

# Theoretical and Experimental Analysis of Bridge Dynamics under Moving Loads

Kankeu Mbefoyo King Jackson<sup>1\*</sup>, Joyce Ursula Nana Pettang<sup>2</sup>, Lezin Seba Minsili<sup>3</sup>, Koumbe Mbock<sup>4</sup>

<sup>1</sup>Ph.D. Student, Department of Civil Engineering, Yaoundé National Advanced School of Engineering ENSPY, The University of Yaoundé 1, P.O. Box 8390 Yaoundé, Cameroon. Email: kankeuleking@yahoo.fr ORCID ID: KING JACKSON KANKEU MBEFOYO (0000-0003-1422-6409)

<sup>2</sup>Lecturer, Department of Civil Engineering, Yaoundé National Advanced School of Engineering ENSPY, The University of Yaoundé 1, P.O. Box 8390 Yaoundé, Cameroon. Email: pettangursu@gmail.com

<sup>3</sup>Associate Professor, Department of Civil Engineering, Yaoundé National Advanced School of Engineering ENSPY, The University of Yaoundé 1, P.O. Box 8390 Yaoundé, Cameroon. Email: lezinsm@yahoo.com

<sup>4</sup>Lecturer, Department of Mathematics, Yaoundé National Advanced School of Engineering ENSPY, The University of Yaoundé 1, P.O. Box 8390 Yaoundé, Cameroon. Email: k.mbock@yahoo.fr

DOI: [10.36348/sjce.2021.v05i11.004](https://doi.org/10.36348/sjce.2021.v05i11.004)

| Received: 18.11.2021 | Accepted: 23.12.2021 | Published: 30.12.2021

\*Corresponding author: Kankeu Mbefoyo King Jackson

## Abstract

Demand for freight to be transported by road increases in many countries, around the world. Consequently, we should determine an interval of vehicle velocities of minimal dynamic effects on a mixed bridge. For this, we describe the dynamic behavior of the type vehicle 12-b on the steel-concrete bridge by combining the model equations of bridge, apron and vehicle and propose a framework in which the overall model equations is solved using the newmark method. In this approach, a criteria of limited deformation on the mixed bridge is required from eurocode 4 to examine the vehicle velocities that lead to minimal dynamical effects. The sensitivity analysis is experimented and this provides a number of admissible velocities of the type vehicle 12-b that satisfies the requirements of our steel-concrete mixed bridge. This type of operation is of great importance in the monitoring of a bridge at the stages of its design, construction or maintenance. Monitoring is an essential step prior to any other management action, and in the context of this monitoring, the measurements represent important issues. This is particularly true for the two special surveillance actions, which are enhanced surveillance and high surveillance; in the latter case, measurements on structures even constitute an essential condition for the very existence of high-level surveillance, because they alone are capable of giving precise information in real time on the unfavorable development of a structure.

**Keywords:** Bridge, interaction, behavior, dynamic, amplification, vehicle.

**Copyright © 2021 The Author(s):** This is an open-access article distributed under the terms of the Creative Commons Attribution **4.0 International License (CC BY-NC 4.0)** which permits unrestricted use, distribution, and reproduction in any medium for non-commercial use provided the original author and source are credited.

## 1. INTRODUCTION

In the absence of the reliable infrastructures there is a great probability in the occurrence of several disasters like the ones observed in Africa and in Cameroon particularly in recent due to an over-exploitation and poor management policies [1, 2]. Bridges constitute an exceptional heritage and asset that need to be preserved and maintained to a satisfactory service level. The knowledge of the response and the behavior of a bridge when subjected to natural and artificial loadings such as traffic loads have created specific interests of several structural engineers and specialized research institutions during the last decades and their founding at different levels have been implemented into design and construction [2, 3].

The trend in bridge engineering is to design and construct a lighter bridge that spans a wider obstacle and carries fast moving heavier traffic [3, 4] to meet the present and future transport demand. The analysis of the dynamic behavior of a highway bridge subjected to the motion of a vehicle is a complicated problem [1, 5-7] to be solved taking into account the interaction of the bridge-vehicle system and their individual structural characteristics such as natural frequencies, the contact interface behavior during the motion, and design ultimate values [8, 9]. In another words how can we predict the behavior of a bridge in the design stage prior of its construction, or what will

be the response of an existing bridge subjected to a new set of loadings that was not used in the design?

The method of resolution of the dynamic interaction between the bridge and vehicles programmed with appropriate numerical tools [10, 11]. This method takes into account the bridge, the vehicle and the profile of the pavement. Each of these components of the interaction leans on a model that is presented briefly. From a criteria of vertical deflection limitation, we provide an organization chart that helps us to determine a targeted speed. This organizational chart is finally used for an analysis of the sensitivity of a reinforced concrete steel bridge loaded with a three axles truck model [12]. A parametric survey is achieved in order to judge the response of the bridge to dynamic loads induced by the road traffic.

The dynamic behavior of the bridge vehicle interaction model is obtained by the coupled model bridge equations and the vehicle numerical moving model in which the interaction model equations are solved using the Newmark method. The allowable output response according to criteria of Eurocode 4 [13] and the bridge mechanical properties are analyzed in our interaction model in order to provide design and construction guidelines subjected to the mixed reinforced concrete steel bridge exploitation requirements. To assess the sustainability of the developed interaction model, simulation results obtained in this paper under MATLAB [10] are compared with satisfaction to experimental results from a local bridge test prior to commissioning, as well as to results obtained by several engineering commercial numerical tools such as EFFEL [14] and the Strength of Material computational theory.

## 2 MATERIAL AND METHODS

The survey of the dynamic behavior of bridges is achieved by a numerical approach that considers the bridge modeled by means of finite elements where the profile of the pavement is included in the contact interface with the heavy weight truck mathematical model. The interaction dynamic system composed of these three subsystems is solved with the help of a software that integrates the differential equations of the motion.

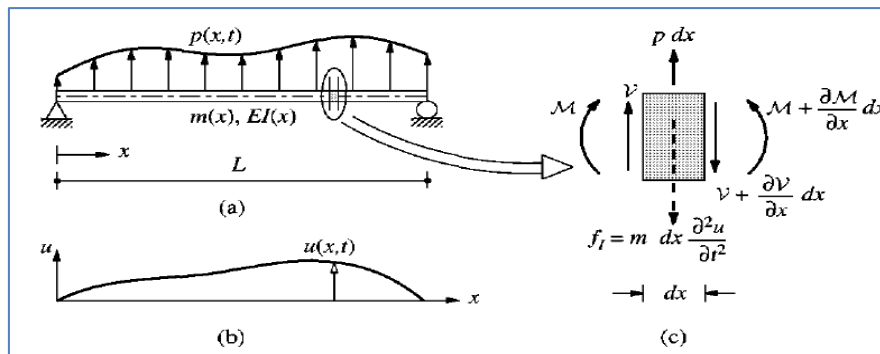
The dynamic model of the of the bridge-vehicle interaction through the motion, general equation in the linear analysis is obtained through the principle of virtual work [15] as:

$$W(u, \dot{u}, \ddot{u}) = W_{int}(u) - W_{ext}(u) + W_{IN}(\ddot{u}) + W_D(\dot{u}) = 0 \quad (1)$$

where  $u$ ,  $\dot{u}$  et  $\ddot{u}$  are respectively the cinematic fields of displacements, speeds and accelerations and subscripts  $int$ ,  $ext$ ,  $IN$  and  $D$  refer respectively to the internal and external quantities of inertia and damping.

### 2.1 The Bridge Model equation

We develop the equation for the transverse vibration of a straight simple supported beam that represents a relatively true representation of a 2D bridge model under specific boundary conditions [13]. Considering a beam of span  $L$  with the flexural rigidity  $EI(x)$  and mass per unit of length  $m(x)$  on which an external forces  $p(x, t)$  is applied at time  $t$ , that produces a transverse displacement  $u(x, t)$  at position  $x$  as seen in figures 1.a) and 1.b).



**Fig-1: Bridge model with variable mass and inertia: a) beam and applied forces; b) displacement; c) beam element**

The system has an infinite number of dynamic degree of freedom (DOF) due to the given distributed properties. For an infinitely small element of length  $dx$  as seen in figure 1.c), we can easily find the system dynamic equation of motion when the damping effect is willingly neglected as:

$$m(x) \frac{\partial^2 u}{\partial t^2} + \frac{\partial^2}{\partial x^2} \left[ EI(x) \frac{\partial^2 u}{\partial x^2} \right] = p(x, t) \quad (2)$$

To get an unique solution to this equation we must specify boundary conditions of the beam as well as initial displacement  $u(x, 0)$  and initial velocity  $\dot{u}(x, 0)$ . Thus when a beam with constant flexural inertia and mass is considered in the presence of a viscous damping model, the system dynamic motion equation is given as

$$M_p \ddot{U} + C_p \dot{U} + K_p U = F_{pv}^{int} \quad (3)$$

Where  $M_p$ ,  $C_p$  et  $K_p$  represent respectively mass, damping and stiffness matrices of the system bridge system ( $p$  indicating the bridge subscript) and  $F_{pv}^{int}$  represents the vector of the bridge vehicle interaction effect.

When we consider a beam element of length  $L$  with a distributed mass  $m(x)$  per unit of length and the flexural rigidity  $EI(x)$ , taking into account that each plane 2-node beam element has four DOF, we use the virtual work principle to express each displacement  $u(x,t)$  as a linear combination of the shape function  $\psi_i(x)$  and nodal displacements  $u_i(t)$  in order to define the system mass ( $M_p$ ), damping ( $C_p$ ) and stiffness ( $K_p$ ) matrices whose influence coefficients (of the stiffness  $k_{ij}$  and mass  $m_{ij}$ ) of  $j$ 's DOF on  $i$ 's DOF are expressed as:

$$k_{ij} = \int_0^L EI(x) \psi_i''(x) \psi_j''(x) dx \quad (4.a)$$

$$m_{ij} = \int_0^L m(x) \psi_i(x) \psi_j(x) dx \quad (4.b)$$

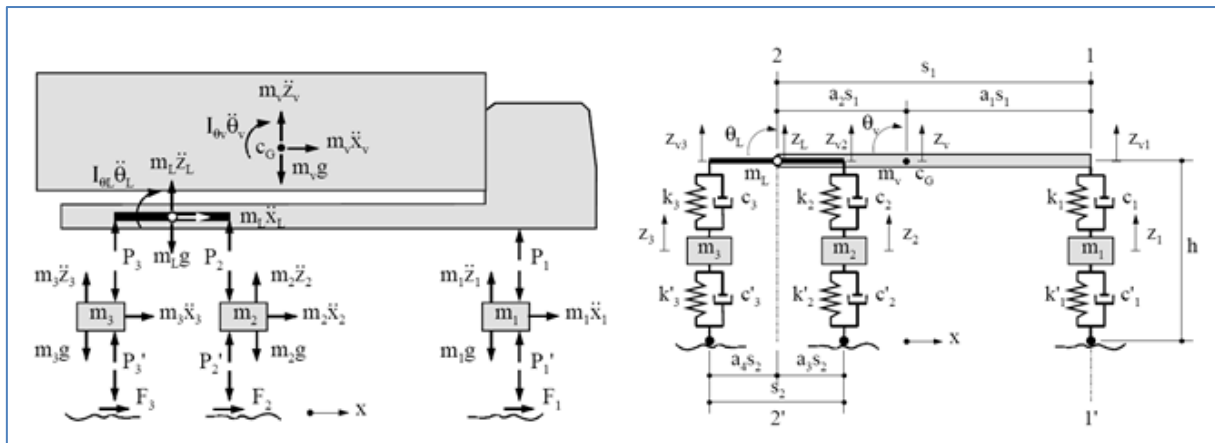


Fig-2: vehicle model with corresponding (a) equilibrium forces and (b) axes characteristics

For the development of our numerical model, the following hypotheses are made on the moving load system:

1. The tires of the axles stay in permanent contact with the pavement. The slope rating  $w_i$  of contact points is expressed as:

$$w_i(t) = w(x_i(t), y_i(t), t) + r(x_i(t), y_i(t)) \quad (5)$$

where  $w(x_i(t), y_i(t), t)$  is the vertical displacement of the bridge to the contact point of coordinate  $(x_i, y_i)$  at time  $t$  and  $r(x_i, y_i)$  is the slope rating of the profile of the pavement at the point  $(x_i, y_i)$ .

2. Axles 1-1' and 2-2' preserve their orientation in relation to the vehicle. In addition, the relative slip of the rigid body in relation to axles is to be neglected.

The Damping matrix  $C$  is taken as a linear combination of the mass and the stiffness matrices according to the Rayleigh formulation [15] using appropriate Rayleigh damping constant  $a_i$ , damping ratio  $\xi_i$  and corresponding natural frequencies  $w_i$ .

$$C = a_0 M + a_1 K \quad (4.c)$$

## 2.2 The Vehicle Model

The vehicle model is taken as the three-axis 12\_B Truck used for the experimental bridge test commissioning of the new bridge that is pictured down in figure 5.a) [12] and represented with the mathematical model represented in figure 2.a). The vehicle has seven independent DOF referred to as time dependent variables  $z_i(t)$ ,  $z_v(t)$ ,  $\theta_v(t)$ ,  $\theta_L(t)$  and  $x_v(t)$  with their corresponding derivatives, subscripts  $i$ ,  $L$ , and  $v$  stand for the three axles, the suspensions, and the car's body represented in figure 2.b).

$$x_v = x_1 - a_1 s_1 \cos(\theta_v) + (h + \Delta_1 + \Delta'_1) \sin(\theta_v) \quad (6.a)$$

Where  $\Delta_1$  is the relative displacement of extremities of the suspension defined by  $k_1$  and  $c_1$ , and  $h$  is the slope rating of the center of gravity of the vehicle measured from the initial level of the pavement.

3. The rotation of the vehicle is too small that the corresponding cosine is close to unity and the sine to the angle value so that the relative slip of the rigid body becomes:

$$x_v = x_1 - a_1 s_1 + h \theta_v \quad (6.b)$$

The vehicle motion equations are obtained from the application of the principle of virtual work that stipulates that for all fields of cinematically admissible displacements, the work done by all internal and external forces remains constant. Thus the general equation of the vehicle motion takes the form:

$$M_v \ddot{Z} + C_v \dot{Z} + K_v Z = F \quad (7)$$

Where  $M_v$  is the mass matrix,  $C_v$  is the damping matrix,  $K_v$  is the stiffness matrix, and  $F$  is the applied force vector.

$$\begin{aligned}
 M_v &= \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 \\ & m_2 & 0 & 0 & 0 & 0 \\ & & m_3 & 0 & 0 & 0 \\ & & & m_v a_1^2 a_4^2 + (m_v + m_L) \frac{h^2}{s_1^2} + \frac{I_{\theta v}}{s_1^2} & m_v a_1 a_2 a_4 - (m_v + m_L) \frac{a_4 h^2}{s_1^2} - \frac{a_4}{s_1^2} I_{\theta v} & m_v a_1 a_2 a_3 - (m_v + m_L) \frac{a_3 h^2}{s_1^2} - \frac{a_3}{s_1^2} I_{\theta v} \\ & & & m_v a_1^2 a_4^2 + m_L a_4^2 + (m_v + m_L) \frac{a_4^2 h^2}{s_1^2} + \frac{a_4^2}{s_1^2} I_{\theta v} + \frac{I_{\theta L}}{s_2^2} & m_v a_1^2 a_3 a_4 + m_L a_3 a_4 + (m_v + m_L) \frac{a_3 a_4 h^2}{s_1^2} + \frac{a_3 a_4}{s_1^2} I_{\theta v} - \frac{I_{\theta L}}{s_2^2} \\ \text{sym.} & & & & & m_v a_1^2 a_3^2 + m_L a_3^2 + (m_v + m_L) \frac{a_3^2 h^2}{s_1^2} + \frac{a_3^2}{s_1^2} I_{\theta v} + \frac{I_{\theta L}}{s_2^2} \end{bmatrix} \\
 C_v &= \begin{bmatrix} c_1 + c_1' & 0 & 0 & -c_1 & 0 & 0 \\ & c_2 + c_2' & 0 & 0 & -c_2 & 0 \\ & & c_3 + c_3' & 0 & 0 & -c_3 \\ & & & c_1 & 0 & 0 \\ & & & & c_2 & 0 \\ \text{sym.} & & & & & c_3 \end{bmatrix} \\
 K_v &= \begin{bmatrix} k_1 + k_1' & 0 & 0 & -k_1 & 0 & 0 \\ & k_2 + k_2' & 0 & 0 & -k_2 & 0 \\ & & k_3 + k_3' & 0 & 0 & -k_3 \\ & & & k_1 & 0 & 0 \\ & & & & k_2 & 0 \\ \text{sym.} & & & & & k_3 \end{bmatrix} \\
 F &= \begin{bmatrix} -m_1 g + k_1' [w(x_1, y_1, t) + r(x_1, y_1)] + c_1' \left[ \dot{w}(x_1, y_1, t) + \left( \frac{\partial w}{\partial x} + \frac{\partial r}{\partial x} \right)_{x=x_1} \dot{x}_1 \right] \\ -m_2 g + k_2' [w(x_2, y_2, t) + r(x_2, y_2)] + c_2' \left[ \dot{w}(x_2, y_2, t) + \left( \frac{\partial w}{\partial x} + \frac{\partial r}{\partial x} \right)_{x=x_2} \dot{x}_1 \right] \\ -m_3 g + k_3' [w(x_3, y_3, t) + r(x_3, y_3)] + c_3' \left[ \dot{w}(x_3, y_3, t) + \left( \frac{\partial w}{\partial x} + \frac{\partial r}{\partial x} \right)_{x=x_3} \dot{x}_1 \right] \\ -a_2 m_v g + (m_v + m_L) \frac{h}{s_1} \ddot{x}_1 \\ -a_1 a_4 m_v g - a_4 m_L g - (m_v + m_L) \frac{a_4 h}{s_1} \ddot{x}_1 \\ -a_1 a_3 m_v g - a_3 m_L g - (m_v + m_L) \frac{a_3 h}{s_1} \ddot{x}_1 \end{bmatrix}
 \end{aligned} \quad (8)$$

The last equation of every system ( $x_i$ ) contains difficult terms to evaluate as one of the components of the friction effect  $F_j$ . These components exercise a little influence on the dynamic behavior of the bridge. To simplify the analysis, the moving speed of the vehicle is supposed to be known so that this component can be disregarded.

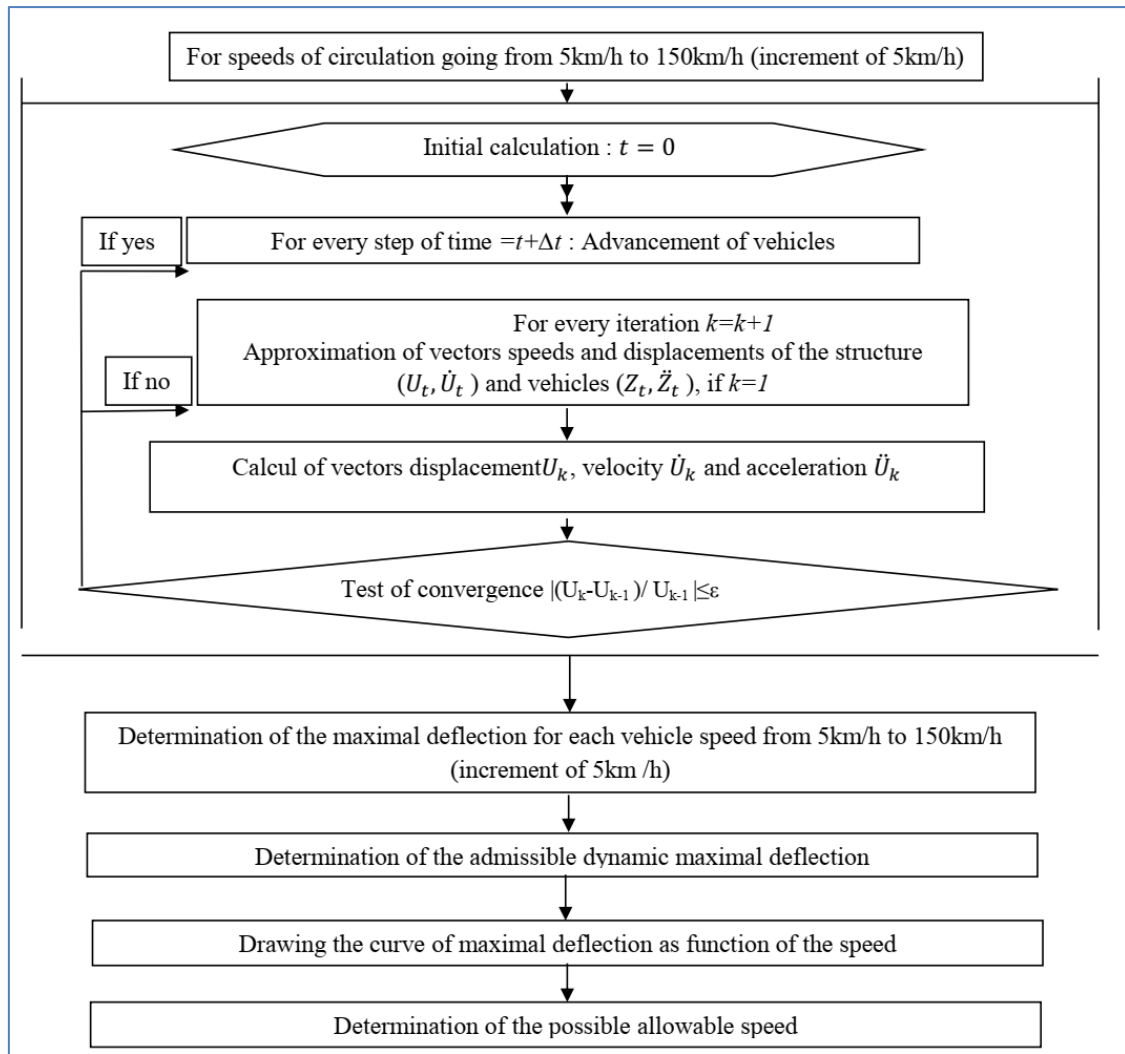
### 2.3 The Proposed Resolution Procedure

Considering the uncertainty and randomness of the pavement profiles  $r(x)$ , they are generated by the MATLAB software [9,10] with the help of spectral density functions  $G_d(n)$  and phase angle  $\theta_j$  randomly chosen between 0 and  $2\pi$ . They are expressed as:

$$r(x) = \sum_{j=1}^N \sqrt{4G_d(n_j)\Delta n} \cos(n_j x - \theta_j) \quad (9)$$

Two resolution methods of the motion equation can be applied to solve for the displacement of

the bridge vehicle system: the coupled method and the uncoupled method. But since the given equation is coupled adequate measures are provided to reach the desired results. The resolution algorithm takes into account the coupling that exists between system variables by adjusting the value of the common DOF defined by the contact point between the wheels and the bridge [16, 17]. This approach is based on a prediction-correction procedure that permits to calculate only in one time independent system matrixes  $M$ ,  $C$ , and  $K$  of the bridge and vehicles. In this adopted methodology enables to save for the resolution cost (space and time of calculation) [18-20], as well as solving for the coupled system differential equations representing the motion of the bridge vehicle interaction [21, 22] based on the Newmark direct numerical integration method. The different stages of resolution method are pictured in the organization chart of figure 3:



**Fig-3: Organization chart of our approach**

The following algorithm explains the different resolution stages they are found in the above diagram:

1. Initial calculation :  $t=0$
1. Calculation of bridge matrices  $M_p$ ,  $C_p$  et  $K_p$  ;
2. Calculation of vehicle matrices  $M_v$ ,  $C_v$  et  $K_v$  ;
3. Calculation of eigen values and vectors (if a modal analysis is required) :  $\Phi_i$  et  $\lambda_i$  ;
4. Generation of the profile of the road  $r(x)$  ;
5. Read and calculation of initials conditions
6. Choice of the time step  $\Delta t$

2. For every time step  $t=t+\Delta t$ , we have a new vehicle position leads to new values as:

$$\begin{cases} v_t = v_{t-\Delta t} + \Delta t \dot{v}_t \\ s_t = s_{t-\Delta t} + \Delta t v_{t-\Delta t} + \dot{v}_t (\Delta t)^2 / 2 \end{cases}$$

3. For every iteration  $k=k+1$
- o Approximation of speeds and displacements vectors of the structure, if  $k=1$

$$\begin{cases} \dot{U}_t = \dot{U}_{t-\Delta t} + \Delta t \ddot{U}_t \\ U_t = U_{t-\Delta t} + \Delta t \dot{U}_{t-\Delta t} + \ddot{U}_t (\Delta t)^2 / 2 \end{cases}$$

- o Approximation of vectors speeds and displacements of vehicles, if  $k=1$
- $$\begin{cases} Z_t = Z_{t-\Delta t} \\ \dot{Z}_t = \dot{Z}_{t-\Delta t} \end{cases}$$

#### 4. For every vehicle

- Initials conditions reading for every vehicle;
- Evaluation of  $\bar{w}_i$ ,  $\dot{\bar{w}}_i$ ,  $r_i$  and  $\dot{r}_i$  in every contact point  $i$ ;
- Calculation of the interaction force in every contact point  $i$ ;

$$\bar{F}_i^{int} = k_i'(\bar{w}_i + r_i) + c_i'(\dot{\bar{w}}_i + \dot{r}_i)$$

- Resolution of the dynamic interaction equation:

$$M_v \ddot{Z} + C_v \dot{Z} + K_v Z = F_g + \bar{F}^{int}$$

#### 2.4 Numerical Data and Parameters

In order to implement the above methodology the present data are used for concrete:  $f_{cj}=30MPa$ ,  $f_{ij}=0.6+0.06f_{cj}=2.4MPa$ ,  $E_{ij}=11000\sqrt[3]{f_{cj}}=34180MPa$ , and for steel E24 with the Young modulus  $E=210KN/mm^2$ . Parameters used for the calculation of mass, damping and stiffness coefficients defined above for the vehicle model for the truck type 12\_B [12]

modeled in figure 2.b) are related to geometry, masses, stiffness and damping. For geometry:  $h=1.80m$ ,  $e=5.55m$ ,  $c_g=3.95m$ ,  $a_1=0.71$ ,  $a_2=0.29$ ,  $a_3=0.50$ ,  $a_4=0.50$ . Concerning masses:  $m_v=24800.00kg$ ,  $m_L=100.00kg$ ,  $m_1=800.00kg$ ,  $m_2=1200.00kg$ ,  $m_3=1200.00kg$ ,  $I_{ev}=241359.00kgm^2$ ,  $I_{eL}=75.00kgm^2$ . For stiffness:  $k_1=520000.00N/m$ ,  $k_2=1174000.00N/m$ ,  $k_3=1174000.00N/m$ ,  $k_4=2000000.00N/m$ ,  $k_5=4000000.00N/m$ ,  $k_6=4000000.00N/m$ . Concerning damping :  $c_1=12194.00Ns/m$ ,  $c_2=20357.00Ns/m$ ,  $c_3=20357.00Ns/m$ ,  $c_4=4.000.00Ns/m$ ,  $c_5=6928.00Ns/m$ ,  $c_6=6928.00Ns/m$ .

The dynamic amplification factor (DAF) constitutes a means to represent the dynamic effect [23, 24] induced on the structure by an abrupt application of an external loading [25-29] and it can be defined as the following ration:

$$DAF = \frac{F_{DYN\ max}}{F_{STA\ max}} \quad (10)$$

Where  $F_{(DYN\ max)}$  represent the maximal dynamic displacement deflection and  $F_{(STA\ max)}$  is the maximal static displacement deflection

Standards in bridge design [13, 25] are giving an upper limitation  $f_{lim}$  for safety reason of the bridge vertical deflection in different states. In the present work the maximal deflection must verify the following condition:

$$f = \delta_1 + \delta_2 \leq f_{lim} \text{ with } f_{lim} = \min(f_{lim1}; f_{lim2}) \quad (11)$$

Where  $f_{lim}$  is the deflection limit,  $f_{lim1}$  is the reinforced concrete deflection limit,  $f_{lim2}$  is the steel beam deflection limit as settled by the proposed standard,  $\delta_1$  is the deflection due to permanent loads,  $\delta_2$  is the deflection due to live loads (vehicles configuration).

Two applications are presented in this work: analytical simulations based on the theory developed above applied on a bridge constructed earlier across the river Nyong in the vicinity of the city of Ayos-Akonolinga in Cameroon with geometrical characteristics of the mixed reinforced concrete steel bridge as shown in figures 4; and the experimental test loading test to check on the bearing capacity of the bridge prior to its commissioning.

The river Nyong in Ayos is one of the most important river of Cameroon with the water stream ranging from 25m in the dry season to 800m in the rainy season. Priors to the construction of this mixed bridge the traffic on the bridge was done through an overused 50 years old 6m width steel bridge of whose replacement became necessarily urgent due to an on growing traffic demand in speed and load. The new constructed mixed steel concrete bridge has three simple supported span of 76.2 m total length and of an average width of 10.3m to support, heavy highway loads (Loads A1, Bc, Bt, Mc120 and exceptional loads) according to local engineering standards [13].

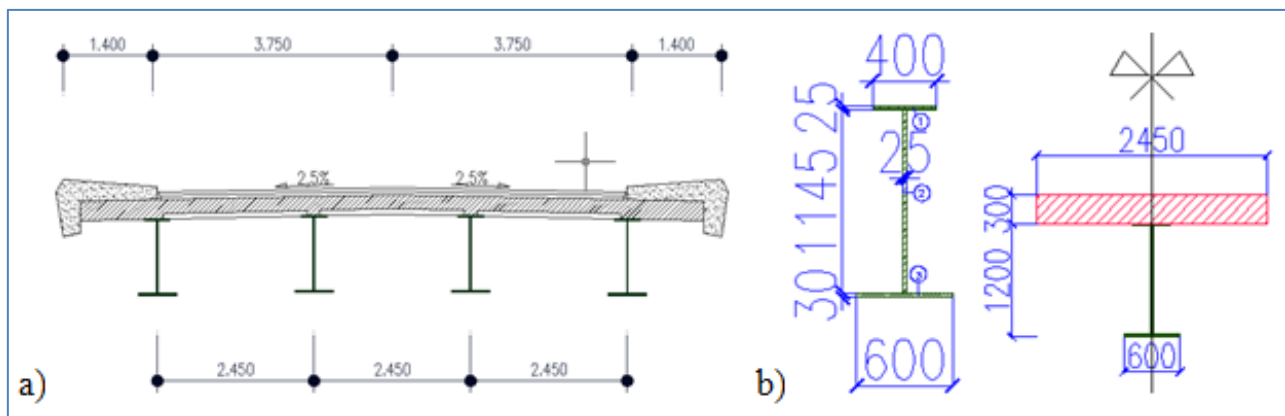


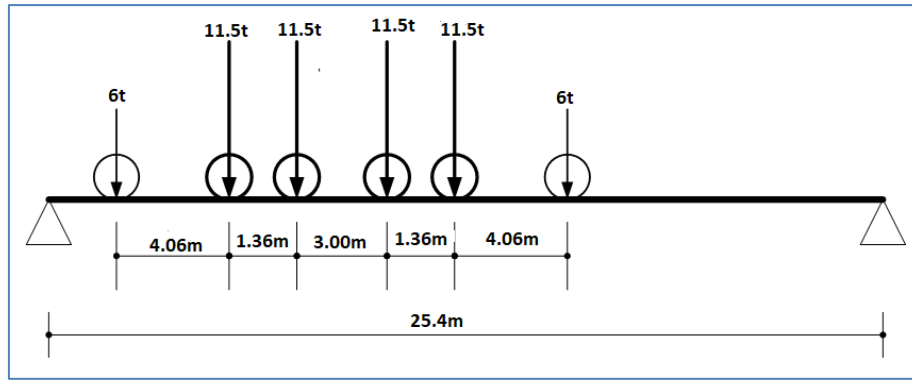
Fig-4: The bridge physical model a) Bridge profile; b) Bridge girder geometry

## 2.5 Static test principle

For static test, the convoy chosen by the enterprise answering to conditions of tests is composed

of four trucks Renault CBH of 29 tonnes (figures 5.a and c) arranged below in two lines like pictorial on the figure:





a)



b)

c)

d)

**Fig-5: Experimental Setting: Trucks a) convoy position; b) comparator position; c) static test loading; and d) the topographic deflectometer**

The static test consists in loading the apron by the convoy of the test known detailed how to determine distortions to the center of bays as well as reactions that will serve to the verification of bruising of support devices. One loads bays then by the four reserved trucks for the test while displacing loads in the longitudinal sense of the work. The position of loads is function of the one of the first axle of the convoy (figure 5-c). The test will be articulated around verification by the topographic method (figure 5-d) of the beam of strand solely for reasons of accessibility to the bridge. For every case of load, one makes surveys of distortions on the apron by a device electronic level as well as of measures of distortions of support devices by comparators of distortions (figure 5-b).

The comparator is a device of length measure. He does not indicate an absolute measure but a relative measure in relation to a point of reference. He is constituted of a dial stepped up with a needle revolving in his center. Around the dial, a telescope including one or several indexes can revolve by hand. The index allows the user to materialize the point zero. The comparator is constituted of a mobile stem in transfer. The displacement of this last is transmitted to a mechanical device transforming the transfer of the stem in rotation of the needle. The total angle of rotation is proportional to the displacement in transfer of the stem. To do a measure one makes a point zero for example to help of a hold stallion. The body of the comparator being stationary one places the piece to measure under the comparator that indicates the difference then

between the point of reference and the dimension of the measured piece.

## 2.6 Dynamic test principle

Among the vehicles used for static test, we keep a number equal to that of the traffic lanes, choosing those with the heaviest axles. These vehicles being arranged in front and in the same direction, they are made to circulate from end to end of the bridge at a speed adapted to the requirements of safety. Location of vehicles on the bridge is limited to the roadway and some parts of the structure cannot be excited up to appropriate level. Vehicles used as source of vibration excitation affect the modal properties of the tested structure by their moving additional mass. The moving live load acts mainly in vertical direction, in case of existence of an important horizontal mode, this kind of excitation can be insufficient. The goal of the test was to verify if it exists the vestigial settlements after loading of support devices. After the passage of the rolling convoy composed of two trucks rolling side by side with a speed of 60km/h comparators indicated some hopeless values. Therefore, we note that devices of supports always function in the elastic domain. In the worry to judge the sensitivity of the bridge opposite, the dynamic solicitations provoked by the road traffic we go done of simulations of the truck passage [26-29].

## 3 RESULTS AND DISCUSSION

### 3.1 Displacements for all the cases

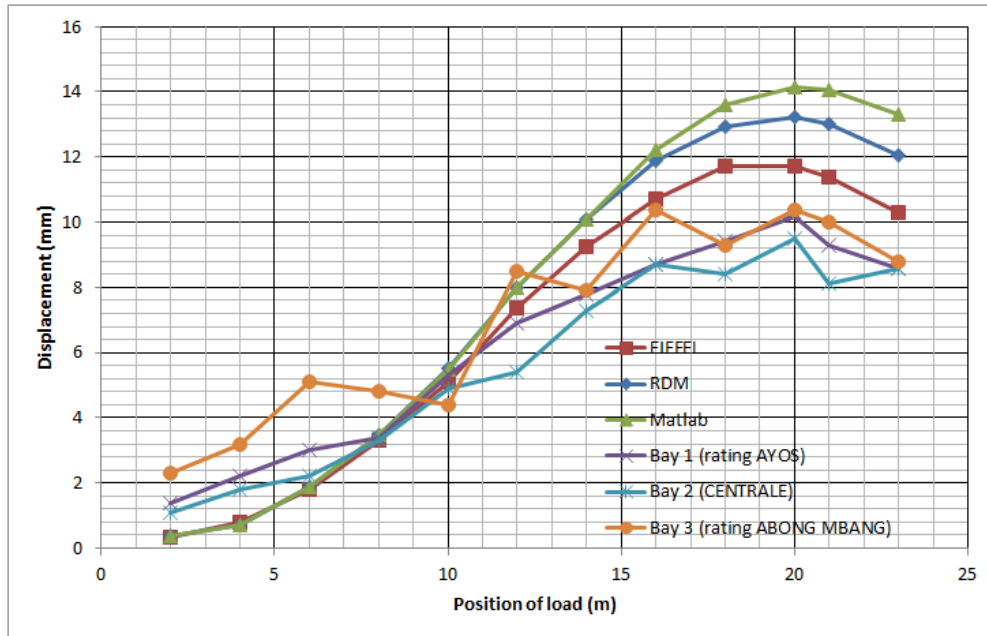
The deflection in middle of bay has been calculated for 12 cases of loading (figure 6). Of the

experimental results, we have the theoretical results following three methods in addition:

- While using the software of structure calculation EFFEL as made by the expert of the control mission of the project ;
- While using principles of the statics of the Material Resistance;

- While using the finites elements method;

These results permitted us to take out again the diagram comparative next one for the different methods:



**Fig-6: Comparatives different methods diagram**

While observing results given by the static test, we note that distortions have the same order of size that those theoretical, the maximal deflections are lower to the calculated deflections. Only one anomaly can be noted, for deflections where position load is understood between 2 and 8m, the real deflections are superior to those theoretical. It can be explained by the elasticity of support devices that generates an instantaneous settlement on the longitudinal axis adding itself to the deflection of the beam, seen that in hypotheses of calculation one considers supports as being rigid. For the other appropriated values, they are distinctly lower to those theoretical explaining itself by the supplementary rigidity generated by struts, sidewalks and ledges that are not considered in the theoretical calculations. Nevertheless, values appropriated on the third bay provided us a curve in teeth of saw compared to the other values for the other bays.

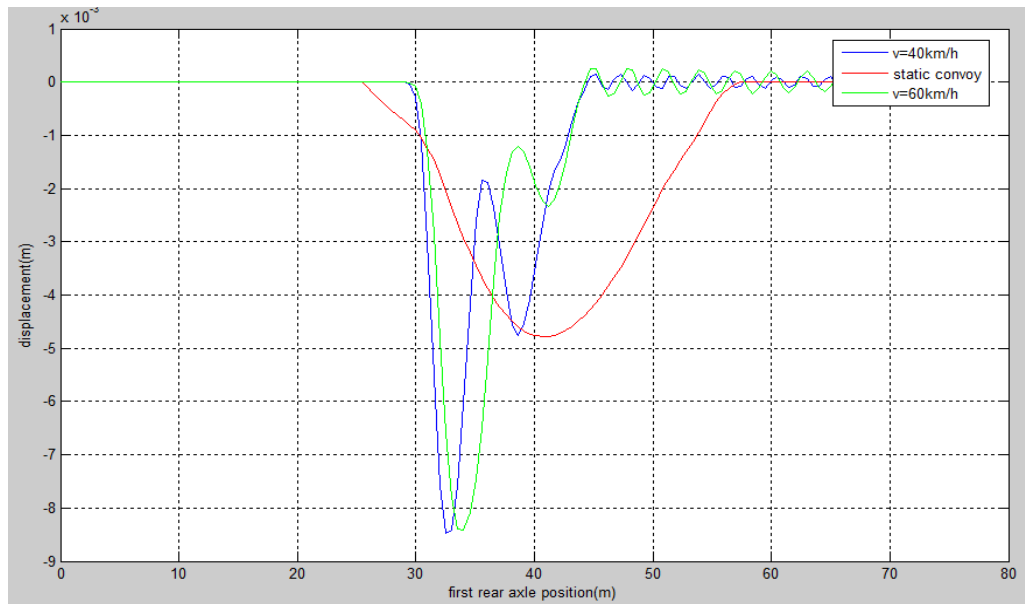
For what is results gotten with the proposed method (finite element method), they are nearly identical those of the software EFFEL for a position of load understood between 2 and 10m, and are thereafter distinctly superior to all the other values for the same position of the load. On the other hand, screw of the method of the materials resistances, values are very neighbors for a position of load understood between 2 and 16m. The calculation of displacements while using

the dynamic features of the bridge, gave us of satisfying results. One also records that after discharge of the bridge (for static and dynamic tests), a second survey has been done on all the length of the work showing the residuals deflection non-existence and that we are always in the elastic domain. Therefore, we conclude that beams normally behave to bending.

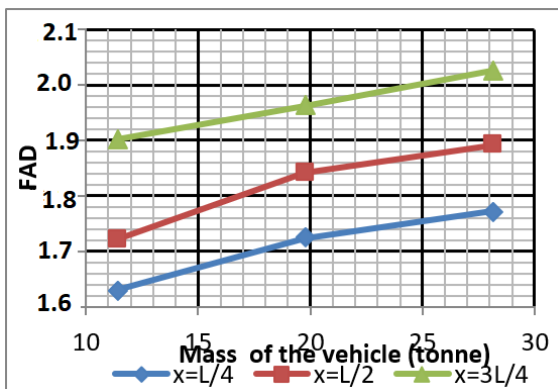
### 3.2 Proposed Analytical Simulation Results

By considering a two-truck convoy, as seen in figure 5.a, travelling at different speeds from the static experimental position to 40 km/h and 60 km/h, deflection analytical results are presented in figure 7.a, as function of the rear heavier axis position on the central span, we notice that the recorded maximal deflection for the three cases is not constant in value and the position in which it occurs is also speed dependent. Figures 7.b) and 7.c) show clearly through the dynamic magnification factor (DAF) that all the points of the bridge span are not subjected to the same amplification factor, depending on which point faces first the heaviest load and at which speed, and on bridge flexibility of that particular point. Since the position of the wheel might not be the unique parameter influencing the DAF, we see also that the vehicle weight as well as the speed too also has a great impact on the DAF at different levels that must be clearly investigated.

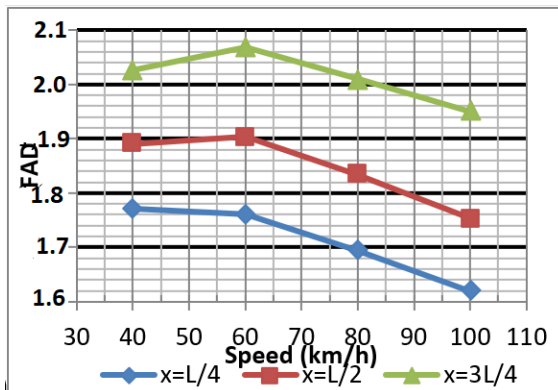




a



b



c

**Fig-7: Analytical results: a) deflection at the quarter span; the dynamic amplification factor DAF as function of b) The weight/mass, and of c) the speed.**

The mass of a truck is, probably, the parameter bound to the vehicle that varies the most. The difference of mass between the empty and loaded truck can entail changes of dynamic behavior and, therefore, of solicitations applied in a bridge. The influence of the mass of the truck 12\_B (of 11.40t mass of an empty vehicle and 19.75t for a full loaded one) on factors of

dynamic amplification is valued with the help of complementary numeric simulations done with the truck charged to the different levels (figure 7-b). Nevertheless, dynamic amplification factors increase with the load of the truck. Indeed, the model of vehicle proposed includes abeyances to constant damping. Dynamic amplification factors increase of 12 on average to 15% between situations where the truck is empty and overloaded. Variations of dynamic amplification factors according to the speed don't permit to clear a tendency at the level (figure 7-c). Although the speed of the vehicle that crosses the bridge influences factors of dynamic amplification, there is not an increasing or decreasing relation of this parameter with dynamic amplification factors.

At the design stage, design codes usually recommend an allowable deflection threshold that should not be reached during exploitation for structural safety reasons, in this work it is also observed that the ultimate standard maximal deflection is set to be 12.1 mm while the analytical value is found to be 42.3 mm corresponding to vehicle moving speeds between 23km/h and 92km/h depending on vehicle loading. We thus recommend for that particular bridge two speeds limitation set as 20 km/h for heavy loaded vehicle and 80 km/h for light vehicles.

## 4. CONCLUSIONS

Tests of dynamic loading especially kept our attention because of his complexity. This complexity is owed to the dynamic behavior of bridge - vehicles system or bridge - vehicles interaction. Indeed, if we consider a circulating vehicle on a bridge, this one distorts itself while modifying the dynamic profile of the pavement thus. One attends a process of successive adjustments of the mass position thus in vibration of the

vehicle and the deformed of the bridge all along the course of the vehicle.

We developed then for the bridge a model including finites elements 2D with two degrees of freedom to every node: a deflection and a rotation. Then, the survey of the interaction has been achieved then by a numerical approach. We developed a model of vehicle thus to three axes with axles tandems to simulate the disposition and the variation of the load intensity exercised on a bridge by these vehicles according to the time. Movements of this vehicle model are described by 7 independent variables. The model of vehicle used is a model 2D including axles of constant damping. For the resolution of the dynamic interaction bridge-vehicle, two methods of resolution can be applied, the coupled method and the uncoupled method. We adopted the uncoupled method.

In the part numeric simulations of this research work, we were interested in the bridge on the NYONG "stream" to Ayos constructs in the setting of the construction of the Ayos-Bonis road. It is a bridge mixed steel - concrete to three isostatic bays. The simulation of loading tests by dead weight as using the dynamic features of the bridge has been compared to those gotten by the software EFFEL, by a RDM calculation and in short to those raised at the time of tests of static loading. The calculated deflections are globally superior to values measured on the land, beams of the bridge normally behave to bending. For the test by rolling weight or dynamic loading test, at the time of tests, he consisted merely in verifying that it doesn't exist some residuals settlements after loading of support devices, what was the case. In the worry to have a bigger knowledge of the dynamic behavior of the bridge we did simulations of this dynamic test nevertheless, while introducing a coefficient for interpretations in applied data absence: the dynamic amplification factor. Of the different done simulations, the speed is not a parameter determining for dynamic amplification factors. This factor increases with the load of the truck. Mistake of data more exhaustive, we were not able to study the influence of the type of vehicle and the quality of the planeity of the pavement on this factor. For a type of constituted convoy of vehicle given we have determined a speed of circulation to assure the sustainability of the bridge.

## REFERENCES

1. Patrice. (2020). Effondrement d'un pont à Maroua : les autorités évaluent les conséquences. Cameroun tribune du 01 septembre 2020.
2. Lufulwabo, F. (2021). Kinshasa : poursuite de la réparation du pont N'djili. Radio Okapi du 16 mars 2021 sur [www.radiookapi.net](http://www.radiookapi.net)
3. Sun, L., Zhao X. (2021). Coupled Dynamics of Vehicle-Bridge Interaction System Using High Efficiency Method. Hindawi Advances in Civil Engineering Volume 2021, Article ID 1964200, 22 pages <https://doi.org/10.1155/2021/1964200>
4. Meyera, M., Canterob, D., Lennera, R. (2021). Dynamics of long multi-trailer heavy vehicles crossing short to medium span length bridges. Engineering Structures, September 2021
5. Vaidya, T., Chatterjee, A. (2017). Vibration of Road Bridges under Moving Vehicles: A Comparative Study between Single Contact Point and Two Contact Point Models. Transactions of the Canadian Society for Mechanical Engineering, 41(1).
6. Zhou, Y., Yang, P., Zhang, K. (2021). Dynamic response analysis of cable-stayed bridge under random traffic flow and fleet. JOURNAL OF VIBROENGINEERING
7. Yang, Y.B., Zhang ,B., Qian ,Y., Wu ,Y. (2017). Contact-Point Response for Modal Identification of Bridges by a Moving Test Vehicle, Int. J. Struct. Stab. Dyn. 18
8. Deng, L., He, W., Yu, Y. (2018). Research progress in theory and applications of highway vehicle-bridge coupling vibration. China J Highw Transp; 31(7); 38–54.
9. Randriatefison, N. (2011). Modelling in 2D the roughness of the roads in the equation of the vehicle movement. Modelling of Physics Systems Laboratory. Department of Physics, University of Antananarivo, Madagascar
10. Casadevall. A. (2013). Introduction à MATLAB. Université Paris-Dauphine. [www.electronique-mixte.fr](http://www.electronique-mixte.fr)
11. Daniel, L., Kortiš, J., Olivier, B., Kouroussis, G. (2016). Modelling of Road Vehicle-Bridge Interaction using a Compound Multibody/Finite Element Approach. 23rd International Congress on Sound & Vibration. Athens. Greece 10-14 July 2016
12. Akoussah, K., Fafard, M., Henchi, K. (2000). Détermination du facteur d'amplification dynamique des ponts par éléments finis. Ecole Nationale Supérieure d'Ingénieurs de Lomé et Université de Laval
13. Afnor. (2000). Calcul des structures mixtes acier-béton et document d'application nationale. Partie 2. Eurocode 4
14. Graitec. (2009). Guide de l'utilisateur. [www.graitec.info](http://www.graitec.info)
15. Clough, R., Penzien, J. (2003). Dynamics of structures. Computers & Structures. University of California, Berkeley, USA
16. Messi, F., Lezin, S., Evina, S. (2020). Influence of Train Speed on the Dynamic Behavior of the Bridge-Train system. 2<sup>nd</sup> International E-Conference on Engineering, Technology and Management – ICETTM2020, 26 july 2020, 35-41.
17. Stoura, D., Zeng, Q., Dimitrakopoulos, E. (2019). Vehicle-bridge interaction analysis using the localized Lagrange multipliers approach. COMPDYN 2019 7th ECCOMAS Thematic

- Conference on Computational Methods in Structural Dynamics and Earthquake Engineering M. Papadrakakis, M. Fragiadakis (eds.) Crete, Greece, 24–26 June 2019
18. Nwoji, C. U, Sopakirite, S., Oguaghamba, O. A, Ibeabuchi, V. T (2021). Deflection of Simply Supported Rectangular Plates under Shear and Bending Deformations Using Orthogonal Polynomial Function. *Saudi Journal of Civil Engineering*. DOI: 10.36348/sjce.2021.v05i05.001
19. Lezin, S., Kankeu, J., Okpwe, R., Baleng, B. (2020). Mass and topology optimization of tensegrity structures with application to footbridges. *Global Journal of Engineering and Technology Advances* , 04(03), 031–044
20. Szurgott, P., Bernacki, P. (2020) Modelling of steel-concrete bridges subjected to a moving high-speed train. *Int J Simul Model*; 19(1): 29–40.
21. Stoura, D., Zeng, Q., Dimitrakopoulos, E. (2020). MDOF extension of the Modified Bridge System method for vehicle–bridge interaction. *Nonlinear Dyn* <https://doi.org/10.1007/s11071-020-06022-6>
22. Wang, J., Pan, J., Zhang J., Ye, G., Xu, R. (2020). Vehicle–bridge interaction analysis by the state-space method and symplectic orthogonality, *Archive of Applied Mechanics*, March 2020
23. Hamidullah, Sharma, H. (2017). Influence of Dynamic Amplification Factor on Response of Steel Highway Bridge. *Proceedings of 29th Research World International Conference*, Las Vegas, USA, 16th-17th March 2017
24. Rahman, T., Ahmed, M. (2016). Dynamic Impact Analysis of Vehicle Bridge Interaction System. *Proceedings of 39th IRF International Conference*, 27th March, 2016, Chennai, India, 2016.
25. Băncilă, R., Bolduş, D., Feier, A., Hernea, S., Malița, M. (2014) Deflection and precambering of steel beams. *International scientific conference cibv*, 2014, 7-8 November 2014, Braşovs
26. Cai C., He, Q., Zhu, S. (2019). Dynamic interaction of suspension-type monorail vehicle and bridge: numerical simulation and experiment. *Mech Syst Signal Process* 2019; 118: 388–407.
27. Okpwe, R., Mamba, P. (2019). BIM review in aec industry and lessons for sub-saharan africa: case of Cameroon. *International Journal of Civil Engineering and Technology (IJCIET)* 10(05), 930-942.
28. Pettang, J., Takam, E., Manjia, M., Lezin, S. (2020). 3D modelling approach for prestressed concrete bridges built by successive corbelling based on building information modeling (bim) concepts. *World Journal of Engineering Research and Technology (wjert)* 2020, Vol. 6, Issue 1, 209-223.
29. Ahmad, H., Malek, H., Majd Al-Deen, M., Ghaida, K., Shaima, K., Doa'a, M. (2021). Evaluation of Urban Public Transport: A Case Study of Yarmouk University. *Saudi Journal of Civil Engineering*. DOI: 10.36348/sjce.2021.v05i01.002.