

Mixture Experiment Model for Predicting Static Modulus of Elasticity of Laterite-Quarry Dust Cement Block

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DOI: [10.36348/sjce.2022.v06i04.003](https://doi.org/10.36348/sjce.2022.v06i04.003)

| Received: 13.02.2022 | Accepted: 24.03.2022 | Published: 11.04.2022

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Abstract

Laterite-quarry dust cement block are masonry unit produce by full replacement of natural sand with appropriate mix of laterite and quarry dust. Static modulus of elasticity is an important parameter in predicting the structural behavior in service under load action and determines the deformations and displacements distribution concrete and similar other structural members like blocks. In this work, a mathematical model is formulated using Mixture experiment for predicting the static modulus of elasticity of laterite-quarry dust block. The model is tested for lack of fit and found adequate.

Keywords: Static modulus of elasticity, laterite, quarry dust, Mixture Experiment, mix proportion, Scheffe's augmented lattice equation.

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1.0 INTRODUCTION

Laterite- Quarry Dust Cement Blocks (LQCB) are currently being used as masonry unit in the construction buildings and other structures in most part of Nigeria and other developing countries. They are produced using appropriate mix of laterite and quarry dust as full replacement of natural sand. To this end [1], observed that "their structural properties and performance, like any other similar structural element, are very key and important not only to user and designer but also for be wider adoption by the construction industry". One of such properties that is considered critical is the Static Modulus of Elasticity (SME) which is the ratio of the applied stress to the corresponding strain. It demonstrate the ability of concrete to withstand deformation due to applied stress as well as its stiffness. Specifically, it reflects the capacity of concrete to deflect elastically. Modulus of elasticity of concrete element is sensitive to aggregate and mixture proportions of concrete. It is possible to predict the structural behavior in service under load action and determine the deformations and displacements distribution of concrete and similar structures like blocks from the elastic modulus of elasticity [2, 3]. Structural designers usually estimate the static modulus of elasticity of structural members using models that associate this property with the

compressive strength. There are growing interests in research for the determination of strength and modulus of elasticity of concrete and block produced using alternative materials considered as waste like quarry dust and laterite. The modulus of elasticity and strength of concrete with alternative materials was investigated by [4]. Also, the elastic modulus and splitting tensile strength of concrete incorporating both fine and coarse recycled aggregates was investigated by Wang *et al.*, [5]. They formulated a model for the predicting the elastic modulus of concrete incorporating both fine and recycled coarse aggregate. The suitability of a mix of sand and quarry dust in the production of concrete blocks was investigated by Febin *et al.*, [6]. The study revealed an increased in Modulus of elasticity of the concrete blocks up to 30% replacement of the natural sand with quarry dust.

There are a number of empirical models used for the estimation of the modulus of elasticity based on the compressive strength. These models according to [7, 8] must be used with reservation and caution being that compressive strength and modulus of elasticity are distinct mechanical properties that are differently influenced by the concrete variables

2.0 MIXTURE EXPERIMENTS AND MODEL EQUATION

Any experiment that involves the mixing of two or more constituents in various proportion with the intent of measuring one or more responses (physical characteristics) of the resulting end products is a mixture experiment. Mixture experiment are special design that are gaining grounds in research because of the ease in proportioning as opposed to the conventional trial and error method. Their use according to Montgomery [9] has resulted in the formation of product that are easier to manufacture having improved field performance and reliability, lower product cost as well as reduction in product design and development time. Consequently, they have gain wide application in engineering, agriculture and other fields in the last years.

Mixture experiments are employed in solving a wide variety of problems involving mix proportion in civil engineering. Okafor and Egbe [10] developed a mixture model for the prediction of static modulus of elasticity of laterite-quarry Dust cement block. Simplex optimization technique was used to formulate mathematical model for optimization of static modulus of elasticity of the blocks. A mathematical model for optimization of static modulus of elasticity of the blocks produced from a combination of sand and laterite using simplex optimization was developed by [11]. Also, a mixture experiment model for predicting the compressive strength and water absorption of sand - quarry dust blocks was developed by [12]

A mixture experiment is describes by Cornel [13] as "that which the response is presumed to be dependent on relative proportions of the constituent materials and not on their total amount." Two basic requirements that must be fulfilled for such experiments are: the sum of the proportions of the constituents must add up to 1 and that none of the constituents will have a negative value. These requirements are represented mathematically as thus:

$$X_1 + X_2 + \dots + X_q = \sum_{i=1}^q X_i = 1 \dots\dots\dots (1)$$

$$0 \leq X_i \leq 1 \dots\dots\dots (2)$$

Where

q is the number of mixture components.

X_i ($i = 1$ to q) is the volume or mass proportion of component i in the mixture.

It should be noted that since the total proportions of the constituents is constrained to 1, only $q-1$ of the variables or constituents can be independently chosen.

A number of mixture experiment models have been developed by researchers and are recently being used in estimation and prediction. One of such, which is very popular is the Scheffe's second degree polynomial

equation for q component reproduced in the canonical form as thus:

$$y = \sum_{1 \leq i \leq q} \beta_i X_i \sum_{1 \leq i \leq j \leq q} \beta_{ij} X_i X_j \dots\dots\dots (3)$$

The number of terms in the Scheffe's polynomial, N is the minimum number of experimental runs necessary to determine the polynomial coefficients and is given as:

$$N = C_n^{(q+n-1)} = \frac{(q+n-1)!}{(q-1)!(n)!} \dots\dots\dots (4)$$

In equation (3) y is the response function and x_i ($i=1$ to q) is the proportion of the component in the mixture. For the fitting mixture experiment data, the second degree is the most commonly used. It is reproduced in the canonical form below:

$$\hat{y} = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{23} X_2 X_3 + \beta_{24} X_2 X_4 + \beta_{34} X_3 X_4 \dots\dots\dots (5)$$

The estimated coefficients in canonical equation are obtained from the regression analysis of the mixture experiment data. The canonical polynomial has fewer terms than the standard polynomial and is often referred to as the $\{q, n\}$ polynomial; n being the degree of polynomial.

Several other researchers [14-20] have utilized mixture experiment in solving mix optimization problems.

In this paper, the scheffe's model equation utilizing mixture experiment was developed and used to predict the Static Modulus of Elasticity of Laterite – quarry dust cement block using values obtained from compressive strength and the model results were tested for lack of fit.

3.0 MATERIALS AND METHOD

The materials and primary data used for this work are taken from author's earlier study [10] on model for predicting the static modulus of Elasticity of Laterite-quarry dust cement blocks.

Cement

Limestone Portland cement (Unicem brand), grade 32.5 obtained from a major dealer in Calabar, Cross River State, Nigeria conforming to BS 12 [21] was used for all the tests.

Water

Potable pipe born water supplied by the Cross River State Water Board (CRSWB) Limited was used for both specimen preparations and curing.

Laterite

The reddish-brown laterite samples used was obtained from an existing borrow pit in Odukpani, Nigeria (Latitude $05^0 07.48'$ and Longitude $08^0 20.5'$), using disturbed sampling technique. Testing was carried out at the Soil Mechanics Laboratory of Civil

Engineering Department of Cross River University of Technology in accordance with BS 1377(BSI, 1990) [22].The specific gravity for the laterite is 2.56.

Quarry dust

Quarry dust was obtained from the abundant deposits at Akamkpa quarry site in Akamkpa Cross River State Nigeria.. The quarry dust had a specific gravity of 2.52.

3.1 THE DESIGN OF THE EXPERIMENT

A statistical software [23] was used in designing the experiment using an augmented [4, 2] schefee’s simplex lattice design. The design simplex is shown in Figure 1 whereas the design is shown in Table 1. The design contained ten mixes at the vertices and edge of tetrahedron, augmented with five more mixes within the simplex. These five points were used to as check point to validate the model developed. There were also replicate points at the vertices and centroid of the tetrahedron, making it a total of twenty point. The design was based on pseudo component and applying randomization.

3.1.1 Components Transformation and moulding of the Blocks

The pseudo ratio was transformed to real component ratio before being used for the moulding of the blocks samples at a particular point in the simples based on the relationship stated below:

$$R = AT \dots\dots\dots (6)$$

Where; **R** is a vector containing the real ratios of the components, **P** is a vector containing the pseudo ratios, **T** is a transformation matrix obtained from trial mixes given as below:

$$T = \begin{pmatrix} 0.53 & 0.63 & 0.80 & 0.9 \\ 1 & 1 & 1 & 1 \\ 5.4 & 3 & 9 & 5 \\ 0.6 & 3 & 1 & 5 \end{pmatrix}$$

The element of each column of [T] represents the components proportions at the vertex in the following order Water(X1), cement (X2), quarry dust(X3) and Laterite (X4).

A total of one hundred and twenty (110) hollow blocks, measuring 450mm x 225mm x 225mm overall dimensions, were moulded using a vibrating block moulding machine. The aggregates were used in their dry state and batching was by weight. Manual mixing was employed. The blocks were cured in open air for 28 days after demoulding by sprinkling them with water in the morning and evening. Sixty blocks each (three for each run) were used to determine the compressive strength. The remaining 10 were for contingencies.

3.1.2 Determination of Compressive strength

A digital Universal Testing (Okhard) Machine (UTM) with a testing range of 0 – 1000kN conforming to the requirements of [24] was used for compression test. The compressive strength of the blocks was determined using equation (7).

$$f_c = \frac{P}{A} \dots\dots\dots (7)$$

Where f_c = the compressive strength, P = crushing load, A = net cross-sectional area of the block.

3.1.3 Determination of Static modulus of Elasticity of Block

The Static modulus of elasticity for the block was computed as a function of compressive strength and density using the relation established by [8] represented as:

$$Ec = 1.7\rho^2 f_c^{0.33 \times 10^{-6}} \dots\dots\dots (8)$$

Where, Ec = Static modulus of Elasticity, ρ = density and f_c = compressive strength

Table 1 below shows the design matrix in the pseudo with the experimental test results for the compressive strength and static modulus of elasticity.

4.0 RESULT AND DISCUSSION

The pseudo components mix and response from compressive strength and static modulus are contained in Table 1.

Model Development for Static Modulus of Elasticity

The second degree model (equation (5), was fitted to the data set of the 20 compressive test responses at 95% confidence limit (α) using Minitab [23] The parameter estimate of the coefficients and analysis of variance tables are shown in table 2 and 3 respectively, while the normal probability plot of the residual is shown in Fig 2.

The model equation for Static Modulus of Elasticity is therefore given as thus:

$$\hat{y} = 8.705X_1 + 7.062X_2 + 7.991X_3 + 7.944X_4 + 2.382X_1X_2 - 1.330X_1X_3 - 0.821X_1X_4 + 5.611 X_2X_3 + 3.421X_2X_4 + 0.850X_3X_4 \dots\dots\dots (9)$$

Test for Adequacy

The p-value for lack-of-fit being 0.103 which is greater than α (0.05). The normal probability plot of the residual in Figure 2, reveals that the residuals fall reasonably close to the reference line, with a p- value of 0.351 (> 0.05), indicating that the data follow a normal distribution, hence justifying the assumption required for use of analysis of variance. The inference drawn from here is that, equation (9) is adequate for predicting the Static Modulus of elasticity of laterite-quarry dust blocks.

Comparison of Experimental and Model Result

The comparison of experimental result, with result predicted by the model is presented Table 4.

Table 1: The pseudo components mix and response from compressive strength and static modulus

Run Order	Std	Pseudo components				Response (y)	
	Order	Water	Cement	Quarry dust	Laterite X ₄	Compressive strength, f_c	Static modulus of Elasticity (GPa)
		(X ₁)	(X ₂)	(X ₃)		(Nmm ⁻²)	
1	5	0	1	0	0	1.87	6.9476
2	11	0.25	0.25	0.25	0.25	2.56	8.2870
3	16	1	0	0	0	2.50	8.8566
4	3	0.5	0	0.5	0	2.24	7.993
5	7	0	0.5	0	0.5	2.37	8.2285
6	4	0.5	0	0	0.5	2.37	8.0286
7	8	0	0	1	0	1.81	7.9795
8	15	0.125	0.125	0.125	0.625	2.42	8.6075
9	2	0.5	0.5	0	0	2.56	8.3193
10	9	0	0	0.5	0.5	2.09	8.1875
11	17	0	1	0	0	1.89	7.0700
12	10	0	0	0	1	2.20	7.6358
13	1	1	0	0	0	2.45	8.4865
14	6	0	0.5	0.5	0	2.54	8.8676
15	19	0	0	0	1	2.20	8.2145
16	12	0.625	0.125	0.125	0.125	2.56	8.9084
17	18	0	0	1	0	1.90	8.0343
18	20	0.25	0.25	0.25	0.25	2.49	8.1386
19	13	0.125	0.625	0.125	0.125	2.5	8.9417
20	14	0.125	0.125	0.625	0.125	2.3	8.4656

Source: Researcher's work

Table 2: Estimated regression coefficient for static modulus (pseudo component)

Term	Coef	SE Coef	T	P	VIF
water	-77261	149409	*	*	21905176262
cement	288	5506	*	*	61869770
quarry dust	-429	503	*	*	15544561
laterite	-724	1145	*	*	17473311
water*cement	88402	210904	0.42	0.684	510570352
water*quarry dust	89565	167144	0.54	0.604	9090126610
water*laterite	93134	176679	0.53	0.610	2461096106
cement*quarry dust	-936	2863	-0.33	0.750	5714098
cement*laterite	-852	1791	-0.48	0.644	481920
quarry dust*laterite	40	138	0.29	0.778	60035
S = 0.335646 PRESS = 5.79223					
R-Sq = 79.83% R-Sq(pred) = 0.00% R-Sq(adj) = 61.67%					

Table 3: Analysis of Variance for Static Modulus of elasticity (scheffe's component proportions)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	9	4.45768	4.45768	0.495298	4.40	0.015
Linear	3	1.37451	0.35277	0.117589	1.04	0.415
Quadratic	6	3.08317	3.08317	0.513862	4.56	0.017
water*cement	1	2.35640	0.01979	0.019793	0.18	0.684
water*quarry d	1	0.44250	0.03235	0.032349	0.29	0.604
water*laterite	1	0.14650	0.03131	0.031305	0.28	0.610
cement*quarry d	1	0.00200	0.01204	0.012043	0.11	0.750
cement*laterite	1	0.12632	0.02550	0.025504	0.23	0.644
quarry d*laterite	1	0.00945	0.00945	0.009449	0.08	0.778
Residual Error	10	1.12658	1.12658	0.112658		
Lack-of-Fit	5	0.87065	0.87065	0.174129	3.40	0.103
Pure Error	5	0.25594	0.25594	0.051188		
Total	19	5.58426				

Table 4: Comparison of Experimental and Model Result

Run Order	Pseudo component				Experimental result (GPa)	Model predicted results (GPa) Scheffe (pseudo unit)
	Water	Cement	Quarry dust	Laterite		
1	0	1	0	0	6.9476	7.0619
2	0.25	0.25	0.25	0.25	8.2870	8.5575
3	1	0	0	0	8.8566	8.7051
4	0.5	0	0.5	0	7.9930	8.0157
5	0	0.5	0	0.5	8.2285	8.3580
6	0.5	0	0	0.5	8.0286	8.1191
7	0	0	1	0	7.9795	7.9914
8	0.125	0.125	0.125	0.625	8.6075	8.3081
9	0.5	0.5	0	0	8.3193	8.4791
10	0	0	0.5	0.5	8.1875	8.1799
11	0	1	0	0	7.0700	7.0619
12	0	0	0	1	7.6358	7.9435
13	1	0	0	0	8.4865	8.7051
14	0	0.5	0.5	0	8.8676	8.9293
15	0	0	0	1	8.2145	7.9435
16	0.625	0.125	0.125	0.125	8.9084	8.4878
17	0	0	1	0	8.0343	7.9914
18	0.25	0.25	0.25	0.25	8.1386	8.5575
19	0.125	0.625	0.125	0.125	8.9417	8.3651
20	0.125	0.125	0.625	0.125	8.4656	8.4371

Source: Researcher's work

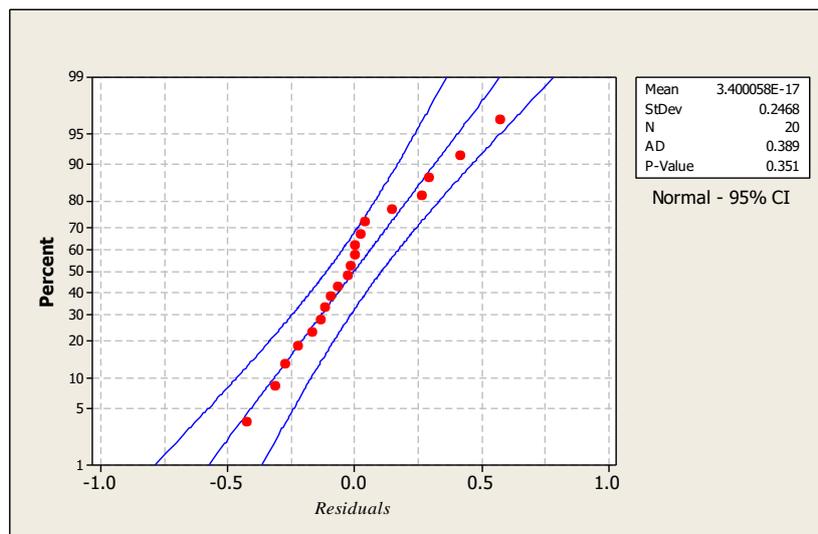


Figure 2: Normal probability plot for Static Modulus of Elasticity (Sheffe's Pseudo component model)

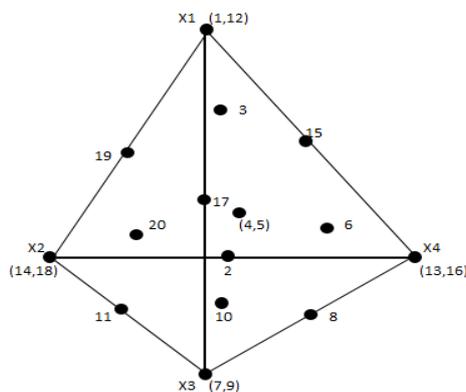


Figure 1: A typical Scheffe's Augmented [4, 2] simplex lattice showing the points and run order

4.1 DISCUSSION OF RESULT

(a) Model coefficient

Equation 8, reveals that:

$\beta_1 > \beta_3 > \beta_4 > \beta_2$, indicating that water contribute most to the strength followed by quarry dust, laterite and cement in that order. There are also binary synergistic effect in the interaction between water and cement, cement and quarry dust, cement and laterite as well as quarry dust and laterite with the highest being that between cement and quarry dust.

(b) Maximum and minimum predictable model values

The model developed can predict values of static Modulus of Elasticity of the block within the range 6.95GPa to 8.94GPa.

(c) Comparison of Experimental and Model Result

The difference between the experimental and predicted results expressed as a percentage of the experimental value for all the models and responses in most cases were all less than 5% which is quite insignificant. This indicate that the model is suitable and adequate for predicting the for static modulus elasticity.

5.0 CONCLUSION AND RECOMMENDATION

A mathematical model for the prediction of static modulus of elasticity of laterite-quarry dust cement blocks was developed in this work. The model can be used to predict the static module of elasticity of laterite-quarry dust cement blocks ranging from 6.95GPa to 8.94GPa. The use of laterite-quarry dust cement will help greatly in reducing the cost associated with providing affordable housing for most sub-Saharan African especially where there is abundant deposit of these materials.

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