OPEN ACCESS
Saudi Journal of Civil Engineering
Abbreviated Key Title: Saudi J Civ Eng
ISSN 2523-2657 (Print) |ISSN 2523-2231 (Online)
Scholars Middle East Publishers, Dubai, United Arab Emirates
Journal homepage: https://saudijournals.com/journal/sjce/home

Review Article

Polynomial Based Nonlinear Analysis of CCCS Thin Isotropic Rectangular Plate

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DOI: 10.36348/sjce.2022.v06i09.002 | **Received:** 18.08.2022 | **Accepted:** 24.09.2022 | **Published:** 30.09.2022

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Abstract

This work is aimed at formulating a polynomial function for the nonlinear analysis of CCCS isotropic rectangular thin plate. The previous researchers used trigonometry function as their shape function on the decoupled Von Karman's equations to obtain particular stress and displacement function respectively. Trigonometry function can only be used effectively for SSSS and CCCC plates; apart from these boundaries conditions its efficiency reduces. This present work hence used a polynomial function to formulate the approximate shape function for the CCCS plate. Direct variational calculus was used applied on Von Karman's equations to obtain the general form of minimized total potential energy which serves as a platform for the determination of coefficient factor(Amplitude or coefficient of deflection). The numerical values of CCCS plate under unit load were obtained using Amplitude equation formulated. These values were obtained for various aspect ratio (ranging from 1 to 1.5 with an increment of 0.1). This work was compared with the previous work [1] and the percentage difference in the results are within the acceptable limit. This results indicate that the approach adopted by the present work is adequate, reliable and satisfactory for the analysis of CCCS rectangular plate.
Keywords: Nonlinear Analysis, Rectangular Thin Plates, Ritz Methods, **v**on Karman's Equation, Variational Principles, Amplitude.

1. INTRODUCTION

In a broader view, a thin plate is a flat structural element bounded by two parallel planes called faces, and a cylindrical surface, called an edge or boundary. The separation between the plane faces is referred to as the thickness (h) of the plate. When the thickness of a plate is divided into equal halves by a plane parallel to its faces, this plane is called the middle plane or midplane. In general, the plate thickness is small compared to other characteristic dimensions of the faces, be it length, width, or diameter. Plate could be bounded geometrically by either straight or curved boundaries as shown in Figure 1.1 where a, and b are principal dimensions, and h is the thickness [2].

Typically, the thickness dimension of the plate is much smaller than its planer dimensions yielding a "thin-walled" type of structure [3]. Among practical examples to describe the dimensions of these plates are roof, building windows, flat part of a table, manhole thin covering and panels. Plates are divided into two categories: thin plates with large deflections and thick plates [4].

Plates are widely used in a broad range of engineering applications and particularly in aeronautical, mechanical, marine, and civil engineering [5]. Plates are subjected to transverse loads, that is, loads normal to its midplane. When plate deforms and the midplane passes into some curvilinear surface, this surface is known as the middle surface. Under transverse loading, plate is considered to be free at its boundaries which will enable it to move in its plane. Plate resist transverse loads by means of bending. Plates may also be subjected to in-plane loading or direct forces which act in the middle plane or the middle surface. These forces have significant contribution to the bending of plate. In-plane loading and their corresponding stresses are known as membrane or in-plane forces and membrane or in-plane stresses respectively. In-plane loads cause stretching and/or contraction of mid-surface [6].

In reality, many plate structures are subjected to high load levels that may cause large deflection. The effect of this large deflection is to stretch the middle plane of the plate, thereby inducing membrane stresses [6]. By this membrane action, the load carrying capacity of the plate is increased to a large extent. For plates of this kind, the governing differential equations are non-linear. The non-linearity of the governing equations may be due to either material non-linearity or geometric non-linearity [6].

The large deflection theory assumes that the deflections are rationally large with respect to the plate thickness but remain small compared to the other characteristic dimensions of the plate. When the deflection is of the order of magnitude of the thickness of the plate, it leads to a pair of coupled non-linear fourth order equations for the transverse displacement and the stress function for the in-plane stress resultants [7].

Von Karman formulated and derived governing differential equations for large deflections of thin plates as shown in Equations (1) and (1):

$$\frac{\partial^4\theta}{\partial x^4}+\frac{2\,\partial^4\theta}{\partial x^2\,\partial\,\partial y^2}+\frac{\partial^4\theta}{\partial y^4}=Eh\left[\left(\frac{\partial^2 w}{\partial x\,\partial_y}\right)^2-\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right] \ldots\ldots 1$$

$$\frac{\partial^4\theta}{\partial x^4}+\frac{2\,\partial^4 w}{\partial x^2\,\partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{1}{D}\left(P+\frac{\partial^2\theta}{\partial x^2}\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2\theta}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\frac{2\,\partial^2\theta}{\partial x\,\partial y}\frac{\partial^2 w}{\partial x\partial y}\right) \ldots\ldots 2$$

Equations 1 and 2 are coupled, non- linear, fourth order partial differential equations. These equations account for membrane deformation and their corresponding stresses. Equation 1 is referred to as an equilibrium equation while Equation 2 is called compatibility equation. The major problem associated with Von Karman equation has been how to decouple the equation because of its indeterminacy. This has made the solution of large deflection of plate problems difficult and hard to come by.

2. PRSVIOUS WORKS

Zenkour [8] used generalized shear deformation theory for bending analysis of functionally graded plates. Woo and Meguid [9] provided an analytical solution for the large deflections of functionally graded plates and shallow shells under transverse mechanical loads and a temperature field using Fourier series. Yang and Shen [10] studied nonlinear bending analysis of shear deformable functionally graded plates subjected to thermo-mechanical loads Tsung and Shukla [11] provided an explicit solution for the nonlinear static and dynamic responses of the functionally graded rectangular plate using the quadratic extrapolation technique for linearization, finite double Chebysher series for spatial discretization of the variables and Houbolt time marching scheme for temporal discretization. Reddy [12] developed Navier's solutions for rectangular dynamic responses of FG plates using the higher-order shear deformation plate theory.

Navazi *et al.*, [13] developed an analytical solution for nonlinear cylindrical bending of a functionally graded plate. Ghannedpour and Alinia [14] obtained an analytical solution for large deflection of rectangular functionally graded plates under pressure loads by minimization of the total potential energy of the plate. Navazi and Haddadpour [15] presented an exact solution for non-linear cylindrical bending of shear deformable functionally graded plates. Zhao and Liew [16] investigated the non-linear response of functionally plates under mechanical and thermal loads using the mesh-free method. Barbosa and Feriera [17] used finite element method for nonlinear analysis of functionally graded plates. Hoa *et al.*, [18] presented an analysis on non-linear dynamic characteristics of a simply supported functionally graded rectangular plate subjected to the transversal and in-plane excitations in time dependent thermal environment.

3. NONLINEAR PLATE THEORY

$$\frac{\partial^4\phi}{\partial x^4}+\frac{2\partial^4\phi}{\partial x^2\partial y^2}+\frac{\partial^4\phi}{\partial y^2}=E\left[\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2-\frac{\partial^2 w\,\partial^2 w}{\partial x^2\partial y^2}\right] \ldots\ldots 3$$

$$\frac{\partial^4 w}{\partial x^4}+\frac{2\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{1}{D}\left[q+h\left(\frac{\partial^2\phi}{\partial y^2}\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2\phi}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\frac{2\partial^2\phi}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}\right)\right] \ldots\ldots 4$$

Equations 3 and 4 define a system of nonlinear, partial differential equations, and they are referred to as the governing differential equations for large deflections theory of plates. The first equation can be described as compatibility equation and, describing the second equation in the same tone as equilibrium equation.

$$\frac{\partial^4\phi}{\propto^4\partial R^4}+\frac{2\partial^4\phi}{\propto^2\partial R^2\partial Q^2}+\frac{\partial^4\phi}{\partial Q^4}=\frac{E}{\propto^2}\left(\left(\frac{\partial^2 w}{\partial R\,\partial Q}\right)^2-\frac{\partial^2 w}{\partial R^2}\frac{\partial^2 w}{\partial Q^2}\right) \ldots\ldots 5$$

$$\frac{\partial^4 w}{\propto^4\partial R^4}+\frac{2\partial^4 w}{\propto^2\partial R^2\partial Q^2}+\frac{\partial^4 w}{\partial Q^4}=\frac{qb^4}{D}+\frac{h}{\propto^2 D}\left(\frac{\partial^2\phi}{\partial Q^2}\frac{\partial^2 w}{\partial R^2}+\frac{\partial^2\phi}{\partial R^2}\frac{\partial^2 w}{\partial Q^2}-\frac{2\partial^2\phi}{\partial R\,\partial Q}\frac{\partial^2 w}{\partial R\,\partial Q}\right) \ldots\ldots 6$$

Equations 5 and 6 are nonlinear differential equation for large deflection of plate under normal load represented in non-dimensional axes. co Equation 6 is

Consider Equation 6 as a functional expressing total potential energy, π of a deformed elastic body and load acting on it. Hence, π consists of potential energy of internal forces and potential energy of external forces. From the elementary physics, potential energy of a body is a measure of work done by external and internal forces in moving the body from its initial position to a final one. Since, all the terms in Equation 6 are in form of force. Equation 6 was therefore converted to full potential energy by multiplying all the terms in it by displacement, w, hence

$$\pi = \frac{1}{2}\int_0^1\int_0^1\left(\frac{\partial^4 w}{\propto^4\partial R^4}.w + \frac{2\partial^4 w}{\propto^2\partial R^2\partial Q^2}.w + \frac{\partial^4 w}{\partial Q^4}.w\right)\partial R\partial Q - \frac{1}{D}\int_0^1\int_0^1\left[qb^4.w + \frac{h}{2\propto^2}\left(\frac{\partial^2\phi}{\partial Q^2}\frac{\partial^2 w}{\partial R^2}.w + \frac{\partial^2\phi}{\partial R^2}\frac{2\partial^2 w}{\partial Q^2}.w - \frac{2\partial^2\phi}{\partial R\partial Q}.\frac{\partial^2 w}{\partial R\partial Q}.w\right)\right]\partial R\partial Q \quad \ldots\ldots 7$$

Letting $w = \Delta H_1$ and $\emptyset = \Delta^2 H_2$

Where Δ is the coefficient factor of the plate. H_1 and H_2 are the profiles of the deflection and stress function respectively. Substituting for w and $\emptyset$ into equation 7 and after the minimization of the total potential energy gave:

$$\frac{\partial\pi}{\partial\Delta} = \Delta\int_0^1\int_0^1\left(\frac{\partial^4 H_1}{\propto^4\partial R^4}.H_1 + \frac{2\partial^4 H_1}{\propto^2\partial R^2\partial Q^2}.H_1 + \frac{\partial^4 H_1}{\partial Q^4}.H_1\right)\partial R\partial Q - \frac{1}{D}\int_0^1\int_0^1 qb^4.H_1\partial R\partial Q - \frac{2\Delta^3 h}{D\propto^2}\int_0^1\int_0^1\left(\frac{\partial^2 H_2}{\partial Q^2}\frac{\partial^2 H_1}{\partial R^2}.H_1 + \frac{\partial^2 H_2}{\partial R^2}\frac{\partial^2 H_1}{\partial Q^2}.H_1 - \frac{2\partial^2 H_2}{\partial R\partial Q}.\frac{\partial^2 H_1}{\partial R\partial Q}.H_1\right)\partial R\partial Q \quad \ldots\ldots 8$$

Equation 8 forms general minimized total potential energy upon which determination of coefficient factor (Amplitude) of CCCS plate was based and with the help of displacement function for CCCS plate which is given as;

$CCCS = \Delta\,(1.5R^2 - 2.5R^3 + R^4)\,(Q^2 - 2Q^3 + Q^4)$ ……………………………………………………….. 9

The stress function for CCCS is given as

$$\emptyset_{CCCS} = \frac{\beta\Delta^2}{25401600}[(126R^6 - 270R^7 + 240.75R^8 - 100R^9 + 16R^{10})(56Q^6 - 144Q^7 + 156Q^8 - 80Q^9 + 16Q^{10}) - (63R^6 - 180R^7 + 175.5R^8 - 75R^9 + 12R^{10})(28Q^6 - 96Q^7 + 114Q^8 - 60Q^9 + 12Q^{10})] \quad \ldots\ldots 10$$

4. AMPLITUDE EQUATION FOR CSCS PLATE

Figure 1 shows a CCCS thin rectangular plate subjected to uniformly distributed transverse load.

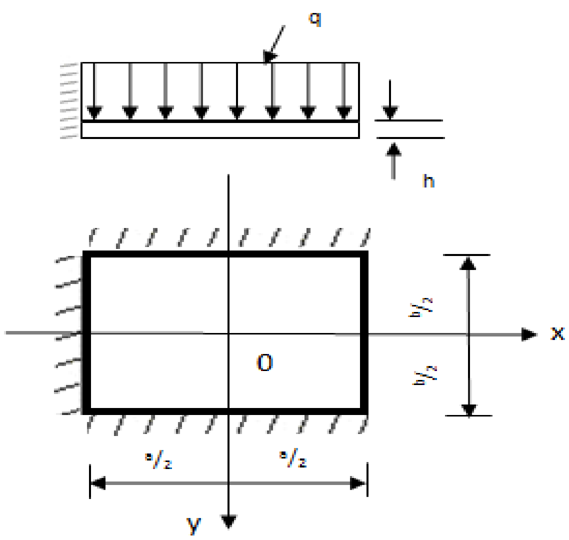

Figure 1: Typical of Rectangular Plate Uniformly Loaded, Simply Supported at the last Edge and Clamped at the other three edges (CCCS)

Deflection $w = \Delta H_1$, from Equation 9 H_1 is:

$$H_1 = (1.5R^2 - 2.5R^3 + R^4)(Q^2 - 2Q^3 + Q^4) \quad \ldots\ldots 11$$

Stress function $\emptyset = \Delta^2 H_2$, from Equation 10 H_2 is:

$$H_2 = \frac{\beta}{25401600}[(126R^6 - 270R^7 + 240.75R^8 - 100R^9 + 16R^{10})(56Q^6 - 144Q^7 + 156Q^8 - 80Q^9 + 16Q^{10}) - (63R^6 - 180R^7 + 175.5R^8 - 75R^9 + 12R^{10})(28Q^6 - 96Q^7 + 114Q^8 - 60Q^9 + 12Q^{10}) \quad \ldots\ldots\ldots\ldots\ldots 12$$

Substituting Equations 10 and 11 into Equation 8 and carrying out the respective differentiation and integration accordingly (it was done in parts).

The first term in Equation 3.310 after differentiation gave:

$$\frac{\partial^4 H_1}{\propto^4 \partial R^4} = \frac{24}{\propto^4}(Q^2 - 2Q^3 + Q^4) \quad \ldots\ldots\ldots\ldots 13$$

Multiplying Equation 3.543 by H_1 resulted to:

$$\frac{\partial^2 H_1}{\propto^4 \partial R^4}.H_1 = \frac{24}{\propto^4}(1.5R^2 - 2.5R^3 + R^4)(Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8) \quad \ldots\ldots\ldots\ldots 14$$

Integrating Equation 14 gave:

$$\int_0^1\int_0^1 \frac{\partial^2 H_1}{\propto^4 \partial R^4}.H_1\,\partial R\partial Q = \int_0^1\int_0^1 \frac{24}{\propto^4}(1.5R^2 - 2.5R^3 + R^4)(Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8)\,\partial R\partial Q \ldots\ldots\ldots\ldots 15$$

Substituting the limits into Equation 15 gave:

$$\int_0^1\int_0^1 \frac{\partial^2 H_1}{\propto^4 \partial R^4}.H_1\,\partial R\partial Q = \frac{24}{\propto^4}\left(\frac{1.5}{3} - \frac{2.5}{4} + \frac{1}{5}\right)\left(\frac{1}{5} - \frac{2}{3} + \frac{6}{7} - \frac{1}{2} + \frac{1}{9}\right) \quad \ldots\ldots\ldots\ldots 16$$

Simplifying Equation 16 further gave:

$$\int_0^1\int_0^1 \frac{\partial^2 H_1}{\propto^4 \partial R^4}.H_1\,\partial R\partial Q = \frac{2.857142857e^{-3}}{\propto^4} \quad \ldots\ldots\ldots\ldots 17$$

The second term in Equation 8 after differentiation gave:

$$2\frac{\partial^4 H_1}{\partial R^2 \partial Q^2} = \frac{2}{\propto^2}(3 - 15R + 12R^2)(2 - 12Q + 12Q^2) \quad \ldots\ldots\ldots\ldots 18$$

Multiplying Equation 18 by H_1 resulted to:

$$2\frac{\partial^4 H_1}{\propto^2 \partial R^2 \partial Q^2}.H_1 = \frac{2}{\propto^2}(4.5R^2 - 30R^3 + 58.5R^4 - 45R^5 + 12R^6)(2Q^2 - 16Q^3 + 38Q^4 - 36Q^5 + 12Q^6) \quad \ldots\ldots\ldots\ldots 19$$

Integrating Equation 19 gave:

$$\int_0^1\int_0^1 2\frac{\partial^4 H_1}{\propto^2 \partial R^2 \partial Q^2}.H_1\,\partial R\partial Q = \int_0^1\int_0^1 \frac{2}{\propto^2}(4.5R^2 - 30R^3 + 58.5R^4 - 45R^5 + 12R^6)(2Q^2 - 16Q^3 + 38Q^4 - 36Q^5 + 12Q^6)\,\partial R\partial Q \quad \ldots\ldots\ldots\ldots 20$$

Substituting the limits into Equation 20 gave:

$$\int_0^1\int_0^1 2\frac{\partial^4 H_1}{\propto^2 \partial R^2 \partial Q^2}.H_1\,\partial R\partial Q = \frac{2}{\propto^2}\left(\frac{4.5}{3} - \frac{30}{4} + \frac{58.5}{5} - \frac{45}{6} + \frac{12}{7}\right)\left(\frac{2}{3} - 4 + \frac{38}{5} - 6 + \frac{12}{7}\right) \quad \ldots\ldots\ldots\ldots 21$$

Simplifying Equation 21 further gave:

$$\int_0^1\int_0^1 2\frac{\partial^4 H_1}{\propto^2 \partial R^2 \partial Q^2}.H_1\,\partial R\partial Q = \frac{3.265306122e^{-3}}{\propto^2} \quad \ldots\ldots\ldots\ldots 22$$

The third term in Equation 8 after differentiation gave:

$$\frac{\partial^4 H_1}{\partial Q^4} = 24\,(1.5R^2 - 2.5R^3 + R^4) \quad \ldots\ldots\ldots\ldots 23$$

Multiplying Equation 23 by H_1 resulted to:

$$\frac{\partial^4 H_1}{\partial Q^4}.H_1 = 24(2.25R^4 - 7.5R^5 + 9.25R^6 - 5R^7 + R^8)(Q^2 - 2Q^3 + Q^4) \quad \ldots\ldots\ldots\ldots 24$$

Integrating Equation 24 gave:

$$\int_0^1\int_0^1 \frac{\partial^4 H_1}{\partial Q^4}.H_1\,\partial R\partial Q = \int_0^1\int_0^1 24(2.25R^4 - 7.5R^5 + 9.25R^6 - 5R^7 + R^8)(Q^2 - 2Q^3 + Q^4)\,\partial R\partial Q \ldots\ldots 25$$

Substituting the limits into Equation 26 gave:

$$\int_0^1\int_0^1 \frac{\partial^4 H_1}{\partial Q^4}.H_1\,\partial R\partial Q = 24\left(\frac{2.25}{5}-\frac{7.5}{6}+\frac{9.25}{7}-\frac{5}{8}+\frac{1}{9}\right)\left(\frac{1}{3}-\frac{1}{2}+\frac{1}{5}\right) \text{ .... } 27$$

Simplifying Equation 27 further gave:

$$\int_0^1\int_0^1 \frac{\partial^4 H_1}{\partial Q^4}.H_1\,\partial R\partial Q = 6.031746032e^{-3} \text{ .... } 28$$

Integrating the fourth term in Equation 8 gave:

$$\int_0^1\int_0^1 .H_1\,\partial R\partial Q = \left(\frac{1.5}{3}-\frac{2.5}{4}+\frac{1}{5}\right)\left(\frac{1}{3}-\frac{1}{2}+\frac{1}{5}\right) \text{ .... } 29$$

Simplifying Equation 29 further gave:

$$\int_0^1\int_0^1 .H_1\,\partial R\partial Q = 2.5e^{-3} \text{ .... } 30$$

Differentiating first part of the fifth term of Equation 8 gave:

$$\frac{\partial^2 H_2}{\partial Q^2} = \frac{\beta}{25401600}[(126R^6 - 270R^7 + 240.75R^8 - 100R^9 + 16R^{10})(1680Q^4 - 6048Q^5 + 8736Q^6 - 5760Q^7 + 1440Q^8) - (63R^6 - 180R^7 + 175.5R^8 - 75R^9 + 12R^{10})(840Q^4 - 4032Q^5 + 6384Q^6 - 4320Q^7 + 1080Q^8)] \text{ .... } 31$$

Differentiating second part of the fifth term of Equation 8 gave:

$$\frac{\partial^2 H_1}{\partial R^2} = (3 - 15R + 12R^2)(Q^2 - 2Q^3 + Q^4) \text{ .... } 32$$

Multiplying Equation 32 by H_1 resulted to:

$$\frac{\partial^2 H_1}{\partial R^2}.H_1 = (4.5R^2 - 30R^3 + 58.5R^4 - 45R^5 + 12R^6)(Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8) \text{ } 3.562$$

Combining Equations 32 and 32, and Integrating gave:

$$\int_0^1\int_0^1 \frac{\partial^2 H_2}{\partial Q^2}\frac{\partial^2 H_1}{\partial R^2}.H_1\,\partial R\partial Q = \int_0^1\int_0^1 \frac{\beta}{25401600}[(567R^8 - 3591R^9 + 10023.3R^{10} - 16074R^{11} + 16161.9R^{12} - 10410R^{13} + 4186.8R^{14} - 960R^{15} + 96R^{16})(1680Q^8 - 17760Q^9 + 71160Q^{10} - 152160Q^{11} + 196440Q^{12} - 159600Q^{13} + 80400Q^{14} - 23040Q^{15} + 2880Q^{16}) - (283.5R^8 - 2079R^9 + 6419.7R^{10} - 11016R^{11} + 11568.6R^{12} - 7650R^{13} + 3121.2R^{14} - 720R^{15} + 72R^{16})(840Q^8 - 9600Q^9 + 42420Q^{10} - 9900Q^{11} + 136620Q^{12} - 115920Q^{13} + 59760Q^{14} - 17280Q^{15} + 2160Q^{16})]\,\partial R\partial Q \text{ .... } 33$$

Substituting the limits into Equation 33 gave:

$$= \frac{\beta}{25401600}\left[\left(\frac{567}{9}-\frac{3591}{10}+\frac{10023.3}{11}-\frac{16074}{12}+\frac{16161.9}{13}-\frac{10410}{14}+\frac{4186.8}{15}-\frac{960}{16}+\frac{96}{17}\right)\left(\frac{1680}{9}-\frac{17760}{10}+\frac{71160}{11}-\frac{152160}{12}+\frac{196440}{13}-\frac{159600}{14}+\frac{80400}{15}-\frac{23040}{16}+\frac{2880}{17}\right) - \left(\frac{283.5}{9}-\frac{2079}{10}+\frac{6419.7}{11}-\frac{11016}{12}+\frac{11568.6}{13}-\frac{7650}{14}+\frac{3121.2}{15}-\frac{720}{16}+\frac{72}{17}\right)\left(\frac{840}{9}-\frac{9600}{10}+\frac{42420}{11}-\frac{99000}{12}+\frac{136620}{13}-\frac{115920}{14}+\frac{59760}{15}-\frac{17280}{16}+\frac{2160}{17}\right)\right] \text{ .... } 34$$

Simplifying Equation 34 gave:

$$= \frac{\beta}{25401600}[(-1.707602025e^{-3}) - (1.596206613e^{-4})] \text{ .... } 35$$

Simplifying Equation 35 further gave:

$$\int_0^1\int_0^1 \frac{\partial^2 H_2}{\partial Q^2}\frac{\partial^2 H_1}{\partial R^2}.H_1\,\partial R\partial Q = -7.350807376\beta e^{-11} \text{ .... } 36$$

Differentiating first part of the sixth term of Equation 8 gave:

$$\frac{\partial^2 H_2}{\partial R^2} = \frac{\beta}{25401600}[(3780R^4 - 11340R^5 + 13482R^6 - 7200R^7 + 1440R^8)(56Q^6 - 144Q^7 + 156Q^8 - 80Q^9 + 16Q^{10}) - (1890R^4 - 7560R^5 + 9828R^6 - 5400R^7 + 1080R^8)(28Q^6 - 96Q^7 + 114Q^8 - 60Q^9 + 12Q^{10})] \text{ .... } 37$$

Differentiating second part of the sixth term of Equation 8 gave:

$$\frac{\partial^2 H_1}{\partial Q^2} = (1.5R^2 - 2.5R^3 + R^4)(2 - 12Q + 12Q^2) \text{ .... } 38$$

Multiplying Equation 38 by H_1 resulted to:

$$\frac{\partial^2 H_1}{\partial Q^2}.H_1 = (2.25R^4 - 7.5R^5 + 9.25R^6 - 5R^7 + R^8)(2Q^2 - 16Q^3 + 38Q^4 - 36Q^5 + 12Q^6) \text{ .... } 39$$

Combining Equations 37 and 39 gave:

$$\frac{\partial^2 H_2}{\partial R^2}\frac{\partial^2 H_1}{\partial Q^2}.H_1 = \frac{\beta}{25401600}[(8505R^8 - 74925R^9 + 248315.625R^{10} - 437062.5R^{11} + 462268.125R^{12} - 306056.25R^{13} + 124875R^{14} - 28800R^{15} + 2880R^{16})(112Q^8 - 851.2Q^9 + 2867.2Q^{10} - 5580.8Q^{11} + 6851.2Q^{12} - 5420.8Q^{13} + 2694.4Q^{14} - 768Q^{15} + 96Q^{16}) - (4253.76R^8 - 40512R^9 + 148172.64R^{10} - 284596.8R^{11} + 321496.48R^{12} - 222140.8R^{13} + 92772.48R^{14} - 21606.4R^{15} + 2160.64R^{16})(56Q^8 - 492.8Q^9 + 1836.8Q^{10} - 3827.2Q^{11} + 4908.8Q^{12} - 3987.2Q^{13} + 2009.6Q^{14} - 576Q^{15} + 72Q^{16})] \ldots\ldots 40$$

Integrating Equation 40 gave:

$$\int_0^1\int_0^1 \frac{\partial^2 H_2}{\partial R^2}\frac{\partial^2 H_1}{\partial Q^2}.H_1\,\partial R\partial Q = \int_0^1\int_0^1 \frac{\beta}{25401600}[(8505R^8 - 74925R^9 + 248315.625R^{10} - 437062.5R^{11} + 462268.125R^{12} - 306056.25R^{13} + 124875R^{14} - 28800R^{15} + 2880R^{16})(112Q^8 - 851.2Q^9 + 2867.2Q^{10} - 5580.8Q^{11} + 6851.2Q^{12} - 5420.8Q^{13} + 2694.4Q^{14} - 768Q^{15} + 96Q^{16}) - (4253.76R^8 - 40512R^9 + 148172.64R^{10} - 284596.8R^{11} + 321496.48R^{12} - 222140.8R^{13} + 92772.48R^{14} - 21606.4R^{15} + 2160.64R^{16})(56Q^8 - 492.8Q^9 + 1836.8Q^{10} - 3827.2Q^{11} + 4908.8Q^{12} - 3987.2Q^{13} + 2009.6Q^{14} - 576Q^{15} + 72Q^{16})]\,\partial R\partial Q \ldots\ldots 41$$

Substituting the limits into Equation 41 gave:

$$\frac{\beta}{25401600}\left[\left(\frac{8505}{9} - \frac{74925}{10} + \frac{248315.625}{11} - \frac{437062.5}{12} + \frac{462268.125}{13} - \frac{306056.25}{14} + \frac{124875}{15} - \frac{28800}{16} + \frac{2880}{17}\right)\left(\frac{112}{9} - \frac{851.2}{10} + \frac{2867.2}{11} - \frac{5580.8}{12} + \frac{6851.2}{13} - \frac{5420.8}{14} + \frac{2694.4}{15} - \frac{768}{16} + \frac{96}{17}\right) - \left(\frac{4253.76}{9} - \frac{40512}{10} + \frac{148172.64}{11} - \frac{284596.8}{12} + \frac{321496.48}{13} - \frac{222140.8}{14} + \frac{92772.48}{15} - \frac{21606.4}{16} + \frac{2160.64}{17}\right)\left(\frac{56}{9} - \frac{492.8}{10} + \frac{1836.8}{11} - \frac{3827.2}{12} + \frac{4908.8}{13} - \frac{3987.2}{14} + \frac{2009.6}{15} - \frac{576}{16} + \frac{72}{17}\right)\right] \ldots\ldots 42$$

Simplifying Equation 42 gave:

$$= \frac{\beta}{25401600}[(-4.141893387e^{-3}) - (-7.116101076e^{-5})] \ldots\ldots 43$$

Simplifying Equation 43 further gave:

$$\int_0^1\int_0^1 \frac{\partial^2 H_2}{\partial R^2}\frac{\partial^2 H_1}{\partial Q^2}.H_1\,\partial R\partial Q = -1.602549594\beta e^{-10} \ldots\ldots 43$$

Differentiating first part of the seventh term of Equation 8 gave:

$$\frac{\partial^2 H_2}{\partial R\partial Q} = \frac{\beta}{25401600}[(756R^5 - 1890R^6 + 1926R^7 - 900R^8 + 160R^9)(336Q^5 - 1008Q^6 + 1248Q^7 - 720Q^8 + 160Q^9) - (378R^5 - 1260R^6 + 1404R^7 - 675R^8 + 120R^9)(168Q^5 - 672Q^6 + 912Q^7 - 540Q^8 + 120Q^9) \ldots\ldots 44$$

Differentiating second part of the seventh term of Equation 8 gave:

$$\frac{\partial^2 H_1}{\partial R\partial Q} = (3R - 7.5R^2 + 4R^3)(2Q - 6Q^2 + 4Q^3) \ldots\ldots 45$$

Multiplying Equation 45 by H_1 resulted to:

$$\frac{\partial^2 H_1}{\partial R\partial Q}.H_1 = (4.5R^3 - 18.75R^4 + 27.75R^5 - 17.5R^6 + 4R^7)(2Q^3 - 10Q^4 + 18Q^5 - 14Q^6 + 4Q^7) \ldots\ldots 46$$

Combining Equations 44 and 46 gave:

$$2\frac{\partial^2 H_2}{\partial R\partial Q}\frac{\partial^2 H_1}{\partial R\partial Q}.H_1 = \frac{\beta}{12700800}[(3402R^8 - 22680R^9 + 65083.5R^{10} - 105840R^{11} + 107140.5R^{12} - 69240R^{13} + 27894R^{14} - 6400R^{15} + 640R^{16})(672Q^8 - 5376Q^9 + 18624Q^{10} - 36768Q^{11} + 45440Q^{12} - 36064Q^{13} + 17952Q^{14} - 5120Q^{15} + 640Q^{16}) - (1701R^8 - 12757.5R^9 + 40432.5R^{10} - 70942.5R^{11} + 75719.25R^{12} - 50591.25R^{13} + 20758.5R^{14} - 4800R^{15} + 480R^{16})(336Q^8 - 3024Q^9 + 11568Q^{10} - 24648Q^{11} + 32136Q^{12} - 26376Q^{13} + 13368Q^{14} - 3840Q^{15} + 480Q^{16})] \ldots\ldots 47$$

Integrating Equation 47 gave:

$$\int_0^1\int_0^1 2\frac{\partial^2 H_2}{\partial R\partial Q}\frac{\partial^2 H_1}{\partial R\partial Q}.H_1\,\partial R\partial Q = \int_0^1\int_0^1 \frac{\beta}{12700800}[(3402R^8 - 22680R^9 + 65083.5R^{10} - 105840R^{11} + 107140.5R^{12} - 69240R^{13} + 27894R^{14} - 6400R^{15} + 640R^{16})(672Q^8 - 5376Q^9 + 18624Q^{10} - 36768Q^{11} + 45440Q^{12} - 36064Q^{13} + 17952Q^{14} - 5120Q^{15} + 640Q^{16}) - (1701R^8 - 12757.5R^9 + 40432.5R^{10} - 70942.5R^{11} + 75719.25R^{12} - 50591.25R^{13} + 20758.5R^{14} - 4800R^{15} + 480R^{16})(336Q^8 - 3024Q^9 + 11568Q^{10} - 24648Q^{11} + 32136Q^{12} - 26376Q^{13} + 13368Q^{14} - 3840Q^{15} + 480Q^{16})]\,\partial R\partial Q \ldots\ldots 48$$

Substituting the limits into Equation 48 gave:

$$= \frac{\beta}{12700800}\left[\left(\frac{3402}{9} - \frac{22680}{10} + \frac{65083.5}{11} - \frac{105840}{12} + \frac{107140.5}{13} - \frac{69240}{14} + \frac{27894}{15} - \frac{6400}{16} + \frac{640}{17}\right)\left(\frac{672}{9} - \frac{5376}{10} + \frac{18624}{11} - \frac{36768}{12} + \frac{45440}{13} - \frac{36064}{14} + \frac{17952}{15} - \frac{5120}{16} + \frac{640}{17}\right) - \left(\frac{1701}{9} - \frac{12757.5}{10} + \frac{40432.5}{11} - \frac{70942.5}{12} + \frac{75719.25}{13} - \frac{50591.25}{14} + \frac{20758.5}{15} - \frac{4800}{16} + \frac{480}{17}\right)\left(\frac{336}{9} - \frac{3024}{10} + \frac{11568}{11} - \frac{24648}{12} + \frac{32136}{13} - \frac{26376}{14} + \frac{13368}{15} - \frac{3840}{16} + \frac{480}{17}\right)\right] \quad \ldots\ldots 49$$

Simplifying Equation 49 gave:

$$= \frac{\beta}{12700800}\left[(2.241227633e^{-3}) - (4.446575736e^{-4})\right] \quad \ldots\ldots 50$$

Simplifying Equation 50 further gave:

$$\int_0^1\int_0^1 \frac{\partial^2 H_2}{\partial R \partial Q}\frac{\partial^2 H_1}{\partial R \partial Q}.H_1\, \partial R \partial Q = 1.41453299\beta e^{-10} \quad \ldots\ldots 51$$

Substituting Equations 17, 22, 28, 30, 36, 43and 51 into Equation 8 gave:

$$\left(\frac{2.857142857e^{-3}}{\propto^4} + \frac{3.265306122e^{-3}}{\propto^2} + 6.031746032e^{-3}\right)\Delta - 2.5e^{-3}\frac{qb^4}{D} - \left[(-1.602549594e^{-10}) + (-7.350807376e^{-11}) - (1.41453299e^{-10})\right]\frac{2\Delta^3\beta h}{\propto^2 D} = 0 \quad \ldots\ldots 52$$

Reducing Equation 52 further gave

$$(9.005191972e^{-9})\frac{(1-\mu^2)\Delta^3}{h^2(1+2\propto^2+\propto^4)} + \left(\frac{2.857142857e^{-3}}{\propto^4} + \frac{3.265306122e^{-3}}{\propto^2} + 6.031746032e^{-3}\right)\Delta - 2.5e^{-3}\frac{qb^4}{D} = 0 \quad \ldots\ldots 53$$

5. RESULTS DISCUSSION

5.1. DISCUSSION ON DEFLECTION COEFFICIENT, Δ OF CCCS PLATE

The numerical values of deflection coefficients for CCCS plate were calculated by solving Equations 54 and 55 with the developed software.

$$(9.005191972e^{-9})\frac{(1-\mu^2)\Delta^3}{h^2(1+2\propto^2+\propto^4)} + \left(\frac{2.857142857e^{-3}}{\propto^4} + \frac{3.265306122e^{-3}}{\propto^2} + 6.031746032e^{-3}\right)\Delta - 2.5e^{-3}\frac{qb^4}{D} = 0 \quad \ldots\ldots 54$$

$$w = \Delta(1.5R^2 - 2.5R^3 + R^4)(Q^2 - 2Q^3 + Q^4) \quad \ldots\ldots 55$$

Table 1: Coefficient of Deflection for CCCS Plate with $v = 0.3$

Aspect ratio ($\propto = \frac{a}{b}$)	$\frac{W(0,0)}{qb^4/D}$		Difference in the results	
	Present	Cui shuang	Difference	% Difference
1	0.0016	0.00157048	0.00002952	1.8796
1.1	0.0018	-	-	-
1.2	0.0019	0.00196902	0.00006902	3.5052
1.3	0.0020	-	-	-
1.4	0.0020	-	-	-
1.5	0.0020	0.00233584	0.00033584	14.3777

The numerical values of this plate under a unit load were calculated using the aforementioned equations. There are limited literature on this boundary condition but, however, Shuang [1] worked on it. His results and the ones gotten in this work are presented on Table 1. The deflections presented in the table are obtained at the center of the plate and they are non-dimensional. The increment in the aspect ratio of the present study is in the arithmetic progression while Shuang has his own in geometric progression. The two studies irrespective of the increment, evaluated deflection at aspect ratio of 1.0, 1.2 and 1.5.

An interesting relationship exist between the results of present study and the results obtained by Shuang. Their results are within the same neighborhood with minor differences and these difference could be attributed to the level of approximations made. Noteworthy, the percentage difference calculated are within the acceptable limit in statistics, hence one can infer that the results of both studies have a great correlation.

5.2. DISCUSSION ON THE DEFLECTION, *w* FOR CCCS PLATE

Numerical studies of deflection for CCCS plate under different loads were carried out to determine how the plate behaves under the load. The values used for this nonlinear analysis of CCCS thin rectangular plate were listed in subsection 5.1.

The numerical studies of CCCS plate under different loading was evaluated and the results presented in Table 4.6. In this analysis, the linear dimension, a, was kept constant while the linear dimension, b, was varied. This variations was made in such a way that the linear dimension, b, was on the decrease. The deflections of each load presented in Table 1 were plotted against the aspect ratio of the plate as shown in Figure 2. Different colours were used to differentiate the loads from each other.

The graph shows that the deflection decreases gradually as the aspect ratio increases. From the tabulated values, the deflection is higher at the aspect ratio 1.0 than the others. This implies that the deflection is quite stable when the aspect ratio is higher. One can therefore conclude that the deflection is a function of the linear dimensions of the plate. And from the three loads used it could also be affirmed that the deflection will continue to decrease as long as there is an increase in the aspect ratio.

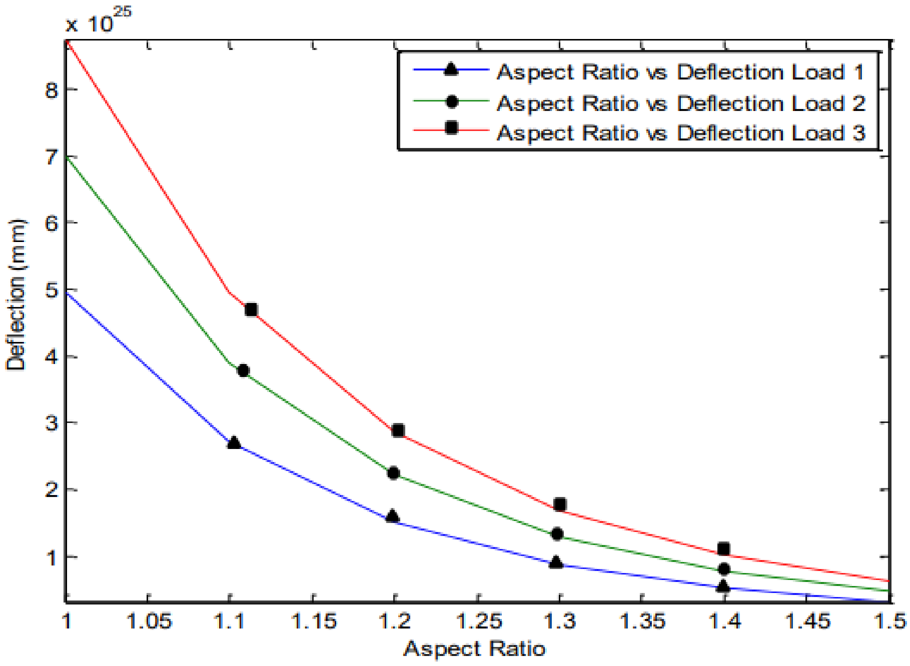

Figure 2: Relationship between the deflection and aspect ratio of CCCS Plate

6. CONCLUSION

The use of polynomial as a shape function has been successfully and effectively carried out in this work against the usual traditional trend of using trigonometric series as shape function. This trigonometric function has its own inadequacy, it can only handle SSSS and CCCC plates but beyond these two plates, its efficiency reduces. Numerical values obtained from this work under a unit load have been compared with the ones in the literature. There is a good agreement with the present results and the previous ones. This results indicate that the method adopted by the present work is adequate, reliable and satisfactory for the analysis of CCCS rectangular plate.

REFERENCES

1. Shuang, C. (2007). Symplectic Elasticity Approach for Exact Bending Solutions of Rectangular Thin Plates. Master Thesis. City University of Hong Kong.
2. Yamaguchi, E. (1999). Basic theory of Plates and Elastic stability. Structural engineering handbook. Ed. Chen Wai-fah Boca Raton.
3. Shufrin, I., Rabinovitch, O., & Eisengerger, M., (2008b). A semi-analytical approach for the nonlinear large deflection analysis of laminated rectangular plates under general out-of plane loading. *International Journal of Non-linear Mechanics*, 43, 328-340.
4. Krysko, V. A., Zhigalov, M. V., Saltykova, O. A., & Krysko, A. V. (2011). Effect of transverse shears on complex nonlinear vibrations of elastic beams. *Journal of applied mechanics and technical physics*, *52*(5), 834-840.
5. Ventsel, E., & Krauthammer, T. (2001). Thin plates and Shells: Theory, Analysis and Applications. New York: Marcel Dekker.
6. Jam, J. E., Maleki, S., & Andakhshideh, A. (2012). Non-linear Bending Analysis of Moderately Thick Functionally Graded Plates Using Generalized Differential Quadrature Method. *International Journal of Aerospace Science*, 1(3), 49-56.
7. Silveira, L. C., & Albuquerque, E. L., (2014). Large deflection of composite of composite laminate thin plates by the Boundary element method. *Blucher Mechanical Engineering Proceedings*, 2(1).

8. Zenkour, A. M. (2006). Generalized shear deformation theory for bending analysis of functionally graded plates. *Applied Mathematical Modelling*, *30*(1), 67-84.
9. Woo, J., & Meguid, S. A. (2001). Nonlinear analysis of functionally graded plates and shallow shells. *International Journal of Solids and structures*, *38*(42-43), 7409-7421.
10. Yang, J., & Shen, H. S. (2003). Nonlinear bending analysis of shear deformable functionally graded plates subjected to thermo-mechanical loads under various boundary conditions. Composites: Part B, 34, 103-115.
11. Wu, T. L., Shukla, K. K., & Huang, J. H. (2006). Nonlinear static and dynamic analysis of functionally graded plates. *International journal of applied Mechanics and Engineering*, *11*(3), 679-698.
12. Reddy, J. (2000). Analysis of functionally graded plates. *International Journal for numerical methods in engineering*, *47*(1-3), 663-684.
13. Navazi, H. M., Haddadpour, H., & Rasekh, M. (2006). An analytical solution for nonlinear cylindrical bending of functionally graded plates. Thin-Walled structures, 44, pp.1129 -1137.
14. GhannadPour, S. A. M., & Alinia, M. M. (2006). Large deflection behavior of functionally graded plates under pressure loads. *Composite Structures*, *75*(1-4), 67-71.
15. Navazi, H. M., & Haddadpour, H. (2008). Nonlinear cylindrical bending analysis of shear deformable functionally graded plates under different loadings using analytical methods. *International Journal of Mechanical Sciences*, *50*(12), 1650-1657.
16. Zhao, X., & Liew, K. (2009). Geometrically nonlinear analysis of functionally graded plates using the element-free kp-Ritz method. *Computer Methods in Applied Mechanics and Engineering*, *198*(33-36), 2796-2811.
17. Barbosa, J. A. T., & Ferreira, A. J. M. (2009). Geometrically nonlinear analysis of functionally graded plates and shells. *Mechanics of Advanced Materials and Structures*, *17*(1), 40-48.
18. Hao, Y. X., Zhang, W., Yang, J., & Li, S. Y. (2011). Nonlinear dynamic response of a simply supported rectangular functionally graded material plate under the time-dependent thermalmechanical loads. *Journal of Mechanical Science and Technology*, *25*(7), 1637-1646.