

Development of Polynomial Based Program for Nonlinear Isotropic Rectangular Thin Plate

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Abstract

The work is aim at the development of a computer program for the nonlinear analysis of rectangular thin isotropic plate on Ritz method. Twelve boundary conditions were analyzed which include: SSSS, CCCC, CSCS, CSSS, CCSS, CCCS, CCFC, SSFS, CCFS, SCFC, CSFS, and SCFS. General expressions for displacement and stress functions for large deflection of isotropic thin rectangular plate under uniformly distributed transverse loading were obtained by direct integration of Von karman's non-linear governing differential compatibility and equilibrium equations. Polynomial function as shape function was on the decoupled Von Karman's equations to obtain particular stress and displacement functions respectively. Non-linear total potential Energy was formulated using Von Karman equilibrium equation and Ritz method was deployed in this formulation. A computer based program was developed using Matlab programming language to circumvent the challenges involved in solving the governing differential equations of thin rectangular plates. The developed program is capable of determining deflection and stresses at any point of the plate against the usual method of evaluating deflection at the center. The results obtained were compared with those of previous researchers The comparison made are only for SSSS, CCCC and CCCS plates. It was so because the remaining boundary conditions considered in this work have not been researched upon by previous researchers. From results obtained, the average percentage differences recorded for SSSS, CCCC, and CCCS plates for the present and previous studies are 4.01978%, 3.7646%, and 5.02% respectively. The percentage differences for the three plates compared are within acceptable limit of 0.05 or 5% level of significance in statistics. From the comparison made, it was obvious that an excellent agreement was observed in all cases thus indicating applicability and validity of the polynomial function and computer program for solving exact plate bending problems.

Keywords: Polynomial, shape function, Nonlinear Analysis, Rectangular Thin Plates, Ritz Methods, von Karman's Equation, Amplitude.

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1. INTRODUCTION

Plate is a solid that consist of two parallel surfaces separated by a small dimension (thickness). Generally, plate is subjected to load condition that can cause deflection transverse to the plate [1]. Plate could be bounded geometrically by either straight or curved boundaries [2]. Among practical examples to describe the dimensions of these plates are roof, building windows, flat part of a table, manhole thin covering and panels. Plates are divided into two categories: thin plates with large deflections and thick plates [3]. Rectangular plates being originally flat develop shear forces, bending and twisting moments to resist transverse loads because the loads are generally resisted

in both directions, the twisting rigidity in isotropic plate is significant considering that plate is considerably stiffer than a beam of comparable span and thickness [4].

Ordinarily, the analysis of the plate subjected to lateral load are readily achieved using a linear theory by assuming that the lateral displacement or deflection as a result of the load are small. However, when the deflections of thin plate gains magnitude beyond a certain level ($\frac{w}{h} > 0.3$) compared to its thickness, the Kirchhoff's linear theory of stiff plate ceases to be valid, hence Three non-linear equations are used to

determine the large deflections and these deflections are followed by stretching of the middle surface [4]. The usual numerical procedure is to reduce the nonlinear differential equations to a system of nonlinear algebraic equations using variational methods, finite difference method, or finite element method.

Chu and Herrmann [5] became the first people to study nonlinear plate dynamics with vibrations of simply supported rectangular plates. Shooshtari and Razavi [6] studied nonlinear free and forced vibration of a transversely isotropic rectangular magneto-electro-elastic (MEE) thin plate with simply supported boundary based on the thin plate theory along with the Von Karman's nonlinear strains. Leung and Mao [7] compared the solution between Movable and immovable edges of simply supported rectangular plate using Galerkin's method. El Kadiri and Benamar [8] extended the work of Chu and Herrmann and created a simplified analytical model. Berger [9] simplified nonlinear plate theory by ignoring terms in the strain energy. Prathap and Pandalai [10] incorporated rotary inertia and the correction for shear in their study of nonlinear plate theory. Chi-Kyung and Myung-Hwan [11] worked on non-linear analysis of skew thin plate by finite difference method which deals with a discrete analysis capability for predicting the geometrically nonlinear behaviour of skew thin plate subjected to uniform pressure.

This work developed a computer program for the analysis for the nonlinear analysis of rectangular thin isotropic plate. Polynomial was used as shape function in Ritz method and the formulated equations were modelled using Matlab as the programming language.

2. HIGHLIGHT ON GAPS IN THE EARLIER STUDY

This research was initiated to bridge the gaps of the previous researchers by formulating a model for the analysis of large deflection of thin rectangular plates. These gaps are listed below:

- Large deflection indeed is an indispensable component in the study of thin plates. But in

3. FUNDAMENTAL EQUATIONS

$$\frac{\partial^4 \phi}{\partial x^4} + \frac{2\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} = E \left[\left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right] \dots\dots\dots 1$$

$$\frac{\partial^4 w}{\partial x^4} + \frac{2\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D} \left[q + h \left(\frac{\partial^2 \phi}{\partial y^2} \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 \phi}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \frac{2\partial^2 \phi}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} \right) \right] \dots\dots\dots 2$$

Equations 1 and 2 define a system of nonlinear, partial differential equations, and they are referred to as the governing differential equations for large deflections theory of plates. The first equation can be described as compatibility equation and, describing the second equation in the same tone as equilibrium equation.

the actual design the deflection is not used unlike bending moment, shear forces and in-plane forces, deflection only provide an insight on the behavior of the plate. The previous researchers have created a serious gap in this regard because their work only ended at the determination of deflections.

- The solution of Von Karman nonlinear elastic plate governing differential equations have been a serious nut to crack and accounted for this was the use of trigonometric series as shape function by the previous researchers. This has also resulted to little availability of literature on the analysis of large deflection of plates.
- It is hard to note that previous researchers have limited their works and analysis for only simply supported plates (SSSS) and clamped plates (CCCC). Other configurations like CCSS, CCSC, SSFS, SCFC, CCFC, SSFS and CCFS plates have not been investigated by the early studies. Thereby creating considerable gap.
- As the deflection has been acclaimed the most valuable characteristic of thin plate. Its effect at every point in the plate ought to be known for efficient judgment. Based on the premises, the previous researchers have only concentrated on the determination of deflections on the centers (central deflection).
- Previous researchers concentrated mostly on the application of uniformly distributed load, therefore creating a yearning demand on the use of the applied point loads.
- The solution of Von Karman nonlinear elastic plate governing differential equations either by trigonometric series function or polynomial function have been a rigorous processes. And in the course of this processes there is a loss of data, a time lag and energy waste. The tedious time consuming, rigorous computation and loss of data could have be circumvented by developing software. The previous studies did not do justice to this development.

With Equations 1 and 2, the stress function ϕ , and deflection w , can be calculated which in return facilitates the determination of the inplane forces N_x, N_y and N_{xy} , as well as the corresponding bending moments M_x, M_y and twisting moment M_{xy} . The major challenge lies in resolving Equations 1 and 2. It is a herculean and mathematical uphill task solving these equations. To lessen the cumbersomeness of the usual

numerical procedures is to reduce the nonlinear differential Equations 1 and 2 to a system of nonlinear algebraic equations using the displacement methods. It is worthwhile to note that the appropriate boundary

condition was used. Figure 1 shows a typical rectangular plate with its characteristic dimensions 'a' and 'b'

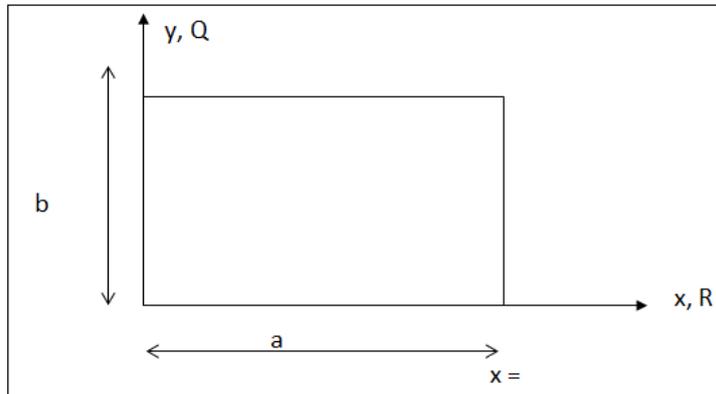


Figure 1: A typical idealized plate

Let $R = \frac{x}{a} : x = aR; 0 \leq R \leq 1; 0 \leq x \leq a$
 $Q = \frac{y}{b} : y = bQ; 0 \leq Q \leq 1; 0 \leq y \leq b$

Where R and Q are non-dimensional parameters, and they are in the x and y-directions respectively. Expressing Equation 1 and 2 in terms of R and Q reduced to:

$$\frac{\partial^4 \phi}{a^4 \partial R^4} + 2 \frac{\partial^4 \phi}{a^2 b^2 \partial R^2 \partial Q^2} + \frac{\partial^4 \phi}{b^4 \partial Q^4} = E \left(\left(\frac{\partial^2 w}{ab \partial R \partial Q} \right)^2 - \frac{\partial^2 w \partial^2 w}{a^2 \partial R^2 b^2 \partial Q^2} \right) \dots \dots \dots 3$$

$$\frac{\partial^4 w}{a^4 \partial R^4} + 2 \frac{\partial^4 w}{a^2 b^2 \partial R^2 \partial Q^2} + \frac{\partial^4 w}{b^4 \partial Q^4} = \frac{1}{D} \left(q + h \left(\frac{\partial^2 \phi}{b^2 \partial Q^2} \frac{\partial^2 w}{a^2 \partial R^2} + \frac{\partial^2 \phi}{a^2 \partial R^2} \frac{\partial^2 w}{b^2 \partial Q^2} - \frac{2 \partial^2 \phi}{ab \partial R \partial Q} \frac{\partial^2 w}{ab \partial R \partial Q} \right) \right) \dots \dots \dots 4$$

The aspect ratio α is equal to a/b ; $a = \alpha b$ and substituting in Equations 3 and 4 respectively gave the following equations:

$$\frac{\partial^4 \phi}{\alpha^4 b^4 \partial R^4} + \frac{2 \partial^4 \phi}{\alpha^2 b^4 \partial R^2 \partial Q^2} + \frac{\partial^4 \phi}{b^4 \partial Q^4} = \frac{E}{\alpha^2 b^4} \left(\left(\frac{\partial^2 w}{\partial R \partial Q} \right)^2 - \frac{\partial^2 w \partial^2 w}{\partial R^2 \partial Q^2} \right) \dots \dots \dots 5$$

$$\frac{\partial^4 w}{\alpha^4 b^4 \partial R^4} + \frac{2 \partial^4 w}{\alpha^2 b^4 \partial R^2 \partial Q^2} + \frac{\partial^4 w}{b^4 \partial Q^4} = \frac{q}{D} + \frac{h}{\alpha^2 D b^4} \left(\frac{\partial^2 \phi}{\partial Q^2} \frac{\partial^2 w}{\partial R^2} + \frac{\partial^2 \phi}{\partial R^2} \frac{\partial^2 w}{\partial Q^2} - \frac{2 \partial^2 \phi}{\partial R \partial Q} \frac{\partial^2 w}{\partial R \partial Q} \right) \dots \dots \dots 6$$

Multiplying both of Equations 5 and 6 by b^4 gave

$$\frac{\partial^4 \phi}{\alpha^4 \partial R^4} + \frac{2 \partial^4 \phi}{\alpha^2 \partial R^2 \partial Q^2} + \frac{\partial^4 \phi}{\partial Q^4} = \frac{E}{\alpha^2} \left(\left(\frac{\partial^2 w}{\partial R \partial Q} \right)^2 - \frac{\partial^2 w \partial^2 w}{\partial R^2 \partial Q^2} \right) \dots \dots \dots 7$$

$$\frac{\partial^4 w}{\alpha^4 \partial R^4} + \frac{2 \partial^4 w}{\alpha^2 \partial R^2 \partial Q^2} + \frac{\partial^4 w}{\partial Q^4} = \frac{qb^4}{D} + \frac{h}{\alpha^2 D} \left(\frac{\partial^2 \phi}{\partial Q^2} \frac{\partial^2 w}{\partial R^2} + \frac{\partial^2 \phi}{\partial R^2} \frac{\partial^2 w}{\partial Q^2} - \frac{2 \partial^2 \phi}{\partial R \partial Q} \frac{\partial^2 w}{\partial R \partial Q} \right) \dots \dots \dots 8$$

Equations 7 and 8 are nonlinear differential equation for large deflection of plate under normal load represented in non-dimensional axes. To obtain the solution of these equations for plates, an approximate method is pertinent.

4. SHAPE FUNCTIONS

Assuming a displacement function of:

$$W = w(x, y) \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} P_m B_n x^m y^n \dots \dots \dots 9$$

Displacement functions in Equation 9 was expressed in terms of non-dimensional parameters (Q and R).

Recall that.

$$X = aR, \text{ and } y = bQ \dots \dots \dots 10$$

Substituting Equation 10 into Equation 9 and terminating the series at $m = n = 4$ gave

$$W = W(R, Q) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} P_m B_n a^m R^m b^n Q^n \dots \dots \dots 11$$

$$\text{Let } a_m = P_m a^m \text{ and } b_n = B_n b^n \dots \dots \dots 12$$

Thus:

$$W = W(R, Q) = \sum_{m=0}^4 \sum_{n=0}^4 a_m R^m b_n Q^n \dots \dots \dots 13$$

Equation 13 is displacement function represented in non-dimensional axes. The displacement function can be expanded by using expansion theorem

over an orthogonal basis, an infinite sum, making the system easy to solve. The expansion was carried out over two bases, one in the x-direction and the other in y-direction. Normalizing the function makes the denominator equal to one. Hence, expanding Equation 13 to 4th series gave:

$$W(a_i R, b_i Q) = (a_0 + a_1 R + a_2 R^2 + a_3 R^3 + a_4 R^4) (b_0 + b_1 Q + b_2 Q^2 + b_3 Q^3 + b_4 Q^4) \dots\dots\dots 14$$

This is polynomial approximation of the shape function. With the proper use of the boundary conditions of the plate, the deflection of the plate will be adequately defined.

$$\begin{aligned} \phi = \beta & \left[\left(\frac{a_1^2}{24} R^4 + \frac{a_1 a_2}{30} R^5 + \frac{1}{180} (3a_1 a_3 + 2a_2^2) R^6 + \frac{1}{210} (2a_1 a_4 + 3a_2 a_3) R^7 + \frac{1}{1680} (16a_2 a_4 + 9a_3^2) R^8 + \frac{a_3 a_4}{126} R^9 + \right. \right. \\ & \left. \frac{a_4^2}{315} R^{10} \right) \left(\frac{b_1^2}{24} Q^4 + \frac{b_1 b_2}{30} Q^5 + \frac{1}{180} (3b_1 b_3 + 2b_2^2) Q^6 + \frac{1}{210} (2b_1 b_4 + 3b_2 b_3) Q^7 + \frac{1}{1680} (16b_2 b_4 + 9b_3^2) Q^8 + \frac{b_3 b_4}{126} Q^9 + \right. \\ & \left. \frac{b_4^2}{315} Q^{10} \right) - \left(\frac{a_0 a_2}{12} R^4 + \frac{1}{60} (a_1 a_2 + 3a_0 a_3) R^5 + \frac{1}{180} (a_2^2 + 3a_1 a_3 + 6a_0 a_4) R^6 + \frac{1}{210} (2a_2 a_3 + 3a_1 a_4) R^7 + \frac{1}{840} (3a_3^2 + \right. \\ & \left. 7a_2 a_4) R^8 + \frac{a_3 a_4}{168} R^9 + \frac{a_4^2}{420} R^{10} \right) \left(\frac{b_0 b_2}{12} Q^4 + \frac{1}{60} (b_1 b_2 + 3b_0 b_3) Q^5 + \frac{1}{180} (b_2^2 + 3b_1 b_3 + 6b_0 b_4) Q^6 + \frac{1}{210} (2b_2 b_3 + \right. \\ & \left. 3b_1 b_4) Q^7 + \frac{1}{840} (3b_3^2 + 7b_2 b_4) Q^8 + \frac{b_3 b_4}{168} Q^9 + \frac{b_4^2}{420} Q^{10} \right) \dots\dots\dots 14 \end{aligned}$$

Equation 14 is the general stress function, $\phi(R, Q)$ for a rectangular plate of any boundary condition. With that the specific (peculiar) stress functions for the various boundary conditions considered in this work were determined.

6. TOTAL POTENTIAL ENERGY

Consider Equation 8 as a functional expressing total potential energy, π of a deformed elastic body and

$$\begin{aligned} \pi = \frac{1}{2} \int_0^1 \int_0^1 & \left(\frac{\partial^4 w}{\alpha^4 \partial R^4} \cdot w + \frac{2\partial^4 w}{\alpha^2 \partial R^2 \partial Q^2} \cdot w + \frac{\partial^4 w}{\partial Q^4} \cdot w \right) \partial R \partial Q - \frac{1}{D} \int_0^1 \int_0^1 \left[q b^4 \cdot w + \frac{h}{2\alpha^2} \left(\frac{\partial^2 \phi}{\partial Q^2} \frac{\partial^2 w}{\partial R^2} \cdot w + \frac{\partial^2 \phi}{\partial R^2} \frac{\partial^2 w}{\partial Q^2} \cdot w - \right. \right. \\ & \left. \left. \frac{2\partial^2 \phi}{\partial R \partial Q} \cdot \frac{\partial^2 w}{\partial R \partial Q} \cdot w \right) \right] \partial R \partial Q \dots\dots\dots 15 \end{aligned}$$

Letting $w = \Delta H_1$ and $\phi = \Delta^2 H_2$

Where Δ is the coefficient factor of the plate. H_1 and H_2 are the profiles of the deflection and stress function respectively. Substituting for w and ϕ into equation 15 gave:

$$\begin{aligned} \pi = \frac{1}{2} \int_0^1 \int_0^1 & \left(\frac{\partial^4 \Delta H_1}{\alpha^4 \partial R^4} \cdot \Delta H_1 + \frac{2\partial^4 \Delta H_1}{\alpha^2 \partial R^2 \partial Q^2} \cdot \Delta H_1 + \frac{\partial^4 \Delta H_1}{\partial Q^4} \cdot \Delta H_1 \right) \partial R \partial Q - \frac{1}{D} \int_0^1 \int_0^1 \left[q b^4 \cdot \Delta H_1 + \frac{h}{2\alpha^2} \left(\frac{\partial^2 \Delta^2 H_2}{\partial Q^2} \frac{\partial^2 \Delta H_1}{\partial R^2} \cdot \Delta H_1 + \right. \right. \\ & \left. \left. \frac{\partial^2 \Delta^2 H_2}{\partial R^2} \frac{\partial^2 \Delta H_1}{\partial Q^2} \cdot \Delta H_1 - \frac{2\partial^2 \Delta^2 H_2}{\partial R \partial Q} \cdot \frac{\partial^2 \Delta H_1}{\partial R \partial Q} \cdot \Delta H_1 \right) \right] \partial R \partial Q \dots\dots\dots 16 \end{aligned}$$

Factorizing coefficient factor Δ out reduced Equation 16 to:

$$\begin{aligned} \pi = \frac{\Delta^2}{2} \int_0^1 \int_0^1 & \left(\frac{\partial^4 H_1}{\alpha^4 \partial R^4} \cdot H_1 + \frac{2\partial^4 H_1}{\alpha^2 \partial R^2 \partial Q^2} \cdot H_1 + \frac{\partial^4 H_1}{\partial Q^4} \cdot H_1 \right) \partial R \partial Q - \frac{1}{D} \int_0^1 \int_0^1 \left[\Delta \cdot q b^4 \cdot H_1 + \frac{\Delta^4 h}{2\alpha^2} \left(\frac{\partial^2 H_2}{\partial Q^2} \frac{\partial^2 H_1}{\partial R^2} \cdot H_1 + \right. \right. \\ & \left. \left. \frac{\partial^2 H_2}{\partial R^2} \frac{\partial^2 H_1}{\partial Q^2} \cdot H_1 - \frac{2\partial^2 H_2}{\partial R \partial Q} \cdot \frac{\partial^2 H_1}{\partial R \partial Q} \cdot H_1 \right) \right] \partial R \partial Q \dots\dots\dots 17 \end{aligned}$$

Equation 17 is Von Karman equilibrium equation expressed as a potential energy. To further reduce coefficient factor in Equation 17, minimization was carried out on it.

5. RATIONALIZATION OF VON KARMAN EQUATIONS

Equations 7 and 8 are nonlinear differential equation for large deflection of plate expressed in terms of non-dimensional axes. They are functions of deflection, w and, stress function, ϕ . Equation 14 describes the shape function, and with this function Equation 7 was integrated in a close domain and rationalized to generate a general (Model) solution for stress function. This was achieved by substituting Equation 14 into the right hand side of Equation 7. Hence the stress function evaluated becomes:

load acting on it. Von Karman equilibrium Equation of 8 which is now taken as total potential energy, π consists of potential energy of internal forces and potential energy of external forces. Equation 8 was therefore converted to full potential energy by multiplying all the terms in it by displacement, w , hence:

7. MINIMIZATION OF TOTAL POTENTIAL ENERGY

The total potential energy was minimized by differentiating total potential energy partially with respect to coefficient factor, which is unknown parameter of the shape function. Resultant partial derivative of the function was therefore equated to zero. This enabled determination of the unknown parameter.

$$\frac{\partial \pi}{\partial \Delta} = \Delta \int_0^1 \int_0^1 \left(\frac{\partial^4 H_1}{\alpha^4 \partial R^4} \cdot H_1 + \frac{2\partial^4 H_1}{\alpha^2 \partial R^2 \partial Q^2} \cdot H_1 + \frac{\partial^4 H_1}{\partial Q^4} \cdot H_1 \right) \partial R \partial Q - \frac{1}{D} \int_0^1 \int_0^1 q b^4 \cdot H_1 \partial R \partial Q - \frac{2\Delta^3 h}{D \alpha^2} \int_0^1 \int_0^1 \left(\frac{\partial^2 H_2}{\partial Q^2} \frac{\partial^2 H_1}{\partial R^2} \cdot H_1 + \frac{\partial^2 H_2}{\partial R^2} \frac{\partial^2 H_1}{\partial Q^2} \cdot H_1 - \frac{2\partial^2 H_2}{\partial R \partial Q} \cdot \frac{\partial^2 H_1}{\partial R \partial Q} \cdot H_1 \right) \partial R \partial Q \dots\dots\dots 18$$

Equation 18 forms general minimized total potential energy upon which determination of coefficient factor (Amplitude) of various boundary conditions was based. Noteworthy, this amplitude determines the extent of deflection of the plate. The larger the amplitude (coefficient factor) the larger the deflection and vice versa.

8. PROGRAM DEVELOPMENT

Analyzing rectangular thin plate is a challenging task because of the enormous data that are involved. When the analysis is carried out, the results come in the full matrix of linear algebraic equations that produce some difficulties in its numerical implementation. Because of these difficulties associated with carrying out nonlinear analysis of thin plate manually, some data will be lost coupled with time and energy wasted.

Nonlinear analysis is a broader method of analysis which entails a repetition of processes to arrive at particular solution. It becomes an issue to carryout nonlinear analysis of thin plate without the use of either an existing software or developing one. Days are gone when one resorts to manual analysis solely and carrying out modelling of any kind in engineering without computer codes has little practical value. High premium is placed on the development of a functional software package for this research work. In the course of this analysis MATLAB was used to write the program which is capable of solving the deflection coefficient and the stress coefficient. The software can predict

deflection, stress function, bending moment, shear force, in-plane force and stress distributions.

The name MATLAB stands for matrix laboratory. It was originally written to provide easy access to matrix software developed by the LINPACK and EISPACK Projects. MATLAB has evolved over a period of years with input from many users in university environments, it is the standard instructional tool for introductory and advanced courses in mathematics, engineering, and science. In industries MATLAB is the tool of choice for high-productivity research, development, and analysis. MATLAB is an interactive system whose basic data element is an array that does not require dimensioning. It allows you to solve many technical computing problems, especially those with matrix and vector formulations, in a fraction of the time it would take to write a program in a scalar non-interactive language such as C or FORTRAN.

9. ALGORITHM

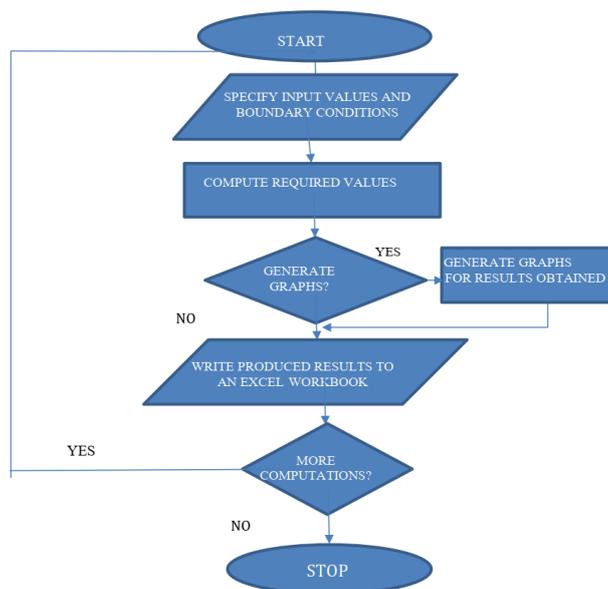
- 10 initiate the program
- 20 input parameters and boundary conditions compute the required values

If graphs are required generate graphs for results obtained write produced results to an excel workbook.

If more values are to be computed go to line 10 else go to line 30.

- 30 terminate the program
- 30 terminate the program

10. FLOWCHART



11. DEVELOPED PROGRAM

Figure 2 to 5 show the developed program with the user interface.

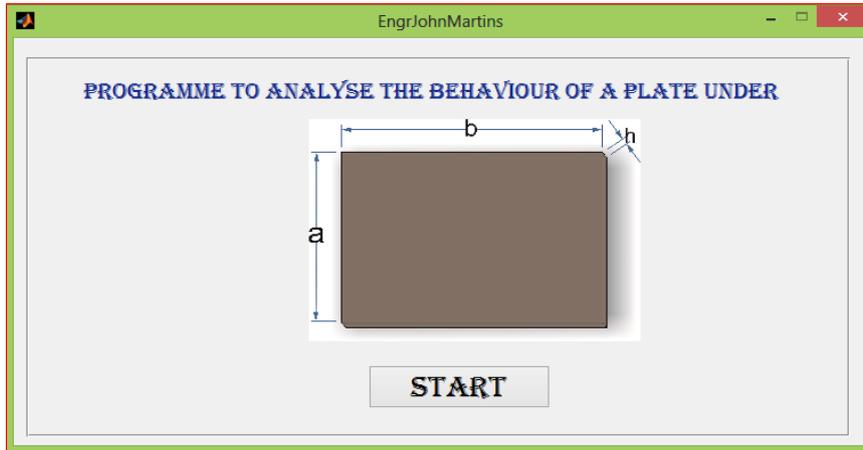


Figure 2: Software Showing the Start Button

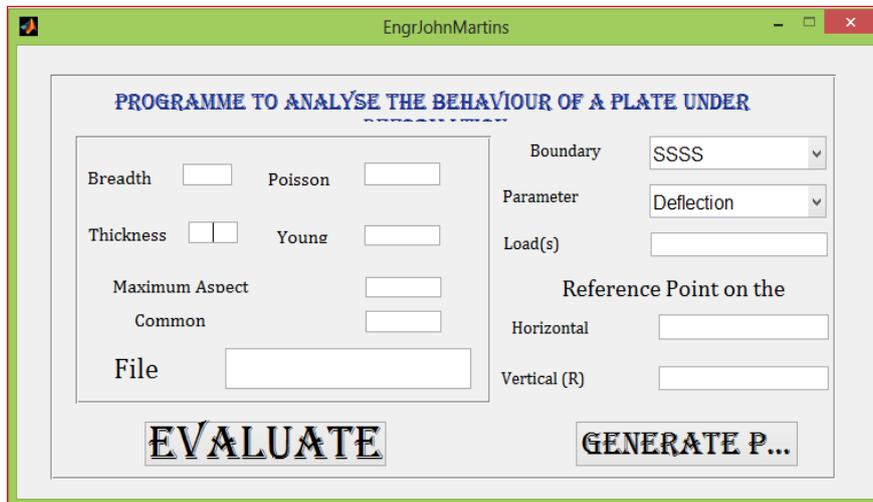


Figure 3: Software Showing the Working Environment

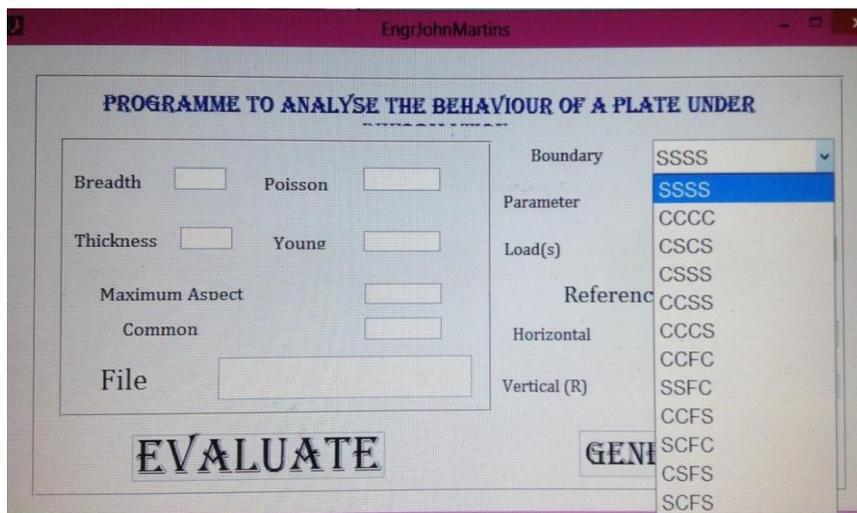


Figure 4: Software Showing the Boundary Conditions

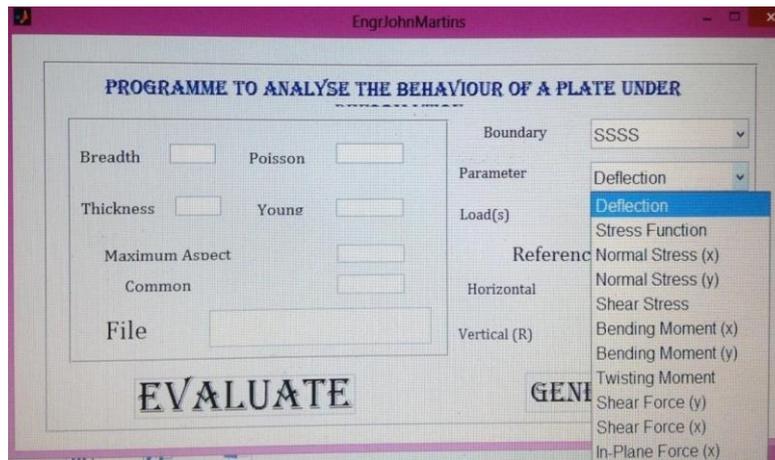


Figure 5: Software Showing the Parameters

Figures 4.1 to 4.4 show the background of the developed program. This program has been designed in such a way that the deflection, stresses, bending moments, shear forces and in-plane force can be determined in any part of plate, be it at the center or at the edges depending on the prerogative of user.

In principle, when the program is turned on, a dialog box appears with a plate showing all its dimensions. The dialog box also contains a "START" button. Clicking on this START button automatically leads to the main working environment. This working environment contains all the commands needed for the execution of the program in the form of inputs and outputs commands as shown in Figure 2

11.1 Inputs Command

The inputs required for execution and successful running of this program are:

- i. Breadth of the plate, b : This defines one of the characteristic dimensions.
- ii. Thickness of the plate, h : With this, thickness of the plate is entered.
- iii. Poisson's Ratio, ν : The required Poisson's ratio of material is entered in this command.
- iv. Young's modulus of elasticity, E : The modulus of elasticity is entered here.
- v. Maximum aspect ratio: This sets the number of aspect ratio to be considered.
- vi. Common: This defines the increment in the aspect ratio.
- vii. Loads: With this, one can input the load(s) as the case may be. Where there are multiple loads to be considered, a comma should be used to separate them
- viii. File: This command enables one to set a name by which the computer can save the analysis with. Accessing the work becomes easy with this file name.

11.2 Output Commands

The output command contains the results of the analysis:

Parameter: This command contains output as shown in Figure 4.3, and are as follows:

- Deflections
- Stress functions
- Bending moments
- Shear forces
- In-plane forces.

Once the analyses are performed, user will therefore select any of the output to view its results.

11.3 Boundary Conditions Command

From this command, the desired boundary condition to be analyzed for is settled as shown in Figure 4.

11.4 Reference Point Command

This command helps to determine part of the plate one wants to carry out the analysis. Horizontal (R) and vertical (Q) help to locate the point of interest on the x and y axis respectively. For example, if the interest is at the center of the plate and the dimensions a and b are 1 each, horizontal point therefore becomes $\frac{1}{2}$. The same applies to vertical point.

11.5 Evaluate Command

Once all the requirements (input) have been made, clicking the evaluate command button automatically executes the programs and prompts the results up.

11.6 Generate P Command

This command enables one plot and generate a graph for the analyzed parameter.

12. CONCLUSION

Carrying out the analysis of large deflection of plate involves too many data which if not properly handled will lead to loss of data, especially when this is done manually. This present study recorded another landmark by developing a software for nonlinear

analysis of large deflection of thin rectangular plates. This software has the capacity to handle different load cases as many as possible. With the application of this software, deflection of the plate can be determined at any point in the plate unlike what was obtained previously, where the deflection was only possible at the center of the plate.

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