Two-Dimensional Modeling of Waste Stabilization Pond with Computational Fluid Dynamics

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Abstract

Waste stabilization ponds (WSP) are used extensively to provide wastewater treatment throughout the world. A review of the literature indicates that, understanding the hydraulics of waste stabilization ponds is critical to their optimization, the research in this area has been relatively limited and that there is a poor mechanistic understanding of the flow behavior that exists within these systems. This explains why there is no generally acceptable model for predicting its performance. The two-dimensional computational fluid dynamics (CFD) model developed in this study was extensively tested on the waste stabilization pond located in the campus of the University of Nigeria, Nsukka which was used as the field pond and also on a laboratory scale waste stabilization pond obtained from literature. Although the model may be solved by several methods, this research was limited to computational method; numerical solution using finite difference method was used in solving the two-dimensional partial differential equations at steady state conditions. In order to validate the quality of the model, its results were compared with the experimental data from the field and the lab-scale ponds. The results obtained were encouraging, prediction of pond performance with measured values shows that correlation coefficient of 0.82 was obtained, representing an accuracy of 82%. The 2-D model was then used in series of investigative studies such as; effect of single inlet and outlet structures at different positions in the pond, effect of multiple inlet and outlets on the pond’s performance, variation of pond performance with depth, effect of short-circuiting on pond treatment efficiency, effect of baffles on pond performance using laboratory-scale pond data and comparison with tracer studies. In all, the results agree with literature.

Keywords: Stabilization pond, modeling, computational fluid dynamics, optimization, hydraulics.

INTRODUCTION

The primary purpose of wastewater treatment is the reduction of pathogenic contamination, suspended solids, oxygen demand and nutrient enrichment. Domestic wastewater can be effectively stabilized by the natural biological process that occurs in shallow ponds. Those treating raw wastewater are referred to as facultative ponds, lagoons or oxidation ponds. Where small ponds are installed after secondary treatment they are referred to as tertiary, maturation, or polishing ponds. Their purpose is to further reduce suspended solids, BOD, faecal micro-organisms and ammonia in the plant effluent. Waste stabilization ponds (WSP) are cheap and effective way to treat waste water in situation where the cost of land is not a factor. Not only has it been found to be one thousand times better in destroying pathogenic bacteria and intestinal parasites than the conventional treatment plants [1], it is also more economical [2]. It is simple to construct, operate and maintain and it does not require any input of external energy. Although a WSP system usually requires large land area because of its long detention time which is attributable to its complete dependence on natural treatment process, it will still be very suitable in several African countries and communities where land acquisition is not a problem. Besides, its efficiency depends on the availability of sunlight and high ambient temperature, which are the prevailing climatic conditions in most cases of these communities.

Waste stabilization ponds (WSP)

In recent years, a rising chorus of concern has developed regarding the quality of the effluent...
discharged from WSPs. The basis for the concern is the algae and coliform organisms, which may be present in the effluent. The parameters used in judging the performance of WSP are bacteria rate of degradation, biochemical oxidation, dispersion, bacteria die-off rate and thermal stratification, which are influenced by temperature gradient. Many models [3-7] have been proposed to describe the process of bacteria degradation. But none has been found acceptable [8]. In terms of predicting the practical performance of the WSPs, hence, the call-in recent times has been to develop more appropriate models that will describe the process accurately [3, 4, 8]. Although WSP system is economical compared with the conventional treatment, no model has yet been found to describe it accurately [4, 8, 3]. WSP are becoming popular for treating wastewater, particularly in tropical and sub-tropical regions where there is an abundance of sunlight, and the ambient temperature is normally high.

Factors affecting Waste Stabilization Pond Performance

There has been little rigorous work done on determining optimal pond shapes. The most common shape is rectangular, although there is much variation in the length to breadth ratio. Several geometrical factors affect wastewater treatment which relates to the hydraulics condition of the pond and influences the mixing characteristics and detention time and ultimately its efficiency. The pond hydraulics is influenced by the presence of unused dead space [9]; length to width ratios, inlet and outlet positions [10], and pond depth. In the design of ponds, it is very essential to choose configurations that will give minimum short-circuiting. Short-circuiting can be reduced and hence hydraulic efficiency increased by introducing baffles [11], and by limiting the length to width ratio to a value not less than 3.0. It should be possible to design ponds, lagoons and channels, especially large ones, more economically if the geometry of the pond is so manipulated as to give the desired dispersion condition to enable optimizing for process requirements.

Hydraulic residence time (HRT) for WSP

Tracer studies of scale model ponds and smaller full scale WSPs have provided an indication of the deviation from both plug and completely mixed behaviour seen in reality. Several authors including [12] have reported that field trials have produced residence time distributions (RTDS) demonstrating a range of behaviour. These range from almost completely mixed behaviour with a majority outflow of the tracer after only a fraction of the theoretical residence time. [13] stated that the circulation of mean residence time can be used to estimate the volume of short circuiting in the ponds [14] presented a realistic hydraulic model by compartmentalizing WSPs into zones of forward and return plug flow. The dispersion model adapted in WSPs by [6] can be used to give an even more efficient description of outflow age. The dispersion number (d) is used as a descriptor of the magnitude of longitudinal dispersion within the pond, a scale ranging from 0 (plug flow) to a (completely mixed conditions). Dispersion number analysis of WSPs was developed from retrospective analysis of pond hydraulic performance. Several researchers such as [15, 9, 16] reported that the validation of these models by application to other sites has not always been particularly successful. A number of expressions have been empirically developed to predict dispersion number based on pond parameters such as length, width, depth and flow rate, [17] also reported that residence time distribution analysis of various pond systems has led researchers including proponents of the dispersed flow model, to conclude that other factors are of significant, if not of primary importance in determining pond dispersion number [12] attributed pond short circuiting mainly to wind position and pond orientation, while [18] reported on a poorly operating WSP in which thermal stratification created significant dead zones and affected flow hydraulics and ultimately treatment performance. These findings indicate that the dispersion number (d) is not the static variable traditionally suggested, but rather a dynamic variable which is a function of pond flows and environmental conditions as well as pond design and layout.

More recently, the popularization of computational fluid dynamics (CFD) techniques has enabled the simulation of RTD studies on ponds of any configuration or scale and under any physical conditions [19, 18, 20] have demonstrated the ability of CFD, RTD analysis to theoretically predict hydraulic short-circuiting in operational WSP systems [18] and [19] have also shown that first order time dependent decay models can be integrated over the distribution area to quantify removal rates and potential improvements available from the simulated results.

Computational Fluid Dynamics Approach to WSPs

The term ‘computational fluid dynamics’, usually abbreviated to ‘CFD’, encompasses computer-based methods for solving the linked partial-differential equation set that governs the conservation of energy, momentum and mass in fluid flow. In order to understand the internal processes and interaction in waste stabilization ponds, the simulation of the hydrodynamics has become a tool worth studying [20]. Pond design involves several physical, hydrological, geometrical and dynamic variables to provide high hydrodynamic efficiency and maximum substrate utilization rates. Computational fluid dynamic modeling (CFD) allows the combination of these factors to predict the behaviour of ponds by using different configurations. The simulation of hydrodynamic in bioreactors supported by modern computing technology is an important tool to gain an improved understanding of the process function and performance [21, 21] provided detailed governing dynamic equations to solving the 2D- depth integrated equations of fluid mass

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and momentum conservation of an incompressible fluid in two horizontal directions.

**RESEARCH METHODS**

Mathematically, a model describes a system of assumptions, equations and procedures intended to describe the performance of a prototype system. Although the model developed in this study may be solved by several methods, this research was limited to computational method; numerical solution using finite difference method was used in solving the three-dimensional partial differential equations at steady state condition and applying the Danckwerts’ boundary conditions [22] and other boundary conditions obtained from the pond surface conditions.

**Sources of data**

The data requirement for the validation of the CFD model developed were obtained from literature of a full-scale field pond (WSP) located at the University of Nigeria, Nsukka, Enugu State [23] and from a published work of a laboratory-scale model [24]. The data analyzed for both the field pond and LSWSP were: temperature (T°C); dissolved oxygen (DO); hydrogen ion concentration (PH); detention time (τ); dispersion number (d); suspended solid (SS); algal concentration (Cs); organic loading rate (OL); faecal coliform per 100ml; the pond settling velocity (V); the maximum pond velocity under no wind (Um); the mean velocity of flow in the pond (U); biochemical oxygen demand (BOD) and chemical oxygen demand (COD).

**Software application**

Since elaborate numerical computations are involved in providing solution to the numerous partial differential equations generated, software application becomes inevitable. One of the software applications used in this study for solving the cumbersome equations generated is the MATLAB. MATLAB is a software package for high-performance numerical computation and visualization [25]. It provides an interactive environment with hundreds of built-in functions for technical computation, graphics, and animation. Best of all, it also provides easy extensibility with its own high-level programming language.

**MODEL DERIVATION**

The principle of conservation of mass

A mass balance can be performed on a finite segment of length Δx, as follows: Accumulation = inflow − outflow − decay reaction

\[
\frac{\partial C}{\partial t} = \left( \frac{Q \partial C}{\partial x} \right) - \left[ Q \left( C \frac{\partial C}{\partial x} \right) - \frac{D}{A} \frac{\partial^2 C}{\partial x^2} \right] - K \frac{\partial C}{\partial x} \Delta x
\]

Where:

- \( V = \) volume (m³), \( Q = \) flow rate (m³/d), \( C = \) concentration (mg/L), \( A = \) tank cross-sectional area (m²) and \( K = \) first-order decay coefficient (d⁻¹)

The dispersion terms are based on Fick’s first law:

\[
\text{Flux} = -D \frac{\partial C}{\partial x} \quad \ldots \quad (4.02)
\]

It specifies that turbulent mixing tends to move mass from regions of high to low concentration.

The parameter D, therefore, reflects the magnitude of turbulent mixing.

By noting that: \( v = ADx \) and \( U = Q/A \)

Equation (4.01) can be simplified, thus:

\[
\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} - \frac{U \partial c}{\partial x} - Kc \quad \ldots \quad (4.03)
\]

Equation (4.03) is a one-dimensional non-steady state advection-dispersion equation for non-conservative contaminants with a first-order decay rate.
At steady state, it reduces to a second–order ODE,

The material balance equation becomes;

\[ Qc - DA \frac{dc}{dx} - Q(c + dc) - DA \frac{dc}{dx} + \frac{d}{dx} \left( DA dc \right) dx - Kcd = 0 \quad \quad \ldots \ldots (4.04) \]

By simplification

\[ -Qdc - DA \frac{d^2c}{dx^2} dx - Kc. Adx = 0 \quad \quad \ldots \ldots (4.05) \]

\[ D \frac{d^2c}{dx^2} - U \frac{dc}{dx} - Kc = 0 \quad \quad \ldots \ldots (4.06) \]

Where; \( U \) = flow velocity

Since we are not only interested in what is happening along the x–axis, we cannot ignore what may happen on the transverse (across) axis, that is, y–axis. Similarly, we can formulate a two-dimensional equation as;

\[ \frac{\partial c}{\partial t} = D_x \frac{\partial^2 c}{\partial x^2} - D_y \frac{\partial^2 c}{\partial y^2} - \frac{U \partial c}{\partial x} - \frac{V \partial c}{\partial y} - Kc \quad \quad \ldots \ldots (4.07) \]

At steady state, equation (4.07) becomes;

\[ D_x \frac{\partial^2 c}{\partial x^2} - D_y \frac{\partial^2 c}{\partial y^2} - \frac{U \partial c}{\partial x} - \frac{V \partial c}{\partial y} - Kc = 0 \quad \quad \ldots \ldots (4.08) \]

Where, \( D_x \) and \( D_y \) are the dispersion coefficient in the x and y axis respectively. U and V are the velocity components in the x and y Cartesian co-ordinate respectively. The principle of conservation of momentum.

According to Newton’s second law of motion;

\[ \sum F = M. a \quad \quad \ldots \ldots (4.09) \]

Considering the forces only in the x–direction, equation (4.09) may be written as

\[ M \frac{ax}{gx} = F_{gx} + F_{px} + F_{vx} \quad \quad \ldots \ldots (4.10) \]

Equation (4.10) is called the Navier–Stokes equation of motion. For complete derivation, it may be presented as:

\[ \rho \left( \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} \right) = -\frac{\partial P}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + S_{mx} \quad \quad \ldots \ldots (4.11) \]

\[ \rho \left( \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} \right) = -\frac{\partial P}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + S_{my} \quad \quad \ldots \ldots (4.12) \]

For incompressible flow, the number of unknowns is three viz; \( u, v \) and \( p \).

The Navier-Stokes equation plus incompressible continuity equation are the sufficient conditions to determine the flow characteristics.

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \quad \ldots \ldots (4.13) \]

The general solution of Navier-Stokes equations has not been found as it is second order non-linear differential equation. However, the solutions have been obtained only for flow situations wherein the boundary configuration is simple and the fluid characteristics such as the density and viscosity are almost constant.

By applying the boundary configuration, equations (4.11), (4.12) and (4.13) are solved
simultaneously using the finite difference method to determine the fluid velocities in the X and Y directions in the CFD model.

Finite difference solution of the 2-D equation

\[
Dx \frac{\partial^2 C}{\partial x^2} + Dy \frac{\partial^2 C}{\partial y^2} - U \frac{\partial C}{\partial x} - V \frac{\partial C}{\partial y} - KC = 0
\]

Writing the finite difference scheme

\[
\frac{\partial^2 C}{\partial x^2} = \frac{C_{i+1,j} - 2C_{i,j} + C_{i-1,j}}{h^2} - \frac{2p^2 \frac{\partial C}{\partial x}}{2h} = \frac{C_{i+1,j} - 2C_{i,j} + C_{i-1,j}}{h^2} - \frac{2p \frac{\partial C}{\partial x}}{2h} = \frac{C_{i+1,j} - C_{i,j}}{2p} + \frac{C_{i,j} - C_{i-1,j}}{2p}
\]

\[
\frac{\partial^2 C}{\partial y^2} = \frac{Dy}{h^2} \left[C_{i+1,j} - 2C_{i,j} + C_{i-1,j}\right] + D_y \left[C_{i,j+1} - 2C_{i,j} + C_{i,j-1}\right] - U \left[C_{i+1,j} - C_{i-1,j}\right] - V \left[C_{i,j+1} - C_{i,j-1}\right] - KC_{i,j} = 0
\]

\[
\left(\frac{Dx}{h^2} - \frac{U}{2h}\right)C_{i+1,j} + \left(\frac{Dy}{h^2} + \frac{U}{2h}\right)C_{i,j+1} + \left(\frac{Dx}{p^2} + \frac{V}{2p}\right)C_{i,j+1} + \left(\frac{Dy}{p^2} + \frac{V}{2p}\right)C_{i,j-1} - \left(2\frac{Dx}{h^2} + \frac{2Dy}{p^2} + K\right)C_{i,j} = 0
\]

Boundary conditions: 

- At the inlet where \(x = 0\), the term \(C_{i-1,j}\) outside the scheme was obtained

\[
\frac{\partial C}{\partial x} = 0, \quad x = 0, \quad 0 < y < B \quad \text{And} \quad \frac{\partial C}{\partial y} = 0, \quad x = L \quad \text{(Outlet end)}, \quad \frac{\partial C}{\partial y} = 0, \quad y = 0, B
\]

At the inlet where \(x = 0\), the term \(C_{i-1,j}\) outside the scheme was obtained

\[
\left(\frac{Dx}{h^2} - \frac{U}{2h}\right)C_{i+1,j} + \left(\frac{Dy}{h^2} + \frac{U}{2h}\right)C_{i,j+1} + \left(\frac{Dx}{p^2} + \frac{V}{2p}\right)C_{i,j+1} + \left(\frac{Dy}{p^2} + \frac{V}{2p}\right)C_{i,j-1} - \left(2\frac{Dx}{h^2} + \frac{2Dy}{p^2} + K\right)C_{i,j} = 0
\]

By invoking the boundary condition for the inlet; \(UC_{in} = UC_o - D \frac{\partial C}{\partial x}\)

A finite divide difference can be substituted for the derivative, where \(C_o = \) concentration at \(x = 0\) at the inlet position.

Thus, \(UC_{in} = UC_o - D \frac{\partial C}{\partial x}\) which can be solved to give

\[
C_{i-1,j} = \frac{2hu}{2n} C_{in} - \frac{2hu}{2n} C_o + C_{i+1,j}
\]

Substitute in equation (4.16)

\[
\left(\frac{Dx}{h^2} + \frac{2Dy}{p^2} + K\right)C_{i+1,j} + \left(\frac{Dx}{h^2} + \frac{2Dy}{p^2} + K\right)C_{i,j+1} + \left(\frac{Dy}{p^2} + \frac{V}{2p}\right)C_{i,j+1} - \left(\frac{Dy}{p^2} + \frac{V}{2p}\right)C_{i,j-1} - \left(2\frac{Dx}{h^2} + \frac{2Dy}{p^2} + K\right)C_{i,j} = 0
\]

\[
\frac{\partial C}{\partial x} = 0, \quad x = 0, \quad 0 < y < B, \quad \text{It implies that,} \quad C_{i+1,j} = C_{i-1,j}
\]

Substitute in equation (4.15)

\[
\left(\frac{Dx}{h^2} + \frac{2Dy}{p^2}\right)C_{i+1,j} + \left(\frac{Dy}{p^2} + \frac{V}{2p}\right)C_{i,j+1} + \left(\frac{Dy}{p^2} + \frac{V}{2p}\right)C_{i,j-1} - \left(2\frac{Dx}{h^2} + \frac{2Dy}{p^2} + K\right)C_{i,j} = 0
\]

At the edge of the pond where \(x = 0, y = 0\); \(\frac{\partial C}{\partial y} = 0, \quad \frac{C_{i+1,j} - C_{i-1,j}}{2p} = 0\)

Hence \(C_{i+1,j} = C_{i-1,j}\) Substitute in equation (4.18)

\[
\left(\frac{Dx}{h^2} + \frac{2Dy}{p^2}\right)C_{i+1,j} + \left(\frac{Dy}{p^2} + \frac{V}{2p}\right)C_{i,j+1} - \left(2\frac{Dx}{h^2} + \frac{2Dy}{p^2} + K\right)C_{i,j} = 0
\]

At the edge of the pond where \(x = 0, y = B\)

\[
\left(\frac{Dx}{h^2} + \frac{2Dy}{p^2}\right)C_{i+1,j} + \left(\frac{Dy}{p^2} + \frac{V}{2p}\right)C_{i,j+1} - \left(2\frac{Dx}{h^2} + \frac{2Dy}{p^2} + K\right)C_{i,j} = 0
\]

For the outlet, the slope must be zero, that is; \(\frac{\partial C}{\partial x} = 0, x = L, \quad 0 < y < B\)

The finite divided difference will yield, \(C_{i+1,j} = C_{i-1,j}\) Substitute in equation (4.15)
\[
\begin{align*}
\left( \frac{2Dx}{h^2} \right) C_{i-1,j} + \left( \frac{Dy}{p^2} - \frac{V}{2p} \right) C_{i,j+1} + \left( \frac{Dy}{p^2} + \frac{V}{2p} \right) C_{i,j-1} - \left( \frac{2Dx}{h^2} + \frac{2Dy}{p^2} + K \right) C_{i,j} &= 0 \quad \ldots \quad (4.21)
\end{align*}
\]

At the pond outlet ends, for which \( y = 0 \), the equation can be written as
\[
\begin{align*}
\left( \frac{2Dx}{h^2} \right) C_{i-1,j} + \left( \frac{2Dy}{p^2} \right) C_{i,j+1} - \left( \frac{2Dx}{h^2} + \frac{2Dy}{p^2} + K \right) C_{i,j} &= 0 \quad \ldots \quad (4.22)
\end{align*}
\]

At the pond outlet ends for which \( y = B \), the equation becomes
\[
\begin{align*}
\left( \frac{2Dx}{h^2} \right) C_{i-1,j} + \left( \frac{2Dy}{p^2} \right) C_{i,j-1} - \left( \frac{2Dx}{h^2} + \frac{2Dy}{p^2} + K \right) C_{i,j} &= 0 \quad \ldots \quad (4.23)
\end{align*}
\]

At the sides of the pond, \( \frac{\partial C}{\partial y} = 0 \). \( y = 0, B \). It implies as before \( C_{i,j+1} = C_{i,j-1} \).

Substitute in equation (4.15) for \( y = 0 \)
\[
\begin{align*}
\left( \frac{Dx}{h^2} - \frac{U}{2h} \right) C_{i+1,j} + \left( \frac{Dx}{h^2} + \frac{U}{2h} \right) C_{i-1,j} + \left( \frac{2Dy}{p^2} \right) C_{i,j+1} - \left( \frac{2Dx}{h^2} + \frac{2Dy}{p^2} + K \right) C_{i,j} &= 0 \quad \ldots \quad (4.24)
\end{align*}
\]

For \( y = B \)
\[
\begin{align*}
\left( \frac{Dx}{h^2} - \frac{U}{2h} \right) C_{i+1,j} + \left( \frac{Dx}{h^2} + \frac{U}{2h} \right) C_{i-1,j} + \left( \frac{2Dy}{p^2} \right) C_{i,j-1} - \left( \frac{2Dx}{h^2} + \frac{2Dy}{p^2} + K \right) C_{i,j} &= 0 \quad \ldots \quad (4.25)
\end{align*}
\]

The general equation for the system’s nodes within the pond may be written using equation (4.15) above.

**RESULTS AND DISCUSSIONS**

**Comparisons of prediction of pond performance in 2-D CFD model with measured values**

These comparisons were made at different depth of the pond and at varying inlets and outlets positions. The following figures (2–4) demonstrate it appropriately.

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**Fig-2:** Comparison between measured value and CFD model using inlet position I

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**Fig-3:** Comparison between measured value and CFD model using inlet position I
Effects of baffles on the performance of WSP

The summary of effluent concentrations and removal efficiency of BOD concentrations for different cases of baffles are presented below.

The study has demonstrated that the BOD removal efficiency increased from 16% to 22% in case 1, from 22% to 92% in case 2 and from 82% to 96% in case 3.

Effect of single inlet and outlet positions on pond performance

As demonstrated previously, position of the inlet and outlet structures affects the pond’s treatment efficiency. Simulation using the model was performed. The result obtained shows variance with respect to inlet/outlet positions used, and this prove the fact.
CONCLUSIONS

Modeling waste stabilization pond is somewhat a daunting course due to the complexity involve in understanding its hydraulics. The results obtained were encouraging, prediction of pond performance with measured values shows that an accuracy of 82% was obtained using the 2-D CFD model. The 2-D model was then used in series of investigation studies such as: effect of single inlet and outlet structures at different positions in the pond, effect of multiple inlet and outlets, variation of pond performance with depth, effect of short- circuiting on pond treatment efficiency, effect of baffles on pond performance using laboratory-scale pond data and comparison with tracer studies. In all, the results were in agreement with established literature.

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