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Original Research Article

Novel Formulas for Displacements and Stresses of Thick Anisotropic Rectangular Plate

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Abstract

This work concentrated on the analysis of thick anisotropic rectangular plate through exact approach using third order shear deformation theory. Refined plate theory assumptions were relied upon to formulate the total potential energy functional. Displacement field, kinematic relations, constitutive relations and stress displacement relations were also obtained from the assumptions. Kinematic relations and Stress-displacement relations were substituted into the universal strain energy equation to formulate the strain energy equation. Total potential energy functional for the analysis of thick anisotropic rectangular plate was obtained by adding the external work and strain energy equation together. The total potential energy functional was differentiated with respect to the out plane deflection (w), shear deformation rotation in x direction (ϕ_x) and shear deformation rotation in y direction (ϕ_y). This yielded the governing equation and two compatibility equations of thick anisotropic rectangular plate. Third order polynomial shear deformation function which was derived by Ibearugbulem *et al.* was relied upon to obtain the displacement functions. From these displacement functions, the unique displacement functions for the SSSS plate boundary condition were determined. Also the stiffness coefficients were calculated for the SSSS plate boundary condition. The formulas for calculating the coefficients of the displacements were combined with elastic equations to determine the novel formulas which were used in calculating for displacements (u, v and w) and stresses ($\sigma_{RR}, \sigma_{QQ}, \tau_{RQ}, \tau_{RS}$ *and* τ_{QS}) at various angle fiber orientation (0^0, 15^0, 30^0, 45^0, 60^0, 75^0 and 90^0) and various span to thickness ratio, α (5, 10, 20, 30, 40, 50, 60, 70, 80, 90 and 100). These formula were used to analyze typical anisotropic rectangular thick plates. The results obtained were shown on Tables 1, 2, 3, 4 and 5. These numerical results obtained showed some level of agreement with previous works by other scholars. Hence the developed method is recommended for analyzing thick rectangular anisotropic plates.

Keywords: Energy; Anisotropic; Thick Plate; Displacement; Compatibility and Governing Equation.

INTRODUCTION

Any material consisting of two or more components with different properties and distinct boundaries between the components can be referred to as a composite material [1]. Anisotropic plate is a composite material with two or more components that possesses different properties. Advances in the application of thick and anisotropic plates over the past few decades have stirrup a revival of the concerned of researchers to formulate appropriate numerical methods and tools to bode the structural behavior of anisotropic plates. Hence, a precise, simple and cost effective numerical method that is suitable to be applied to anisotropic composite thick plate is of exceptional interest and therefore, various anisotropic composite plate theories and numerical method have already been formulated to predict the behavior of anisotropic composite plates. Among these methods is the polynomial shear deformation theory that is well meant for the effective analysis of anisotropic composite plate structures.

Anisotropic composite plate is one of the most important structural materials used in engineering industries such as marine, astronautic, aeronautics, electronics, computer, etc. This is as a result of high strength to weight and stiffness to weight ratios of anisotropic composite materials, which makes them conforming to an ultimate standard of perfection in weight sensitive structures. The dynamic distinguishing quality of the anisotropic composite plate have a great act of alluring engineers and material scientist.

Various solutions and models established in the literature for analysis of anisotropic plate did not derive specific formulas for calculation of displacement and stresses. Yang et al analyzed for the bending, free vibration and buckling analyses of anisotropic layered micro-plates based on a new size dependent model [2]. Lisboa and Marczak applied adomian decomposition method to anisotropic thick plate in bending [3]. Ghugal and Shimpi reviewed a refined shear deformation theories of isotropic and anisotropic laminated plates [4]. Nelson and Lorch worked on a refined theory for laminated orthotropic plates [5]. Phan and Reddy applied a higher order shear deformation theory to obtain the stability and vibration of isotropic, orthotropic and laminated plates [6].

One common observation is that most of these works are based mainly on trigonometric and assumed displacement functions. These works could not determine specific formulas for calculation of displacements and stresses which has been a major obstacle in easy understanding of thick anisotropic plate analysis. It is believed that deriving specific formulas for displacements and stresses will enhance the understandability of the solution and as well guide the engineers to easy usability of the solution in thick anisotropic rectangular plate. Another observation is that earlier works on anisotropic thick plates had relied on assumed displacement functions (which are mainly trigonometric) and could not determine specific formulas for calculation of displacements and stresses of thick anisotropic rectangular plate. Those works always have to follow a rigorous processes/method each time to calculate for displacements and stresses. This long processes can be very difficult and tiring in thick anisotropic plate analysis. Hence, the need to derive formulas which can easily determine displacements and stresses is very necessary in thick anisotropic plate analysis. To cover this gap in anisotropic thick plate analysis is the primary motivation of the present study.

Based on the above observations, it can be concluded that there is need for the development of formulas for displacements and stresses of thick rectangular anisotropic plate subjected under bending loading which can accurately calculate for in-plane displacements, out-plane displacement, in-plane stresses and out-plane stresses. With above points, the main objective of the present study is to develop novel formulas for bending responses of anisotropic rectangular plate.

Hence, this paper gives the results for displacements and stresses of thick anisotropic and as well isotropic plates of SSSS boundary condition.

Theoritical formulation

To obtain the novel formulas for the analysis of thick anisotropic plate, third order shear deformation theory and Ritz energy method were adopted. The method is described in this research work as follows;

Displacement field

The refined plate theory (RPT) in-plane displacements, u and v are defined mathematically as presented:

$$u = u_c + u_s \qquad 1$$

$$v = v_c + v_s \qquad 2$$

The non-dimensional forms of the orthogonal axes are defined as: R = x/a; Q = y/b; S = z/t. aspect ratio, denoted as β is defined as β = b/a.

The classical part of the in-plane displacements u_c and v_c are defined as follows:

$$u_c = -z\theta_{cx} = -z\frac{dw}{dx} = -\frac{St}{a}\frac{dw}{dR} \qquad 3$$

$$v_c = -z\theta_{cy} = -z\frac{dw}{dy} = -\frac{St}{b}\frac{dw}{dR} = -\frac{St}{\beta a}\frac{dw}{dQ} \qquad 4$$

Where w, is the out-plane displacement. Transverse displacements u_s and v_s are defined as:

$$u_s = F(z)\theta_{sx} \qquad 5$$

$$v_s = F(z)\theta_{sy} \qquad 6$$

Where: F(z) is the third order shear deformation model defined as:

$$F(z) = z - \frac{4}{3}.\frac{z^3}{t^2} = z\left(1 - \frac{4}{3}\left[\frac{z}{t}\right]^2\right) \qquad 7a$$

The non-dimensional form of the model is:

$$F = F(s) = t\left(S - \frac{4}{3}S^3\right) \qquad 7b$$

That is:

$$F = tH \qquad 7c$$

Where:

$$H = S - \frac{4}{3}S^3 \qquad 7d$$

Adding Equations 3 and 5 gives:

$$u = -\frac{St}{a}\frac{dw}{dR} + F(z).\emptyset_x \qquad 8a$$

Similarly, adding Equations 4 and 6 gives:

$$v = -\frac{St}{\beta a}\frac{dw}{dQ} + F(z).\emptyset_y \qquad 8b$$

Substituting Equation 7c into Equations 8a and 8b gives:

$$u = \frac{t}{a}\left[-S\frac{\partial w}{\partial R} + Ha.\emptyset_x\right] \qquad 8c$$

$$v = \frac{t}{a\beta}\left[-S\frac{\partial w}{\partial Q} + \beta Ha.\emptyset_y\right] \qquad 8d$$

Strain - displacement relations (kinematic relations)

The strain – displacement relations equations are:

$$\varepsilon_R = \frac{\partial u}{\partial x} = \frac{\partial u}{a\partial R} = \frac{t}{a^2}\left[-S\frac{\partial^2 w}{\partial R^2} + Ha.\frac{\partial \phi_x}{\partial R}\right] \qquad 9$$

$$\varepsilon_Q = \frac{\partial v}{\partial y} = \frac{\partial v}{a\beta\partial Q} = \frac{t}{\beta^2 a^2}\left[-S\frac{\partial^2 w}{\partial Q^2} + Ha\beta.\frac{\partial \phi_y}{\partial Q}\right] \qquad 10$$

$$\gamma_{RQ} = \varepsilon_{RQ} + \varepsilon_{QR} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{t}{\beta a^2}\left[-S\frac{\partial^2 w}{\partial R\partial Q} + Ha.\frac{\partial \phi_x}{\partial Q}\right] + \frac{t}{\beta a^2}\left[-S\frac{\partial^2 w}{\partial R\partial Q} + H\beta a.\frac{\partial \phi_y}{\partial R}\right]$$. That is:

$$\gamma_{RQ} = \frac{t}{\beta a^2}\left[-2S\frac{\partial^2 w}{\partial R\partial Q} + Ha.\left(\frac{\partial \phi_x}{\partial Q} + \beta.\frac{\partial \phi_y}{\partial R}\right)\right] \qquad 11$$

$$\gamma_{RS} = \varepsilon_{RS} + \varepsilon_{SR} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \frac{1}{a}\left[-\frac{\partial w}{\partial R} + a\frac{\partial H}{\partial S}.\emptyset_x\right] + \frac{1}{a}\frac{\partial w}{\partial R}$$. That is:

$$\gamma_{RS} = \frac{\partial H}{\partial S}.\emptyset_x \qquad 12$$

$$\gamma_{QS} = \frac{\partial H}{\partial S}.\emptyset_y = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \frac{1}{\beta a}\left[-\frac{\partial w}{\partial Q} + \beta a\frac{\partial H}{\partial S}.\emptyset_y\right] + \frac{1}{\beta a}.\frac{\partial w}{\partial Q}$$. That is:

$$\gamma_{QS} = \frac{\partial H}{\partial S}.\emptyset_y \qquad 13$$

Constitutive relations (Stress – Strain Relations)

$$\begin{bmatrix}\sigma_R\\ \sigma_Q\\ \tau_{RQ}\\ \tau_{RS}\\ \tau_{QS}\end{bmatrix} = \frac{E_0}{1-\mu_{12}\mu_{21}}\begin{bmatrix}B_{11} & B_{12} & B_{13} & 0 & 0\\ B_{21} & B_{22} & B_{23} & 0 & 0\\ B_{31} & B_{32} & B_{33} & 0 & 0\\ 0 & 0 & 0 & B_{44} & 0\\ 0 & 0 & 0 & 0 & B_{55}\end{bmatrix}\begin{bmatrix}\varepsilon_R\\ \varepsilon_Q\\ \gamma_{RQ}\\ \gamma_{RS}\\ \gamma_{QS}\end{bmatrix} \qquad 14$$

Where:

E_0 is the reference Elastic modulus. It can be E_1 or E_2; $m = Cos\ \theta$; $n = Sin\ \theta$

$$B_{11} = m^4 d_{11} + 2m^2n^2(d_{12} + 2d_{33}) + n^4 d_{22} \qquad 15$$

$$B_{12} = d_{12}(n^4 + m^4) + m^2n^2(d_{11} + d_{22} - 4d_{33}) \qquad 16$$

$$B_{13} = m^3 n(d_{11} - d_{12} - 2d_{33}) + mn^3(d_{12} - d_{22} + 2d_{33}) \qquad 17$$

$$B_{22} = n^4 d_{11} + 2m^2n^2(d_{12} + 2d_{33}) + m^4 d_{22} \qquad 18$$

$$B_{23} = mn^3 d_{11} - m^3 n d_{22} + (m^3 n - mn^3)(d_{12} + 2d_{33}) \qquad 19$$

$$B_{33} = m^2n^2(d_{11} - 2d_{12} + d_{22} - 2d_{33}) + d_{33}(m^4 + n^4) \qquad 20$$

$$B_{44} = d_{44};\ B_{55} = d_{55}; B_{21} = B_{12}; B_{31} = B_{13};\ B_{32} = B_{23} \qquad 21$$

$$d_{11} = E_1/E_0 \qquad 22$$

$$d_{12} = E_2\ .\mu_{12}/E_0 \qquad 23$$

$$d_{21} = E_1\ .\mu_{21}/E_0 \qquad 24$$

$$d_{22} = E_{22}/E_0 \qquad 25$$

$$d_{33} = G_{12}(1 - \mu_{12}\mu_{21})/E_0 \qquad 26$$

$$d_{44} = G_{13}(1 - \mu_{12}\mu_{21})/E_0 \qquad 27$$

$$d_{55} = G_{23}(1-\mu_{12}\mu_{21})/E_0 \qquad 28$$

Substituting Equations 9 to 13 into Equation 14 gives each stress component as:

$$\sigma_R = \frac{E_0 t}{[1-\mu_{12}\mu_{21}]a^2}.\left(B_{11}.\left[-S\frac{\partial^2 w}{\partial R^2} + Ha.\frac{\partial\phi_x}{\partial R}\right] + \frac{B_{12}}{\beta^2}.\left[-S\frac{\partial^2 w}{\partial Q^2} + Ha\beta.\frac{\partial\phi_y}{\partial Q}\right] + \frac{B_{13}}{\beta}.\left[-2S\frac{\partial^2 w}{\partial R\partial Q} + Ha.\left(\frac{\partial\phi_x}{\partial Q} + \beta.\frac{\partial\phi_y}{\partial R}\right)\right]\right) \qquad 29$$

$$\sigma_Q = \frac{E_0 t}{[1-\mu_{12}\mu_{21}]a^2}.\left(B_{21}.\left[-S\frac{\partial^2 w}{\partial R^2} + Ha.\frac{\partial\phi_x}{\partial R}\right] + \frac{B_{22}}{\beta^2}.\left[-S\frac{\partial^2 w}{\partial Q^2} + Ha\beta.\frac{\partial\phi_y}{\partial Q}\right] + \frac{B_{23}}{\beta}.\left[-2S\frac{\partial^2 w}{\partial R\partial Q} + Ha.\left(\frac{\partial\phi_x}{\partial Q} + \beta.\frac{\partial\phi_y}{\partial R}\right)\right]\right) \qquad 30$$

$$\tau_{RQ} = \frac{E_0 t}{[1-\mu_{12}\mu_{21}]a^2}.\left(B_{31}.\left[-S\frac{\partial^2 w}{\partial R^2} + Ha.\frac{\partial\phi_x}{\partial R}\right] + \frac{B_{32}}{\beta^2}.\left[-S\frac{\partial^2 w}{\partial Q^2} + Ha\beta.\frac{\partial\phi_y}{\partial Q}\right] + \frac{B_{33}}{\beta}.\left[-2S\frac{\partial^2 w}{\partial R\partial Q} + Ha.\left(\frac{\partial\phi_x}{\partial Q} + \beta.\frac{\partial\phi_y}{\partial R}\right)\right]\right) \qquad 31$$

$$\tau_{RS} = \frac{E_0}{1-\mu_{12}\mu_{21}}.B_{44}.\left[\frac{\partial H}{\partial S}\right].\emptyset_x = \frac{E_0 t}{[1-\mu_{12}\mu_{21}]a^2}.B_{44}.\left[\frac{a^2}{t}.\frac{\partial H}{\partial S}\right].\emptyset_x \qquad 32$$

$$\tau_{QS} = \frac{E_0}{1-\mu_{12}\mu_{21}}.B_{55}.\left[\frac{\partial H}{\partial S}\right].\emptyset_y = \frac{E_0 t}{[1-\mu_{12}\mu_{21}]a^2}.B_{55}.\left[\frac{a^2}{t}.\frac{\partial H}{\partial S}\right].\emptyset_y \qquad 33$$

Total potential energy functional

The total potential energy functional is given as:

$$\Pi = \frac{abt}{2}\int_0^1\int_0^1\int_{-0.5}^{0.5}\left(\sigma_R\varepsilon_R + \sigma_R\varepsilon_R + \tau_{RQ}\gamma_{RQ} + \tau_{RS}\gamma_{RS} + \tau_{QS}\gamma_{QS}\right) dR\, dQ\, dS - qab\int_0^1\int_0^1 w\, dR\, dQ \qquad 34$$

Substituting Equations 9 to 13 and Equations 29 to 33 into Equations 34 gives:

$$\Pi = \frac{abD_0}{2a^4}.\int_0^1\int_0^1\left\{\left\{B_{11}.\left[\left(\frac{\partial^2 w}{\partial R^2}\right)^2 - 2g_2 a.\frac{\partial^2 w}{\partial R^2}.\frac{\partial\phi_x}{\partial R} + g_3 a^2.\left(\frac{\partial\phi_x}{\partial R}\right)^2\right]\right.\right.$$
$$+ \frac{B_{12}}{\beta^2}.\left[2\left(\frac{\partial^2 w}{\partial R\,\partial Q}\right)^2 - g_2\frac{a}{\beta}\frac{\partial^2 w}{\partial Q^2}.\frac{\partial\phi_y}{\partial Q} - g_2 a\beta^2\frac{\partial^2 w}{\partial R^2}.\frac{\partial\phi_x}{\partial R} - g_2 a.\frac{\partial\phi_x}{\partial R}.\frac{\partial^2 w}{\partial Q^2} - g_2 a\beta.\frac{\partial^2 w}{\partial R^2}.\frac{\partial\phi_y}{\partial Q} + 2g_3 a^2\beta.\frac{\partial\phi_x}{\partial R}.\frac{\partial\phi_y}{\partial Q}\right]$$
$$+ \frac{B_{13}}{\beta}.\left[4\frac{\partial^2 w}{\partial R\partial Q}.\frac{\partial^2 w}{\partial R^2} - 2g_2 a.\left(\frac{\partial\phi_x}{\partial Q} + \beta.\frac{\partial\phi_y}{\partial R}\right).\frac{\partial^2 w}{\partial R^2} - 4g_2 a\frac{\partial^2 w}{\partial R\partial Q}.\frac{\partial\phi_x}{\partial R} + 2g_3 a^2.\left(\frac{\partial\phi_x}{\partial Q} + \beta.\frac{\partial\phi_y}{\partial R}\right).\frac{\partial\phi_x}{\partial R}\right] + \frac{B_{22}}{\beta^4}.\left[\left(\frac{\partial^2 w}{\partial Q^2}\right)^2 - 2g_2 a\beta.\frac{\partial^2 w}{\partial Q^2}.\frac{\partial\phi_y}{\partial Q} + g_3 a^2\beta^2.\left(\frac{\partial\phi_y}{\partial Q}\right)^2\right]$$
$$+ \frac{B_{23}}{\beta^3}.\left[4\frac{\partial^2 w}{\partial R\partial Q}.\frac{\partial^2 w}{\partial Q^2} - 2g_2 a.\left(\frac{\partial\phi_x}{\partial Q} + \beta.\frac{\partial\phi_y}{\partial R}\right).\frac{\partial^2 w}{\partial Q^2} - 4g_2 a\beta.\frac{\partial^2 w}{\partial R\partial Q}.\frac{\partial\phi_y}{\partial Q} + 2g_3 a^2\beta.\left(\frac{\partial\phi_x}{\partial Q} + \beta.\frac{\partial\phi_y}{\partial R}\right).\frac{\partial\phi_y}{\partial Q}\right]$$
$$+ \frac{B_{33}}{\beta^2}.\left[4\left(\frac{\partial^2 w}{\partial R\partial Q}\right)^2 - 2g_2 a.\left(\frac{\partial^2 w}{\partial R\partial Q}.\frac{\partial\phi_x}{\partial Q} + \beta\frac{\partial^2 w}{\partial R\partial Q}.\frac{\partial\phi_y}{\partial R}\right) + g_3 a^2.\left(\left(\frac{\partial\phi_x}{\partial Q}\right)^2 + 2\beta\frac{\partial\phi_x}{\partial Q}.\frac{\partial\phi_y}{\partial R} + \beta^2.\left(\frac{\partial\phi_y}{\partial R}\right)^2\right)\right] + a^2 B_{44}.\left(\frac{a}{t}\right)^2.g_4.\emptyset_x^{\,2}$$
$$\left.\left. + a^2 B_{55}.\left(\frac{a}{t}\right)^2.g_4.\emptyset_y^{\,2}\right\} - 2\frac{qa^4}{D_0}w\right\} dR\, dQ \qquad 35$$

Where:

$$D_0 = \frac{E_0 t^3}{12[1-\mu_{12}\mu_{21}]} \quad 36$$

Governing equation and compatibility equations

Differentiating Equation 35 with respect to w, θ_x and θ_y gives the governing equation and compatibility equations respectively.

$$\frac{d\prod}{dw} = \frac{d\prod}{d\emptyset_x} = \frac{d\prod}{d\emptyset_y} = 0 \quad 37$$

That is:

$$\frac{d\prod}{dw} = \int_0^1\int_0^1 \Bigg\{ B_{11}.\frac{\partial^4 w}{\partial R^4} + \frac{2}{\beta^2}.B_{xy}\frac{\partial^4 w}{\partial R^2\,\partial Q^2} + \frac{B_{22}}{\beta^4}.\frac{\partial^4 w}{\partial Q^4} + 4\frac{B_{13}}{\beta}.\frac{\partial^4 w}{\partial R^3\,\partial Q} + 4\frac{B_{23}}{\beta^3}.\frac{\partial^4 w}{\partial R\,\partial Q^3} - \frac{g_2 a}{2}[2B_{11}+B_{12}]\frac{\partial^3\phi_x}{\partial R^3} - \frac{g_2 a}{2\beta^2}.B_{xy}\frac{\partial^3\phi_x}{\partial R\,\partial Q^2} - 3g_2 a.\frac{B_{13}}{\beta}\frac{\partial^3\phi_x}{\partial R^2\,\partial Q} - \frac{g_2 a}{2\beta^3}[B_{12}+2B_{22}]\frac{\partial^3\phi_y}{\partial Q^3} - \frac{g_2 a}{2\beta}B_{xy}\frac{\partial^3\phi_y}{\partial R^2\,\partial Q} - 3g_2 a.\frac{B_{23}}{\beta^2}\frac{\partial^3\phi_y}{\partial R\,\partial Q^2} - g_2 a.B_{13}.\frac{\partial^3\phi_y}{\partial R^3} - \frac{g_2 a}{\beta^3}.B_{23}.\frac{\partial^3\phi_x}{\partial Q^3} - \frac{qa^4}{D_0}\Bigg\} dR\,dQ = 0 \quad 38$$

$$\frac{d\prod}{d\emptyset_x} = B_{11}.\left[-g_2 a.\frac{\partial^3 w}{\partial R^3} + g_3 a^2.\frac{\partial^2\phi_x}{\partial R^2}\right] + \frac{B_{12}}{2\beta^2}.\left[-g_2 a\beta^2\frac{\partial^3 w}{\partial R^3} - g_2 a.\frac{\partial^3 w}{\partial R\,\partial Q^2} + 2g_3 a^2\beta.\frac{\partial^2\phi_y}{\partial R\partial Q}\right] + \frac{B_{13}}{\beta}.\left[-g_2 a.\frac{\partial^3 w}{\partial Q\,\partial R^2} - 2g_2 a\frac{\partial^3 w}{\partial Q\,\partial R^2} + 2g_3 a^2.\frac{\partial^2\phi_x}{\partial R\partial Q} + g_3 a^2.\beta.\frac{\partial^2\phi_y}{\partial R^2}\right] + \frac{B_{23}}{\beta^3}.\left[-g_2 a.\frac{\partial^3 w}{\partial Q^3} + g_3 a^2\beta.\frac{\partial^2\phi_y}{\partial Q^2}\right] + \frac{B_{33}}{\beta^2}.\left[-g_2 a.\frac{\partial^3 w}{\partial R\,\partial Q^2} + g_3 a^2.\frac{\partial^2\phi_x}{\partial Q^2} + g_3 a^2.\beta\frac{\partial^2\phi_y}{\partial R\partial Q}\right] + a^2 B_{44}.\left(\frac{a}{t}\right)^2.g_4.\emptyset_x = 0 \quad 39$$

$$\frac{d\prod}{d\emptyset_y} = \frac{B_{12}}{2\beta^2}.\left[-g_2\frac{a}{\beta}\frac{\partial^3 w}{\partial Q^3} - g_2 a\beta.\frac{\partial^3 w}{\partial R^2\partial Q} + 2g_3 a^2\beta.\frac{\partial^2\phi_x}{\partial R\partial Q}\right] + \frac{B_{13}}{\beta}.\left[-g_2 a.\beta.\frac{\partial^3 w}{\partial R^3} + g_3 a^2.\beta.\frac{\partial^2\phi_x}{\partial R^2}\right] + \frac{B_{22}}{\beta^4}.\left[-g_2 a\beta.\frac{\partial^3 w}{\partial Q^3} + g_3 a^2\beta^2.\frac{\partial^2\phi_y}{\partial Q^2}\right] + \frac{B_{23}}{\beta^3}.\left[-g_2 a.\beta.\frac{\partial^3 w}{\partial R\,\partial Q^2} - 2g_2 a\beta.\frac{\partial^3 w}{\partial R\partial Q^2} + g_3 a^2\beta.\frac{\partial^2\phi_x}{\partial Q^2} + 2g_3 a^2\beta^2.\frac{\partial^2\phi_y}{\partial R\partial Q}\right] + \frac{B_{33}}{\beta^2}.\left[-g_2 a.\beta\frac{\partial^3 w}{\partial R^2\partial Q} + g_3 a^2.\beta\frac{\partial^2\phi_x}{\partial R\partial Q} + g_3 a^2.\beta^2.\frac{\partial^2\phi_y}{\partial R^2}\right] + a^2 B_{55}.\left(\frac{a}{t}\right)^2.g_4.\emptyset_y = 0 \quad 40$$

Equations 38, 39 and 40 are the governing equation of equilibrium of forces, compatibility equation of displacements in x-z plane and compatibility equation of displacements in y-z plane respectively.

Solutions of governing equation and compatibility equations

Solving Equations 38, 39 and 40 gives:

$$w = A_1 h \quad 41a$$

$$w = (\alpha_0 + \alpha_1 R + \alpha_2 R^2 + \alpha_3 R^3 + \alpha_4 R^4)(\lambda_0 + \lambda_1 Q + \lambda_2 Q^2 + \lambda_3 Q^3 + \lambda_4 Q^4) \quad 41b$$

$$\phi_x = \frac{A_2}{a}.\frac{\partial h}{\partial R} \quad 42$$

$$\emptyset_y = \frac{A_3}{a\beta}.\frac{\partial h}{\partial Q} \quad 43$$

Satisfying the boundary condition for SSSS plates gives their distinct deflection equation respectively as:

$$w = A_1 h = \alpha_3\alpha_4(R - 2R^3 + R^4)(Q - 2Q^3 + \lambda_4 Q^4) \quad \{for\ SSSS\} \quad 41c$$

Substituting Equations 41a, 42 and 43 into Equation 35 gives:

$$\Pi = \frac{ab}{2a^4}.\left\{\left\{B_{11}.[A_1{}^2 - 2g_2A_1A_2. + g_3A_2{}^2]k_1 + \frac{(B_{12}+2B_{33})}{\beta^2}.[2A_1{}^2 - g_2A_1A_2 - g_2A_1A_3].k_2 + 2\frac{[B_{12}+B_{33}]}{\beta^2}g_3\,A_2A_3.k_2 + \frac{B_{12}}{\beta^2}.g_2\left[-\frac{A_1A_3}{\beta^2}k_3 - A_1A_2\beta^2k_1\right] + \frac{B_{33}}{\beta^2}.[+g_3A_2{}^2 + g_3A_3{}^2]k_2 + \frac{B_{13}}{\beta}.[4A_1{}^2 - 2g_2(A_1A_2 + A_1A_3) - 4g_2A_1A_2 + 2g_3(A_2{}^2 + A_2A_3)]k_4 + \frac{B_{22}}{\beta^4}.[A_1{}^2 - 2g_2A_1A_3 + g_3A_3{}^2]k_3 + \frac{B_{23}}{\beta^3}.[4A_1{}^2 - 2g_2(A_1A_2 + A_1A_3) - 4g_2A_1A_3 + 2g_3(A_2A_3 + A_3{}^2)]k_5 + B_{44}.\left(\frac{a}{t}\right)^2.g_4.A_2{}^2k_6 + \frac{B_{55}}{\beta^2}.\left(\frac{a}{t}\right)^2.g_4A_3{}^2k_7\right\} - 2A_1\frac{qa^4}{D_0}k_8\right\} \quad 44$$

Note:

$$k_1 = \int_0^1\int_0^1\left(\frac{d^2h}{dR^2}\right)^2 dR\,dQ;\ k_2 = \int_0^1\int_0^1\left(\frac{d^2h}{dRdQ}\right)^2 dR\,dQ\,;\ k_3 = \int_0^1\int_0^1\left(\frac{d^2h}{dQ^2}\right)^2 dR\,dQ$$

$$k_4 = \int_0^1\int_0^1\left(\frac{d^2h}{dR^2}\right)\left(\frac{d^2h}{dRdQ}\right) dR\,dQ;\ k_5 = \int_0^1\int_0^1\left(\frac{d^2h}{dQ^2}\right)\left(\frac{d^2h}{dRdQ}\right) dR\,dQ$$

$$k_6 = \int_0^1\int_0^1\left(\frac{dh}{dR}\right)^2 dR\,dQ;\ k_7 = \int_0^1\int_0^1\left(\frac{dh}{dQ}\right)^2 dR\,dQ;\ k_8 = \int_0^1\int_0^1 h\,dR\,dQ$$

To obtain the quasi equations of equilibrium of forces and quasi compatibility equations, Equation 44 must be differentiated with respect to A1, A2 and A3. That is:

$$\frac{d\Pi}{dA_1} = \frac{d\Pi}{dA_2} = \frac{d\Pi}{dA_3} = 0 \quad 45$$

$$\frac{d\Pi}{dA_1} = L_{11}A_1 - L_{12}A_2 - L_{13}A_3 - \frac{qa^4}{D_0}\,k_8 = 0 \quad 46$$

$$\frac{d\Pi}{dA_2} = L_{12}A_1 - L_{22}A_2 - L_{23}A_3 = 0 \quad 47$$

$$\frac{d\Pi}{dA_3} = L_{13}A_1 - L_{23}A_2 - L_{33}A_3 = 0 \quad 48$$

Where:

$$L_{11} = B_{11}k_1 + \frac{2\{2B_{33}+B_{12}\}}{\beta^2}k_2 + \frac{B_{22}}{\beta^4}k_3 + 3\frac{B_{13}}{\beta}k_4 + 3\frac{B_{23}}{\beta^3}k_5 \quad 49$$

$$L_{12} = B_{11}g_2k_1 + \frac{\{2B_{33}+B_{12}\}}{\beta^2}g_2k_2 + 2.25\frac{B_{13}}{\beta}g_2k_4 + 0.75\frac{B_{13}}{\beta}g_2k_5 \quad 50$$

$$L_{13} = \frac{\{2B_{33}+B_{12}\}}{\beta^2}g_2k_2 + \frac{B_{22}}{\beta^4}g_2k_3 + 0.75\frac{B_{13}}{\beta}g_2k_4 + 2.25\frac{B_{23}}{\beta}g_2k_5 \quad 51$$

$$L_{12} = B_{11}g_2k_1 + \frac{\{2B_{33}+B_{12}\}}{\beta^2}g_2k_2 + 2.25\frac{B_{13}}{\beta}g_2k_4 + 0.75\frac{B_{13}}{\beta}g_2k_5 \quad 52$$

$$L_{22} = B_{11}g_3k_1 + \frac{B_{33}}{\beta^2}g_3k_2 + 1.5\frac{B_{13}}{\beta}g_3k_4 + B_{44}\propto^2 g_4k_6 \quad 53$$

$$L_{23} = \frac{B_{12}}{\beta^2}g_3k_2 + \frac{B_{33}}{\beta^2}g_3k_2 + 0.75\frac{B_{13}}{\beta}g_3k_4 + 0.75\frac{B_{23}}{\beta^3}g_3k_5 + \propto^2 g_4\frac{B_{45}}{\beta}k_8 \quad 54$$

$$L_{13} = \frac{\{2B_{33}+B_{12}\}}{p^2}g_2k_2 + \frac{B_{22}}{p^4}g_2k_3 + 0.75\frac{B_{13}}{p}g_2k_4 + 2.25\frac{B_{23}}{p}g_2k_5 \quad 55$$

$$L_{23} = \frac{B_{12}}{p^2}g_3k_2 + \frac{B_{33}}{p^2}g_3k_2 + 0.75\frac{B_{13}}{p}g_3k_4 + 0.75\frac{B_{23}}{p^3}g_3k_5 + \propto^2 g_4\frac{B_{45}}{p}k_8 \quad 56$$

$$L_{33} = \frac{B_{33}}{p^2}g_3k_2 + \frac{B_{22}}{p^4}g_3k_3 + 1.5\frac{B_{23}}{p^3}g_3k_5 + \propto^2 g_4\frac{B_{55}}{p^2}k_7 \quad 57$$

Solving Equations 47 and 48 simultaneously gives:

$$A_2 = \left(\frac{L_{12}L_{33} - L_{13}L_{23}}{L_{22}L_{33} - L_{23}L_{23}}\right) A_1 = P_2\, A_1 \qquad 58$$

$$A_3 = \left(\frac{L_{13}L_{22} - L_{12}L_{23}}{L_{22}L_{33} - L_{23}L_{23}}\right) A_1 = P_3\, A_1 \qquad 59$$

Substituting Equations 57 and 58 into Equation 46 gives:

$$A_1 = \frac{qa^4}{D_0}.\frac{k_8}{(L_{11} - L_{12}P_2 - L_{13}P_3)} = \frac{qa^4}{D_0}.k_9 \qquad 60$$

Novel formulas for analysis

Substituting Equation 60 into Equation 41a and substituting Equation 36 into the resulting equation and simplifying gives:

$$w\frac{E_0t^3}{qa^4} = 12[1 - \mu_{12}\mu_{21}].k_9h \qquad 61$$

Substituting Equations 41a, 42 and 43 into Equations 8c, 8d, 29, 30, 31, 32 and 33, where appropriate and simplifying gives:

$$u\frac{E_0}{qa}\left(\frac{t}{a}\right)^2 = 12[1 - \mu_{12}\mu_{21}].k_9.[-S + HP_2].\frac{\partial h}{\partial R} \qquad 62$$

$$v\frac{E_0}{qa}\left(\frac{t}{a}\right)^2 = 12[1 - \mu_{12}\mu_{21}].\frac{[-S + H.P_3]}{\beta}.\frac{\partial h}{\partial Q}.k_9 \qquad 63$$

$$\frac{\sigma_R}{q}\left(\frac{t}{a}\right)^2 = 12.k_9\left(B_{11}[HP_2 - S]\frac{\partial^2 h}{\partial R^2} + \frac{B_{12}}{\beta^2}[HP_3 - S]\frac{\partial^2 h}{\partial Q^2} + \frac{B_{13}}{\beta}H(P_2 + P_3 - 2S)\frac{\partial^2 h}{\partial R\partial Q}\right) \qquad 64$$

$$\frac{\sigma_Q}{q}\left(\frac{t}{a}\right)^2 = 12.k_9.\left(B_{21}[HP_2 - S]\frac{\partial^2 h}{\partial R^2} + \frac{B_{22}}{\beta^2}[HP_3 - S]\frac{\partial^2 h}{\partial Q^2} + \frac{B_{23}}{\beta}H(P_2 + P_3) - 2S\frac{\partial^2 h}{\partial R\partial Q}\right) \qquad 65$$

$$\frac{\tau_{RQ}}{q}\left(\frac{t}{a}\right)^2 = 12k_9.\left(B_{31}.[HP_2 - S]\frac{\partial^2 h}{\partial R^2} + \frac{B_{32}}{\beta^2}.[HP_3 - S]\frac{\partial^2 h}{\partial Q^2} + \frac{B_{33}}{\beta}.H(P_2 + P_3 - 2S)\frac{\partial^2 h}{\partial R\partial Q}\right) \qquad 66$$

$$\overline{\tau_{RS}} = \frac{\tau_{RS}}{q}\left(\frac{t}{a}\right)^3 = 12\,k_9\left(B_{44}.P_2.\frac{\partial H}{\partial S}\right).\frac{\partial h}{\partial R} \qquad 67$$

$$\overline{\tau_{QS}} = \frac{\tau_{QS}}{q}\left(\frac{t}{a}\right)^3 = 12.k_9.\left(B_{55}.\frac{P_3}{\beta}.\frac{\partial H}{\partial S}\right).\frac{\partial h}{\partial Q} \qquad 68$$

Typical example problems for SSSS thick anisotropic plate

The formulas were used to analyze typical anisotropic rectangular thick plates. The numerical values for In-plane displacements, u and v, out-plane displacement (central deflection), w, in-plane stresses, σ_x, σ_y and τ_{xy}, and out-plane stresses, τ_{xz} and τ_{yz}, were determined for angles fiber orientations of 0^0, 15^0, 30^0, 45^0, 60^0, 75^0 and 90^0 at span to thickness ration (α), 5, 10, 20, 30, 40, 50, 60, 70, 80, 90 and 100 for plate that is simply supported at all edges (SSSS). The plate was analyzed at various meaningful points along the length, width and depth axis; in-plane displacement (u and v) at $(x = 0.5, y = 0.5, z = 0.5)$, transverse displacement (w) at $(x = 0.5, y = 0.5, z = 0.5)$, In-plane normal stresses $(\sigma_x \text{ and } \sigma_y)$ at $(x = 0.5, y = 0.5, z = 0.5 \text{ or } z = 0.25)$, in-plane shear stress (τ_{xy}) at $(x = 0, y = 0, z = 0.5)$, out-plane shear stress (τ_{xz}) at $(x = 0, y = 0.5, z = 0.5)$ and out-plane shear stress (τ_{yz}) at $(x = 0.5, y = 0, z = 0.5)$ respectively. The plate is subjected to uniformly distributed load. The following non dimensionalizations that were applied by [7, 8, 9, 10, 11, 12, 13] were used to present the results; $\left[\overline{w} = w\frac{E_0t^3}{qa^4}x100, \overline{u}, \overline{v} = u, v\frac{E_0t^2}{qa^3}, (\overline{\sigma_{xx}}, \overline{\sigma_{yy}}, \overline{\tau_{xy}}) = (\frac{\sigma_x,\sigma_y,\tau_{xy}t^2}{qa^2}), (\overline{\tau_{xz}}, \overline{\tau_{yz}}) = (\frac{\tau_{xz}t}{qa})\right]$. The material properties used were as follows; for Table 1 to 7, $(E_1/E_2 = 25, G_{12}/E_2 =$

$0.5, G_{13}/E_2 = 0.5, G_{23}/E_2 = 0.2, v_{12} = 0.25)$ and for Table 8, $\left(E_1 = E_2 = 210\text{GPa}, v_{12} = 0.3, G = \frac{E}{2(1+\mu)}\right)$ respectively.

i. Analyze an orthotropic thick square SSSS plate with the following information:

$(E1 = 25;\ E2 = 1;\ G12 = 0.5;\ G13 = 0.5;\ G23 = 0.2, \mu12 = 0.25)$

ii. Analyze an orthotropic thick square SSSS plate with the following information:

$\left(E_1 = E_2 = 210\text{GPa}, v_{12} = 0.3, G = \frac{E}{2(1+\mu)}\right)$

RESULTS AND DISCUSSIONS

Presentation of Results

The results of SSSS rectangular thick anisotropic plate were presented here.

Displacements and stresses formulas

The novel formulas derived in the present study for the determination of displacements and stresses are presented as follows:

$$u\frac{E_0}{qa}\left(\frac{t}{a}\right)^2 = 12[1-\mu_{12}\mu_{21}].k_9.[-S+HP_2].\frac{\partial h}{\partial R} \quad 69$$

$$v\frac{E_0}{qa}\left(\frac{t}{a}\right)^2 = 12[1-\mu_{12}\mu_{21}].\frac{[-S+H.P_3]}{\beta}.\frac{\partial h}{\partial Q}.k_9 \quad 70$$

$$\frac{\sigma_R}{q}\left(\frac{t}{a}\right)^2 = 12.k_9\left(B_{11}[HP_2-S]\frac{\partial^2 h}{\partial R^2}+\frac{B_{12}}{\beta^2}[HP_3-S]\frac{\partial^2 h}{\partial Q^2} + \frac{B_{13}}{\beta}H(P_2+P_3-2S)\frac{\partial^2 h}{\partial R\partial Q}\right) \quad 71$$

$$\frac{\sigma_Q}{q}\left(\frac{t}{a}\right)^2 = 12.k_9.\left(B_{21}[HP_2-S]\frac{\partial^2 h}{\partial R^2}+\frac{B_{22}}{\beta^2}[HP_3-S]\frac{\partial^2 h}{\partial Q^2}+\frac{B_{23}}{\beta}H(P_2+P_3) - 2S\frac{\partial^2 h}{\partial R\partial Q}\right) \quad 72$$

$$\frac{\tau_{RQ}}{q}\left(\frac{t}{a}\right)^2 = 12k_9.\left(B_{31}.[HP_2-S]\frac{\partial^2 h}{\partial R^2}+\frac{B_{32}}{\beta^2}.[HP_3-S]\frac{\partial^2 h}{\partial Q^2} + \frac{B_{33}}{\beta}.H(P_2+P_3-2S)\frac{\partial^2 h}{\partial R\partial Q}\right) \quad 73$$

$$\overline{\tau_{RS}} = \frac{\tau_{RS}}{q}\left(\frac{t}{a}\right)^3 = 12\,k_9\left(B_{44}.P_2.\frac{\partial H}{\partial S}\right).\frac{\partial h}{\partial R} \quad 74$$

$$\overline{\tau_{QS}} = \frac{\tau_{QS}}{q}\left(\frac{t}{a}\right)^3 = 12.k_9.\left(B_{55}.\frac{P_3}{\beta}.\frac{\partial H}{\partial S}\right).\frac{\partial h}{\partial Q} \quad 75$$

RESULTS OF NUMERICAL PROBLEMS

The numerical values for displacements (u, v and w) and stresses (σ_x, σ_y, τ_{xy}, τ_{xz} and τ_{yz}) are presented on Tables (1) to (7).

Table-1: Displacements and stresses for SSSS anisotropic rectangular thick plate for 0^0 @ ($\alpha = 5$ to $100, \beta = 1$)

α	$\overline{w}$	$\overline{u}$	$\overline{v}$	$\overline{\sigma_{xx}}$	$\overline{\sigma_{yy}}$	$\overline{\tau_{xy}}$	$\overline{\tau_{xz}}$	$\overline{\tau_{yz}}$
5	0.0180	-0.32005	-0.60615	0.980772	0.0364545	0.0592765	0.59966	0.06820
10	0.01005	-1.15660	-1.51589	0.881023	0.0259871	0.0427599	0.67721	0.05531
20	0.00775	-4.4924	-4.87597	0.853597	0.0222705	0.0374734	0.70067	0.05011
30	0.00731	-10.0509	-10.4394	0.848404	0.0215162	0.0364274	0.70524	0.04903
40	0.00715	-17.8329	-18.2231	0.846577	0.0212470	0.0360560	0.70686	0.04864
50	0.00708	-27.8382	-28.2292	0.845730	0.0211214	0.0358832	0.70761	0.04846
60	0.00704	-40.0669	-40.4583	0.845270	0.0210529	0.0357890	0.70802	0.04836
70	0.00701	-54.5190	-54.9107	0.844992	0.0210115	0.0357321	0.70827	0.04830
80	0.0070	-71.1945	-71.5863	0.844811	0.0209846	0.0356952	0.70843	0.04826
90	0.00699	-90.0934	-90.4853	0.844688	0.0209662	0.0356699	0.70854	0.04823
100	0.00698	-111.216	-111.608	0.844599	0.0209530	0.0356517	0.70862	0.04821

Table-2: Displacements and stresses for SSSS anisotropic rectangular thick plate for 15^0 @ ($\alpha = 5$ to $100, \beta = 1$)

α	$\bar{w}$	$\bar{u}$	$\bar{v}$	$\overline{\sigma_{xx}}$	$\overline{\sigma_{yy}}$	$\overline{\tau_{xy}}$	$\overline{\tau_{xz}}$	$\overline{\tau_{yz}}$
5	0.01245	-0.2212	-0.379625	0.661806	0.0977385	0.1517064	0.41532	0.071053
10	0.00791	-0.9118	-1.103948	0.658215	0.0849413	0.1272449	0.53056	0.096888
20	0.00634	-3.6773	-3.879932	0.656304	0.0807469	0.1192681	0.57014	0.106484
30	0.00602	-8.2868	-8.491418	0.655874	0.0799134	0.1176856	0.57813	0.108468
40	0.00591	-14.740	-14.94539	0.655717	0.0796172	0.1171235	0.58098	0.109179
50	0.00586	-23.037	-23.24277	0.655643	0.0794793	0.1168619	0.58231	0.109512
60	0.00583	-33.178	-33.38378	0.655603	0.0794042	0.1167193	0.58304	0.109693
70	0.00581	-45.163	-45.36852	0.655578	0.0793588	0.1166332	0.58347	0.109803
80	0.00580	-58.991	-59.19701	0.655562	0.0793293	0.1165773	0.58376	0.109874
90	0.00579	-74.663	-74.86926	0.655551	0.0793091	0.1165390	0.58395	0.109923
100	0.00579	-92.179	-92.38529	0.655543	0.0792946	0.1165115	0.58409	0.109958

Table-3: Displacements and stresses for SSSS anisotropic rectangular thick plate for 30^0 @ ($\alpha = 5$ to $100, \beta = 1$)

α	$\bar{w}$	$\bar{u}$	$\bar{v}$	$\overline{\sigma_{xx}}$	$\overline{\sigma_{yy}}$	$\overline{\tau_{xy}}$	$\overline{\tau_{xz}}$	$\overline{\tau_{yz}}$
5	0.00774	-0.15608	-0.179078	0.374328	0.1433016	0.2109912	0.23050	0.07840
10	0.00568	-0.68334	-0.656346	0.391837	0.1468940	0.2108444	0.33873	0.15169
20	0.00471	-2.75267	-2.695600	0.396415	0.1489391	0.2143670	0.38959	0.19008
30	0.00448	-6.19068	-6.126367	0.397230	0.1494259	0.2153889	0.40116	0.19905
40	0.00440	-11.0020	-10.93497	0.397511	0.1496063	0.2157821	0.40540	0.20236
50	0.00436	-17.1873	-17.11908	0.397641	0.1496919	0.2159707	0.40740	0.20393
60	0.00434	-24.7471	-24.67808	0.397711	0.1497388	0.2160750	0.40850	0.20479
70	0.00433	-33.6811	-33.61173	0.397752	0.1497673	0.2161385	0.40916	0.20531
80	0.00432	-43.9896	-43.91996	0.397780	0.1497858	0.2161799	0.40960	0.20565
90	0.00432	-55.6726	-55.60270	0.397799	0.1497986	0.2162085	0.40989	0.20588
100	0.00431	-68.7299	-68.65994	0.397812	0.1498077	0.2162290	0.41011	0.20605

Table-4: Displacements and stresses for SSSS anisotropic rectangular thick plate for values; 45^0 @ ($\alpha = 5$ to $100, \beta = 1$)

α	$\bar{w}$	$\bar{u}$	$\bar{v}$	$\overline{\sigma_{xx}}$	$\overline{\sigma_{yy}}$	$\overline{\tau_{xy}}$	$\overline{\tau_{xz}}$	$\overline{\tau_{yz}}$
5	0.00680	-0.19077	-0.089217	0.229241	0.2170548	0.22904	0.12201	0.10974
10	0.00521	-0.68901	-0.484018	0.236799	0.2306494	0.23989	0.21532	0.20913
20	0.00425	-2.54258	-2.275540	0.241002	0.2389991	0.24634	0.27041	0.26839
30	0.00402	-5.59693	-5.314405	0.242033	0.2410916	0.24794	0.28410	0.28315
40	0.00393	-9.86670	-9.578364	0.242420	0.2418793	0.24855	0.28925	0.28870
50	0.00389	-15.3545	-15.06343	0.242604	0.2422543	0.24883	0.29169	0.29135
60	0.00386	-22.0612	-21.76853	0.242705	0.2424610	0.24899	0.29305	0.29280
70	0.00385	-29.9868	-29.69326	0.242766	0.2425866	0.24909	0.29387	0.29368
80	0.00384	-39.1316	-38.83746	0.242806	0.2426685	0.24915	0.29440	0.29426
90	0.00383	-49.4956	-49.20106	0.242834	0.2427248	0.24919	0.29477	0.29466
100	0.00383	-61.0789	-60.78399	0.242854	0.2427652	0.24922	0.29503	0.29494

Table-5: Displacements and stresses for SSSS anisotropic rectangular thick plate for 60^0 @ ($\alpha = 5$ to $100, \beta = 1$)

α	$\bar{w}$	$\bar{u}$	$\bar{v}$	$\overline{\sigma_{xx}}$	$\overline{\sigma_{yy}}$	$\overline{\tau_{xy}}$	$\overline{\tau_{xz}}$	$\overline{\tau_{yz}}$
5	0.00896	-0.30878	-0.086577	0.144820	0.3249868	0.2488920	0.07430	0.16304
10	0.00643	-0.93244	-0.533919	0.147495	0.364930	0.2307812	0.14551	0.29731
20	0.00497	-3.05655	-2.556298	0.149099	0.3875076	0.2208424	0.18736	0.37509
30	0.00461	-6.50804	-5.982777	0.149497	0.3930315	0.2184275	0.19769	0.39423
40	0.00448	-11.3243	-10.78976	0.149646	0.3950991	0.2175250	0.20156	0.40140
50	0.00441	-17.5121	-16.97309	0.149717	0.3960814	0.2170964	0.20341	0.40481
60	0.00438	-25.0731	-24.53166	0.149757	0.3966220	0.2168606	0.20442	0.40668
70	0.00436	-34.0080	-33.46506	0.149780	0.3969503	0.2167174	0.20504	0.40782
80	0.00434	-44.3171	-43.77311	0.149796	0.3971644	0.2166241	0.20544	0.40857
90	0.00433	-56.0004	-55.45574	0.149806	0.3973116	0.2165599	0.20571	0.40908
100	0.00432	-69.0580	-68.51289	0.149814	0.3974171	0.2165139	0.20591	0.40945

Table-6: Displacements and stresses for SSSS anisotropic rectangular thick plate for 75^0 @ $(\alpha = 5 \text{ to } 100, \beta = 1)$

α	$\bar{w}$	$\bar{u}$	$\bar{v}$	$\overline{\sigma_{xx}}$	$\overline{\sigma_{yy}}$	$\overline{\tau_{xy}}$	$\overline{\tau_{xz}}$	$\overline{\tau_{yz}}$
5	0.01683	-0.61624	-0.188913	0.123480	0.6256783	0.2033115	0.08573	0.29069
10	0.01018	-1.56106	-0.851163	0.097464	0.6419197	0.1522786	0.10206	0.46676
20	0.00706	-4.44752	-3.602851	0.084644	0.6513665	0.1270507	0.10788	0.54996
30	0.00636	-9.08430	-8.209154	0.081723	0.6535931	0.1212998	0.10910	0.56872
40	0.00610	-15.5475	-14.66125	0.080651	0.6544162	0.1191885	0.10953	0.57560
50	0.00598	-23.8493	-22.95775	0.080146	0.6548053	0.1181932	0.10974	0.57884
60	0.00591	-33.9927	-33.09829	0.079869	0.6550188	0.1176475	0.10985	0.58061
70	0.00587	-45.9789	-45.08273	0.079701	0.6551483	0.1173168	0.10992	0.58169
80	0.00585	-59.8083	-58.91103	0.079592	0.6552327	0.1171015	0.10996	0.58239
90	0.00583	-75.4812	-74.58315	0.079517	0.6552906	0.1169536	0.10999	0.58287
100	0.00582	-92.9977	-92.09909	0.079463	0.6553322	0.1168476	0.11001	0.58321

Table-7: Displacements and stresses for SSSS anisotropic rectangular thick plate for values; 90^0 @ $(\alpha = 5 \text{ to } 100, \beta = 1)$

α	$\bar{w}$	$\bar{u}$	$\bar{v}$	$\overline{\sigma_{xx}}$	$\overline{\sigma_{yy}}$	$\overline{\tau_{xy}}$	$\overline{\tau_{xz}}$	$\overline{\tau_{yz}}$
5	0.02864	-1.06997	-0.350592	0.139262	1.08659	0.0909160	0.11324	0.47692
10	0.01410	-2.20768	-1.204006	0.075449	0.92186	0.0545870	0.07184	0.63094
20	0.00889	-5.65573	-4.546235	0.051070	0.86518	0.0408079	0.05475	0.68760
30	0.00783	-11.2375	-10.10619	0.045995	0.85368	0.0379445	0.05113	0.69929
40	0.00745	-19.0278	-17.88861	0.044173	0.84957	0.0369165	0.04983	0.70348
50	0.00727	-29.0370	-27.89414	0.043321	0.84765	0.0364360	0.04922	0.70544
60	0.00717	-41.2678	-40.12295	0.042856	0.84660	0.0361737	0.04889	0.70651
70	0.00711	-55.7212	-54.57511	0.042575	0.84597	0.0360151	0.04869	0.70715
80	0.00707	-72.3975	-71.25064	0.042392	0.84556	0.0359121	0.04856	0.70757
90	0.00705	-91.2970	-90.14955	0.042267	0.84528	0.0358413	0.04847	0.70786
100	0.00703	-112.419	-111.2718	0.042177	0.84508	0.0357906	0.04840	0.70807

Table-8: Comparison of present study results with various results from isotropic square SSSS plate subjected to uniformly distributed load

a/t	Author	Theory	$\bar{u}$	$\bar{w}$	$\overline{\sigma_{xx}}$	$\overline{\tau_{xy}}$	$\overline{\tau_{xz}}$
4	Present	TOSDT	0.07213	5.4893	0.3091	0.1775	0.2898
	Sayyad *et al.*	ESDT	0.079	5.816	0.300	0.223	0.481
	%difference		**-9.52%**	**-5.95%**	**2.94%**	**-25.6%**	**-65.9%**
	Reddy	HSDT	0.079	5.869	0.299	0.218	0.482
	Ghugal&Sayyad	TSDT	0.074	5.680	0.318	0.208	0.483
	Ghugal&Pawar	HPSDT	0.079	5.858	0.297	0.185	0.477
	Mindlin	FSDT	0.074	5.633	0.287	0.195	0.330
	%difference		**-2.59%**	**-2.61%**	**7.14%**	**-9.85%**	**-13.8%**
	Kirchhoff	CPT	0.074	4.436	0.287	0.195	-
	Pagano	Elasticity	0.072	5.694	0.307	-	0.460
10	Present	TSDT	0.07225	4.6881	0.3097	0.1775	0.3177
	Sayyad *et al.*	ESDT	0.075	4.658	0.289	0.204	0.494
	%error		**-3.73%**	**0.642%**	**6.68%**	**-14.6%**	**-55.3%**
	Reddy	HSDT	0.075	4.666	0.289	0.203	0.492
	Ghugal&Sayyad	TSDT	0.073	4.625	0.307	0.195	0.504
	Ghugal&Pawar	HPSDT	0.074	4.665	0.289	0.193	0.489
	Mindlin	FSDT	0.074	4.670	0.287	0.195	0.330
	%error		**-2.35%**	**0.38%**	**7.33%**	**-9.55%**	**-3.77%**
	Kirchhoff	CPT	0.074	4.436	0.287	0.195	-
	Pagano	Elasticity	0.073	4.639	0.289	-	0.487

RESULTS ANALYSIS

Displacements and stresses formulas

The combination of the elastic equations, displacement functions equations, stiffness equations, governing equations and the compatibility equations yielded the required formulas for the calculation of displacements and stresses. The displacements follows similar pattern and they are generally related by the following terms $\left[12(k_9)(\frac{a}{t})^2.\frac{q}{E_0}.(1-\mu_{12}.\mu_{21})\right]$ Where $\frac{a}{t}$ is the ratio of span to thickness which is majorly used to classify the plate, q is the pure bending loading on the plate, E_0 is the elastic modulus while k_9 is the stiffness coefficient which is calculated by dividing k_8 with a combination of different parameters as shown in equations 49 to 60. Although the in-plane displacements, u and v, are more closely related with the different being the derivative $\left(\frac{dh}{dR},\frac{dh}{dQ}\right)$ and aspect ratio (P_2, P_3). When these displacement formulas are applied in a problem, the values obtained with the formulas of u and v are more closely related than the values obtained with the out-plane displacement w.

The in-plane stresses $(\sigma_{RR}, \sigma_{QQ}, \tau_{RQ})$ have these terms in common, $\left[12q.\left(\frac{a}{t}\right)^2(k_9),(HP_2-S).\frac{\partial^2 h}{\partial R^2},(HP_3-S).\frac{\partial^2 h}{\partial Q^2} \text{ and } H(P_2+P_3-2S).\frac{\partial^2 h}{\partial R\partial Q}\right]$. The common terms improves the applicability and usability of the solutions for easy solution of thick anisotropic plate. Also the out-plane displacement (τ_{Rs} & τ_{Qs}) have $\left[12q.(k_9)\left(\frac{a}{t}\right)^3.\frac{\partial H}{\partial S}\right]$ in common. These formulas are new and very easy to apply when analyzing thick anisotropic rectangular plate. It only requires the user to substitute the formulas data as provided in the problem at hand. These equations are the novel equations for displacements and stresses used for thick anisotropic rectangular plate analysis.

Numerical values of SSSS plate at angle fiber orientation of 0^0

From Table 4.2, it is observed that out-plane displacement values (w) decreases toward the positive direction as the thickness of the plate decreases while the in-plane displacements (u & v) decrease toward the negative direction as the thickness of the plate decreases. The decreases were very visible at the thick plate zone (α = 5 to 10) but becomes very small and slightly negligible at the thin plate zone (α = 20 to 100). This is a confirmation that the effects of the displacement are more on thick plate than thin plate. The in-plane stresses, $\overline{\sigma_{xx}}$, $\overline{\sigma_{yy}}$, and τ_{xy} decrease as the plate thickness decreases. A close look will reveal a sharp decrease at thick and moderately thick plate section while the thin plate section decreased lightly. The out-plane stress τ_{xz} increases as the plate thickness decreases while the out-plane stress τ_{yz} decreases as the plate thickness decreases. This decrease of displacements values and increase and decrease of the stresses values of the plate can be explained from the fact that anisotropic plates have different properties in different directions.

Numerical values of SSSS plate at angle fiber orientation of 15^0

Table 4.3 shows that out-plane displacement values (w) and the in-plane displacements values (u & v) decrease as the thickness of the plate decreases. These effects act more at the thick plate zone but becomes very negligible at the thing plate zone. The in-plane stresses, $\overline{\sigma_{xx}}$, $\overline{\sigma_{yy}}$, and τ_{xy} decrease as the plate decreases in thickness vice versa. Also, it is observed that the decrease is more effective at thick and moderately thick plate section but very negligible at the thin plate section. The values of out-plane stresses τ_{xz} and τ_{yz} increase as the plate thickness decreases. These decrease of displacements values and increase and decrease of the stresses values of the plate can be explained from the fact that anisotropic plates have different properties in different directions.

Numerical values of SSSS plate at angle fiber orientation of 30^0

We observed from Table 4.3 that out-plane displacement values (w) and in-plane displacement values (u & v) decrease as the thickness of the plate decreases. The decrease was very noticeable at the thick plate zone but gradually diminishes as the thickness decreases. This shows that the displacements are more effective on thick plate than thin plate. The in-plane stresses, $\overline{\sigma_{xx}}$, $\overline{\sigma_{yy}}$ and τ_{xy} decrease as the plate thickness decreases vice versa. It is observed that the decrease is more noticeable at the thick plate section but gradually decreases at the thin plate section. The values of out-plane stresses τ_{xz} and τ_{yz} increase as the plate thickness decreases. These decrease of displacements values and increase and decrease of the stresses values of the plate can be explained from the fact that anisotropic plates have different properties in different directions.

Results of numerical values of SSSS plate at angle fiber orientation of 45^0

It is observed from Table 4.2d that out-plane displacement values (w) and in-plane displacements values (u & v) decrease as the thickness of the plate decreases. The decrease are more effective at the thick plate zone but was gradual at the thin plate zone. The in-plane stresses, $\overline{\sigma_{xx}}$, , $\overline{\sigma_{yy}}$, and τ_{xy} and the out-plane stresses τ_{xz} and τ_{yz} increases as the plate thickness decreases vice versa. This increase in stresses as the plate thickness decreases are very obvious at the thick plate section but gradually decreases as the thickness of the plate decrease. At 45^0 angle fiber orientation, the out-plane and the in-plane displacements decrease as the plate thickness decreases while both the out-plane and the in-plane stresses increase as the plate thickness decreases.

Results of numerical values of SSSS plate at angle fiber orientation of 60^0

Table 4.5 shows that out-plane displacement (w) values and in-plane displacements (v) values decrease as the thickness of the plate decreases. The decrease was much at the thick plate section but was gradual at the thin plate section. This shows that the out-plane displacement are more effective on thick plate than thin plate. The values of the in-plane stresses, $\overline{\sigma_{xx}}$, and $\overline{\sigma_{yy}}$ and the out-plane stresses τ_{xz} and τ_{yz} increase as the plate thickness decreases vice versa while the values of the in-plane stress τ_{xy} decreases as the thickness of the plate decreases. Both the increase and decrease of the displacements and stresses are more effective at the thick plate section but becomes gradual as the thickness of the plate decreases. At 60^0 angle fiber orientation both the out-plane and the in-plane displacements decreases as the plate thickness decreases while both the out-plane and the in-plane stresses increases as the plate thickness decreases except the values of the in-plane stress τ_{xy} which decreases as the plate thickness decreases. These decrease of displacements values and increase and decrease of the stresses values of the plate can be explained from the fact that anisotropic plates have different properties in different directions.

Results of numerical values of SSSS plate at angle fiber orientation of 75^0

We observed from Table 4.6 that out-plane displacement values (w) and in-plane displacements (u & v) decrease as the thickness of the plate decreases. The decrease was more effective at the thick plate zone than at the thin plate zone. The in-plane stresses, $\overline{\sigma_{xx}}$, and τ_{xy} decrease in values as the plate thickness decreases while the in-plane stress $\overline{\sigma_{yy}}$ and out-plane stresses τ_{xz} and τ_{yz} increase in values as the plate thickness decreases vice versa. These decrease of displacements values and increase and decrease of the stresses values of the plate can be explained from the fact that anisotropic plates have different properties in different directions.

Results of numerical values of SSSS plate at angle fiber orientation of 90^0

Table 4.7 shows that out-plane displacement values (w) and in-plane displacements (u & v) values decrease as the thickness of the plate decreases. The decrease was more at the thick plate section than at the thin plate section. This shows that the out-plane displacements are more active on thick plate than thin plate. The values of the in-plane stresses, $\overline{\sigma_{xx}}$, $\overline{\sigma_{yy}}$, and τ_{xy} and the out-plane stress τ_{xz} decrease as the plate thickness decreases while the values of the out-plane stress τ_{yz} increases as the thickness of the plate decreases. This increase or decrease in stresses as the plate thickness decreases are very obvious at the thick plate section but gradually decreases as the thickness of the plate decreases. At 90^0 angle fiber orientation both the out-plane and the in-plane displacements and stresses values decrease as the plate thickness decreases except the values of the out-plane stress τ_{yz} which increases as the plate thickness decreases.

Comparison of present study with those from other authors

The result from present study was compared with the results from SSSS isotropic square plate as obtained by various authors: [7-13]. At, α = a/t = 4, the percentage difference between present study and [8] for displacement (u) was -9.52% which seems mildly under-estimated but gave -2.59% when compared with [12]. Also, at a/t = 10, for displacement (u), the percentage differences between present study, [8] and [12] are -3.73% and -2.35% respectively. For displacements (w) at a/t = 4, the percentage differences between [8] and [12] with present study are -5.95% and -2.61%. At a/t = 10, the percentage differences for [8] and [12] with present study gave 0.64% and 0.38% for displacement (w). For in-plane stress ($\overline{\sigma_{xx}}$), at a/t = 4, comparing present study with [8] and [12] gave 2.94% and 7.14% respectively. For in-plane stress ($\overline{\sigma_{xx}}$), at a/t = 10, the percentage differences between present study, [8] and [12] are 6.68% and 7.33%. For In-plane stress ($\overline{\tau_{xy}}$) at a/t = 4, the percentage differences between the present study, [8] and [12] are -25.6% and -9.85% respectively. Also for In-plane stress ($\overline{\tau_{xy}}$) at a/t = 10, the percentage differences with present study, [8] and [12] are mildly underestimated as -14.6% and -9.55%. The out-plane stress ($\overline{\tau_{xz}}$), at a/t = 4 and a/t = 10, for [8] gave -65.9% and -55.3% and gave 13.8% and -3.77% when compared with [12]. From Table 4.8, It is observed that similar methods yield closer values than non-similar methods. The table has confirmed the similarity of present study with previous works, even though the similarity were not very high but, it is good enough to validate the present study.

CONCLUSIONS

The study presents novel formulas for the analysis of thick rectangular anisotropic plates based on refined plate theory and assumptions. Third order shear deformation theory and Ritz energy method was employed for the analysis. The solution derived novel formulas for the out-plane displacement (w), in-plane displacements (u & v), in-plane stresses ($\overline{\sigma_{xx}}$, $\overline{\sigma_{yy}}$ and τ_{xy}) and out-plane stresses (τ_{xz} and τ_{yz}) which were used in the analysis of thick anisotropic rectangular plate. The shear deformation function derived by [14] was employed. The formulas was employed to determine the deflection (u, v & w) at the center of the anisotropic rectangular plate and also stresses (σ_x, σ_y, τ_{xy}, τ_{xz} and τ_{yz}) at the meaningful points on the thick anisotropic rectangular plate for various angles and various span to thickness ratio (alpha, α) of the two boundary conditions; all four edges simply supported (ssss).

From the numerical and compared results obtained, the following conclusion was drawn. The novel formulas developed for prediction of displacements and stresses in the present study can be used to satisfactory analyze thick anisotropic rectangular plate problems of SSSS boundary condition. The method is simple and can be employed to analyze rectangular plates of other boundary conditions.

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