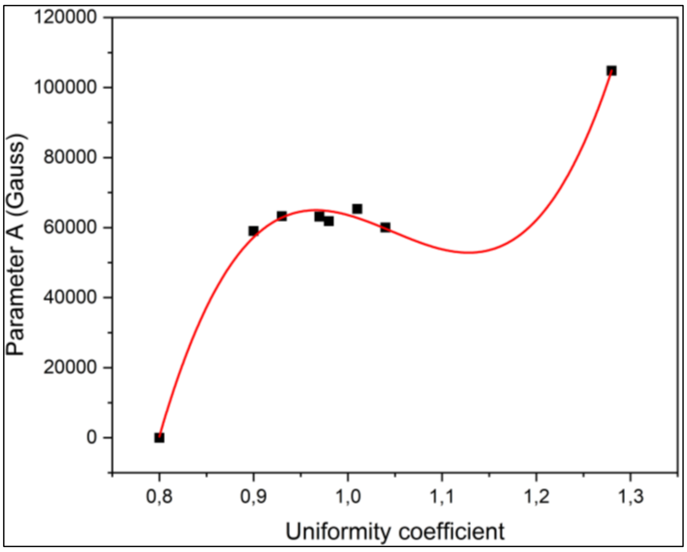

Figure 8: Evolution of parameter A of the Gauss model as a function of the uniformity coefficient of natural sands improved with S9 crusher sand

Figure 8 shows the evolution of parameter A as a function of the uniformity coefficient in the form of an increasing polylinear fit:

$Y = 6E^{+06}X^3 - 2E^{+07}X^2 + 2E^{+07}X - 6E^{+06}$ ………………….. (16)

$A = 6E^{+06}(UC)^3 - 2E^{+07}(UC)^2 + 2E^{+07}(UC) - 6E^{+06}$ ……….. (17)

$R^2 = 0.996$

With, A - parameter of the Gauss model, UC - uniformity coefficient, R^2 - coefficient of determination.

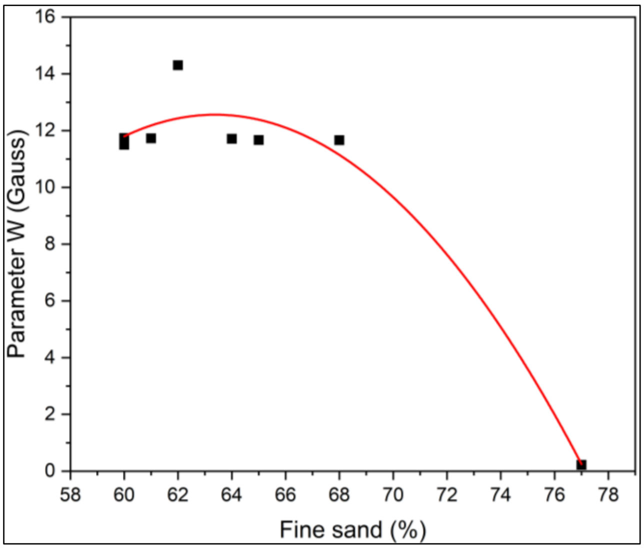

Figure 9: Evolution of the parameter W of the Gauss model as a function of the fine sand content of natural sands improved with sand S9

Figure 9 shows the evolution of the parameter was a function of the fine sand fraction, with a decreasing polylinear fit:

$Y = -0.066X^2 + 8.417X - 254.19$; …….. (18)

$W = -0.066(FS)^2 + 8.417(FS) - 254.19$ …………….. (19)

$R^2 = 0.959$

With, W - parameter of the Gauss model, Fsa - fine sand, -coefficient of determination

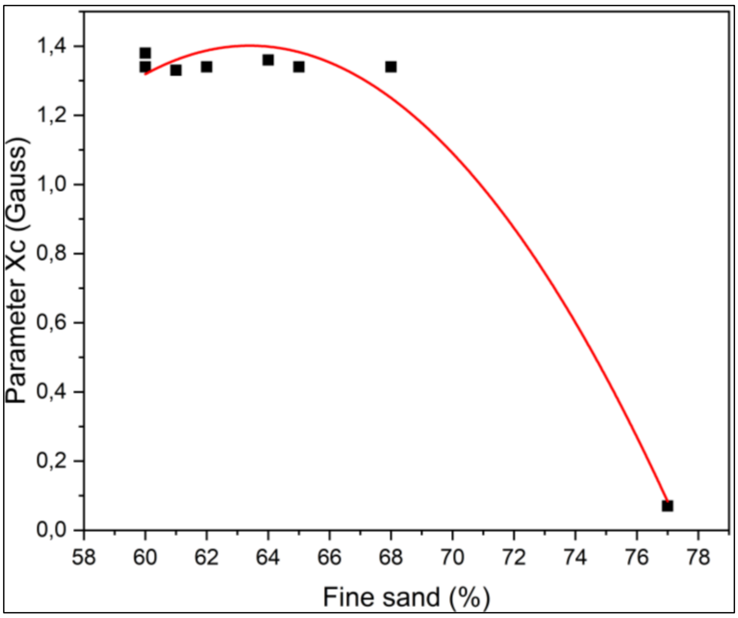

Figure 10: Evolution of the Xc parameter of the Gauss model as a function of the fine sand content of the improved sands

Figure 10 shows the evolution of the parameter Xc as a function of the fine sand fraction in the form of a decreasing polylinear fit:

$Y = -0.007X^2 + 0.903X - 27.219$; …….. (20)

$Xc = -0.007(FS)^2 + 0.903(FS) - 27.219$ …………….. (21)

$R^2 = 0.987$

Where, Xc - Gauss model parameter, Fsa - fine sand, -coefficient of determination

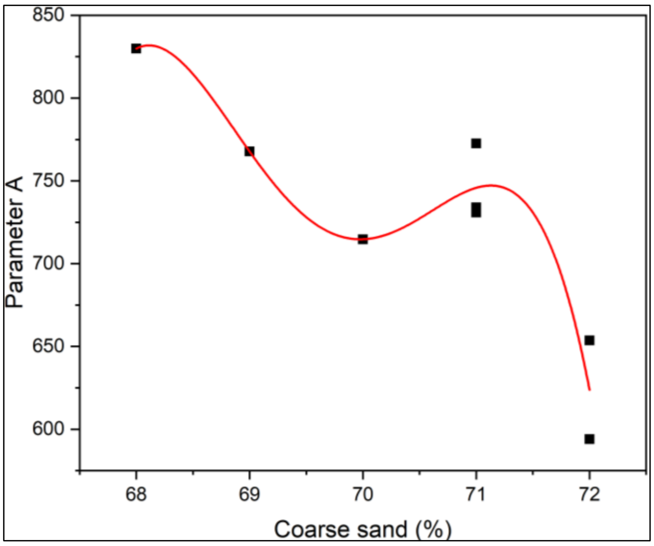

Figure 11: Evolution of the A parameter of the Gauss model as a function of the coarse sand content of the improved sands

Figure 11 shows the evolution of the parameter A as a function of the coarse sand fraction in the form of an decreasing polylinear fit:

$$Y = -13.003X^4 + 3627.2X^3 - 379382X^2 + 2E^{+08}X - 3E^{+08} \ldots\ldots\ldots\ldots\ldots (22)$$

$$Y = -13.003(Csa)^4 + 3627.2(Csa)^3 - 379382(Csa)^2 + 2E^{+08}(Csa) - 3E^{+08} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

$$R^2 = 0.924$$

Where, A - Gauss model parameter, Csa - coarse sand, - coefficient of determination

3.2. DISCUSSION

The maximum grain size of crushed sand S9 is 5 mm, whereas the maximum grain diameter of natural sands (S1-S8) is 1 mm, and they all have more or less the same appearance, i.e., they may have the same eolian origin [30, 31].

Les courbes granulométriques des sables naturels (Figure 1) sont des sables très fins présentent des concavités orientées vers le bas. A la Figure 2, les courbes de fréquence des sables naturels sont identiques et présentent deux modes entre 0.125 - 0.25 mm (sable fin) et 0.375 - 1 mm. Ces deux modes sont justifiées par la répartition des fractions granulaires (sable fin, sable moyen, sable grossier). Cependant, la répartition des grains dans le sable normalisé ou standard se situé dans la plage 0.625 - 0.75 mm proche du sable grossier situé dans la plage 0.5 - 1 mm.

L'amélioration des sables naturels (S1-S8) par ajout du sable de concassage S9 (Figure 3) permet d'obtenir la gamme des sables normalisés conformément à la norme EN 196-1. Les courbes granulométrique des sables naturels (S1-S8) améliorés (Figure 3) ont une concavité ascendante ce qui laisse présager la présence des grains grossiers provenant du sable de concassage. Les courbes de fréquences sont proches à celles des sables standards EN 196-1 [4], présentant deux modes dont le plus élevé se situe dans la plage 0.625 - 0.75 mm (Figure 4).

The bulk densities of the natural sands in Table 3 do not comply with the current standard, while the enhanced sands (S2+S9, S2+S9, S7+S9) have bulk densities within the permitted range (2.63-2.67 T/m3) (Table 10) [32]. The bulk densities of sands S3, S6 and S9 are similar to those of dune sands [30]. The absolute densities of natural sands are close, these sands may have similar mineralogy and chemical composition.

Table 3 also shows that the fineness moduli of the sands (S1-S9) are less than 2, i.e., the sands are fine [30]. The uniformity coefficients of the sands are greater than 2, i.e., the grain size of the natural sands is spread out (Table 3, 10). The coefficient of curvature of sands S2-S5, S7, S9 is less than 1 [4]. In other words, the grain size of these sands is poorly graded, with the absence of certain grain diameters corresponding to D10 and D60 [33]. Poorly graded sands produce mortars with poor mechanical properties [30, 34]. On the other hand, the curvature coefficients of sands S1, S6 and S8 are located in the CC range (1-3) of well-graded natural sands [33]. The grain size of natural sands varies from one sampling site to another. The uniformity coefficient of S9 crusher sand used to improve the grain size of natural sands is of the same order as that of standard sands, with a smaller average diameter. This smaller average diameter depends on its fines content, which is higher than the permitted values.

Tables 11-12 show the values of the correlation coefficients between the model parameters and the geotechnical quantities and between the model parameters and the different grain size fractions of fine, medium and coarse sands. The results obtained show that there are relationships between the model parameters and the geotechnical quantities, as well as between the model parameters and the different granulometric fractions. However, most of the relationships are positive or negative, reflecting a tendency for the model parameter and the quantity concerned to increase or decrease. The results of the modelling of the particle size curves of the eight natural sands show that the curves of the mathematical models selected follow the experimental curves well. The values of the statistical parameters of the eight natural sands used to smooth the experimental curves are presented in Table 11, and those of the eight natural sands improved with S9 crusher sand are presented in Table 12. A model performs well when it has a large number of parameters.

4. CONCLUSION

The aim of this study was to model the particle size distribution of natural sands, natural sands enhanced with S9 sand and to study the links between the parameters of the best models selected with the particle size fractions and the geotechnical properties of the sands. The results show that of the four models studied, the five-parameter exponential model performs best for natural sands, followed by the Gaussian model. For natural sands upgraded to S9 crusher sands, the four-parameter Gaussian model performed best, followed by the exponential model. A mathematical model performs best when it has several parameters. This is why the three-parameter Li Yong model and the two-parameter Weibull model performed less well than the exponential and Gaussian models. For natural sands, the correlations between the exponential model parameters (A1, Yo) and the geotechnical properties (uniformity coefficient, fraction of fine sand) are polylinear fits with R2(0.816 - 0.876). Then, the correlation between the exponential function parameter (t1) and the geotechnical property (coarse sand fraction) is a linear fit with R2(0.988). Natural sands are upgraded with S9 crushed sand for standard sand production. Correlations between Gauss model parameters (A, Xc, W) and geotechnical properties (UC, Fsa, Csa) are polylinear fits with R2 (0.924 - 0.996).

Declarations Conflict of Interest: The authors declare no competing interests.

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