

Stochastic Analysis of a Simply Supported All-in-Laterite Concrete Beam in Flexure: A Model-Based Reliability Study for a Single Laterite Source

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Abstract

All-in-laterite concrete, in which lateritic material supplies both aggregate fractions, is a low-cost indigenous alternative to conventional concrete, but reinforced use is resisted because its strength variability makes structural performance unpredictable. This paper tests that objection for one laterite source and beam configuration. A published strength regression for this concrete from Idah, Kogi State, Nigeria (twelve mixes, forty-eight standard-cured cubes) was reproduced and embedded in a probabilistic resistance model, with cube-to-in-place strength transfer carried as a random variable. Strength variance used an assumed bounding model of predictive error, batching scatter and ACI 214R-11 control tiers. Fifteen basic variables followed the JCSS Probabilistic Model Code, with Gaussian-copula dependence. The first-order reliability method, verified by importance sampling, assessed sections designed to EN 1992-1-1 and BS 8110 against four limit states. The material supports strength class C12/15. The reference beam attained flexural reliability indices of 5.018 and 5.149 against a target of 3.8; the code-compliant shear-reinforced member attained 8.763, with links governed by minimum-ratio and maximum-spacing rules rather than strength, and 3.297 for quasi-permanent deflection. Concrete strength carried 0.0022 of the flexural limit-state variances, against 0.6332 for imposed load, and the flexural index differed by only 0.029 from conventional concrete of identical class. A concrete-only shear mechanism, screened for an unlinked member, returned 3.103. The flexural conclusion holds while transferred-resistance model uncertainty stays below a coefficient of variation of about 0.15. Strength variability therefore does not govern flexural reliability here, and raising the concrete partial factor would be an ineffective safeguard.

Keywords: All-in-laterite concrete, structural reliability, first-order reliability method, flexural limit state, partial safety factors, model uncertainty, serviceability.

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1. INTRODUCTION

Aggregate scarcity is reshaping concrete practice across the tropics. River sand and crushed granite together occupy 60 to 80 per cent of the volume of conventional concrete, and both are increasingly costly and environmentally contentious to extract. Lateritic soils and laterite rock are abundant across West Africa and have accordingly attracted sustained attention as substitute aggregates. Most published work treats laterite as a partial replacement for fine aggregate while retaining granite as the coarse fraction. The proposition examined here, all-in-laterite concrete, in which lateritic material supplies both fractions and conventional aggregate is eliminated entirely, has been characterized only recently.

1.1 Terminology

This literature uses two terms inconsistently; this paper distinguishes them throughout. *Laterized concrete* denotes concrete in which lateritic fines partially or wholly replace the sand fraction while a conventional coarse aggregate, normally crushed granite, is retained; this is the system studied by most of the works cited below. *All-in-laterite concrete* denotes concrete in which lateritic material supplies both the fine and coarse fractions, so no conventional aggregate is present. The distinction matters because the two systems differ in grading, aggregate strength and water demand, and results are not transferable between them. Where a cited study concerns laterized concrete, the term is used accordingly.

1.2 Background

The permissible level of laterite in structural concrete remains contested. Udoeyo et al. (2006) found that replacing sand with laterite up to 40 per cent still attained a 20 N/mm² design strength, whereas Ukpata et al. (2024) reported that only replacements up to 10 per cent met a 25 N/mm² requirement. At the permissive extreme, Ettu et al. (2013) argued from forty mix ratios that laterite can serve as the sole fine aggregate in structural concrete, and Balogun and Adepegba (1982) reported strengths at or above 25 MPa for an optimum 1:1½:3 mix. This divergence itself shows variability: nominally identical mixes from different deposits behave differently, a point emphasized by reviews of laterite characterization [Shuaibu et al., 2014; Kumar et al., 2022]. Falade (1994) established that the absorption of lateritic fines governs water demand and hence the effective water–cement ratio, the mechanism later invoked to reconcile high nominal ratios with respectable strengths, and consistent with the classical dependence of strength on effective water–cement ratio [Abrams, 1918; Neville, 2011]. Work on the all-in system specifically is sparse; Ephraim et al. (2016) reported on laterite rock concrete from a related source, and recent studies have applied response-surface optimization to lateritized mixes with supplementary binders [Khalid et al., 2024].

The deterministic behaviour of the all-in system is now reasonably well established for one source. Rowland-Lato (2026) cast 108 cubes across three mix proportions and water–cement ratios from 0.65 to 1.25 using laterite from Idah, Kogi State, and reported 28-day strengths of 18.8 to 23.4 MPa rising to 21.0 to 26.9 MPa at 64 days. Twelve mixes yielding forty-eight strength observations were fitted by ordinary least squares to a three-predictor model in water–cement ratio, aggregate–cement ratio and the logarithm of curing age, achieving a coefficient of determination of 0.936; leave-one-mix-out cross-validation returned a cross-validated coefficient of determination of approximately 0.91. The apparently high nominal water–cement ratios were resolved as an artefact of the exceptional aggregate absorption, 13.50 per cent for the fine and 7.95 per cent for the coarse fraction, which leaves the effective ratio governing hydration in the conventional 0.4 to 0.7 band.

Treating strength as a random variable rather than a deterministic value is long-standing practice in concrete specification. ACI 214R-11 [ACI Committee 214, 2011] relates the mean strength a producer must target to the achieved standard deviation and the specified characteristic strength, and tabulates control standards as bands of achievable standard deviation; BS EN 206 [BSI, 2021] embeds equivalent logic in its conformity criteria. The relationship between standard-cured specimen strength and the strength actually present in a member is itself a subject of codified guidance [ACI Committee 214, 2010; CEN, 2019] and statistical study [Bartlett & MacGregor, 1996], and this paper treats it

explicitly rather than assuming it away. Forsdyke et al. (2023) coupled cross-validated strength models with probabilistic specification for conventional and recycled concretes, establishing a template that has not been transferred to laterite. On the structural side, the modern reliability framework rests on the invariant second-moment formulation of Hasofer and Lind (1974), extended to non-normal variables by Rackwitz and Fiessler (1978) and set out comprehensively by Melchers and Beck (2018). EN 1990 [CEN, 2005] adopts a reliability index of 3.8 over a fifty-year reference period for reliability class RC2, with an indicative value of 1.5 for irreversible serviceability, and ISO 2394 [ISO, 2015] provides the equivalent international basis. The calibration studies underpinning those targets, synthesised by Holický (2009) and by Nowak and Collins (2013), together with the JCSS Probabilistic Model Code [JCSS, 2001] and the fib Model Code [fib, 2013], supply the probabilistic models adopted here.

1.3 Research Gap, Aim and Research Questions

What remains unresolved is the structural consequence of the material's variability. The strength of all-in-laterite concrete is not merely lower than that of conventional concrete; it is more scattered, because lateritic deposits vary in mineralogy, clay content and absorption, and because the absorptive aggregate makes effective water content sensitive to batching and pre-wetting practice. This scatter is the principal reason cited for confining the material to non-structural use, yet the argument is made qualitatively. Reliability analysis of reinforced concrete members is mature for conventional concrete, and strength prediction is now validated for all-in-laterite concrete from one source, but the two have not been joined. The source study identified this need, concluding that further work should yield a distribution of prediction error suitable for reliability-based target-strength specification [Rowland-Lato, 2026].

This paper aims to quantify, for one laterite source and one beam configuration, how much of the failure probability of a reinforced all-in-laterite beam is attributable to concrete strength, and to locate the flexural limit state within the hierarchy of limit states that govern such a member. Five research questions are addressed.

1. What characteristic strength and EN 1992-1-1 strength class does all-in-laterite concrete from this source support, once predictive error, batching scatter and production control are represented explicitly?
2. What reliability index does a simply supported all-in-laterite beam designed to EN 1992-1-1, and separately to BS 8110, attain in flexure under the assumed probabilistic inputs, and how does it compare with the EN 1990 target?
3. What share of the flexural limit-state variance is carried by concrete strength, and how does that share compare with the shares carried by actions, geometry, reinforcement and model uncertainty?

4. Does the elevated strength variability of all-in-laterite concrete measurably reduce flexural reliability relative to conventional concrete of the same strength class?
5. How does flexural reliability compare with that of the same member assessed for code-compliant shear and for quasi-permanent deflection, and where therefore does concrete strength actually govern?

The study is explicitly model-based. It uses one published strength model from one deposit, one member configuration, and probabilistic models for actions, geometry, steel, and model uncertainty transferred from conventional-concrete calibrations. No laterite beams were tested. The conclusions are therefore conditional, and Section 3.10 reports how far they move when each transferred assumption is varied.

2. MATERIALS AND METHODS

2.1 Source Data and Verification

The analysis is based on the pooled dataset of Rowland-Lato (2026): twelve mixes spanning water–cement ratios of 0.65 to 1.25 and aggregate–cement ratios of 2.99 to 6.02, each tested at 7, 14, 28, and 64 days, for a total of 48 observations. All specimens were 150 mm cubes, moist-cured by immersion and tested saturated. The dataset was reconstructed from the published mix table and the combined model refitted by ordinary least squares to the design matrix $\mathbf{X} = [1, w/c, a/c, \ln t]$.

Reconstruction was verified against the normal-equation matrices published in the source appendix. The reconstructed $\mathbf{X}^T\mathbf{X}$ and $\mathbf{X}^T\mathbf{y}$ agreed with the published values to within 0.02 and 0.05, respectively, and the residual sum of squares reproduced as 75.775 against the published 75.774. The refitted coefficients, their standard errors, the Shapiro–Wilk statistic on the residuals and the cross-validated error all reproduced the published values to the printed digits, as reported in Section 3.1.

2.2 Strength Transfer Chain

The quantity the source regression predicts, and the quantity a limit state requires are not the same, and the steps between them are set out explicitly here rather than left implicit.

The regression predicts $f_{c,cube}$, the mean compressive strength of a standard-cured 150 mm cube at a given age: that is what was cast, cured and tested in the source programme. EN 1992-1-1 strength classes and design expressions are defined on the standard-cured cylinder strength, obtained as $f_{c,cyl} = k_{cyl} f_{c,cube}$ with $k_{cyl} = 0.80$ for normal-strength concrete [Neville, 2011]. The strength actually present in a member differs from both, because in-place concrete is compacted, cured and loaded differently from a laboratory specimen; codified assessment procedures [ACI Committee 214, 2010; CEN, 2019] and the statistical study of Bartlett and MacGregor (1996) indicate in-place-to-standard ratios

below unity for cast-in-place members. This step is therefore carried as an explicit random variable,

$$f_{c,is} = k_{is} k_{cyl} f_{c,cube} \quad (1)$$

with k_{is} lognormal of mean 0.90 and coefficient of variation 0.06. Two points about Equation (1) should be stated plainly. First, k_{is} has not been measured for all-in-laterite concrete; its mean and scatter are transferred from conventional-concrete practice, and Section 3.10 reports the sensitivity of every conclusion to that transfer. Second, the design step uses the standard-cured characteristic cylinder strength because EN 1992-1-1 strength classes and γ_c are defined on that basis, whereas the limit states use $f_{c,is}$; separating the two prevents double-counting the in-place transfer within γ_c .

No cores were taken from all-in-laterite members in the source programme, so Equation (1) is an assumption, not a validated relationship. Establishing k_{is} for this material by core testing is identified in Section 3.11 as required further work.

2.3 Assumed Bounding Model for Strength Variance

The variance of in-place strength is represented by a three-component bounding model. This is not a measured variance decomposition: only the first component is estimated from data, the second propagates assumed batching tolerances, and the third is assigned from published control bands. The model is used to bracket the plausible range of strength scatter, and every reliability result is reported across that range in Section 3.7.

The first component is the predictive error of the mix-design model. For a new batch at predictor vector $\mathbf{x}$,

$$\text{Var}_{\text{model}} = s^2(1 + \mathbf{x}^T(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{x}) \quad (2)$$

where s is the leave-one-mix-out cross-validated root-mean-square error of 1.471 MPa rather than the in-sample residual standard deviation, because the model extrapolates to an unbuilt mix, which cross-validation estimates.

The second is batching scatter. Site weigh-batching does not reproduce nominal proportions exactly, so w/c and a/c are themselves random. Propagating to first order,

$$\text{Var}_{\text{batch}} = (\beta_1 V_{w/c} \cdot w/c)^2 + (\beta_2 V_{a/c} \cdot a/c)^2 \quad (3)$$

with assumed coefficients of variation $V_{w/c} = 0.05$ and $V_{a/c} = 0.04$ representing weigh-batching tolerance. Because $\beta_1 = -26.878$ MPa per unit w/c is large, even modest scatter in the water–cement ratio propagates appreciably.

The third is production and testing variance, encompassing cement and aggregate variability between deliveries, mixing efficiency, compaction, curing and testing error. This cannot be estimated from a single

controlled laboratory programme and is assigned from the ACI 214R-11 standards of control for general construction testing [ACI Committee 214, 2011], which specify overall standard deviations of 2.8, 3.4, 4.1 and 4.8 MPa for excellent, very good, good and fair control at characteristic strengths below 34.5 MPa. The total standard deviation is taken as the tier value subject to a floor at the irreducible model-plus-batching value,

$$\sigma_{\text{total}} = \max(\sigma_{\text{tier}}, \sqrt{\text{Var}_{\text{model}} + \text{Var}_{\text{batch}}}) \quad (4)$$

The characteristic cube strength is the 5 per cent fractile, $f_{\text{ck,cube}} = \mu - 1.645 \sigma_{\text{total}}$. In-place strength is modelled as lognormal, consistent with the JCSS Probabilistic Model Code [JCSS, 2001]; the justification for that choice, and the effect of replacing it with a normal marginal, are given in Sections 3.2 and 3.10.

2.4 Reference Beam, Code Basis and the Design Step

The reference member is a simply supported rectangular beam of 4.5 m span, 230 mm width and 450 mm overall depth, with an effective depth of 410 mm and 25 mm nominal cover, reinforced with Grade 460 ribbed steel. The span and section represent the lightly loaded low-rise application for which the material was judged suitable [Rowland-Lato, 2026]. The characteristic total uniformly distributed load is 12 kN/m, with self-weight computed from the section and measured hardened density, and the load ratio is $\chi = Q_k/(G_k + Q_k) = 0.30$. Figure 1 shows the geometry, loading arrangement, assumed reinforcement, and the four limit states examined.

The code basis is EN 1992-1-1:2004+A1:2014 [CEN, 2014] with EN 1990:2002+A1:2005 [CEN, 2005] actions, using CEN recommended values throughout; no national annex is applied, and readers applying a national annex should expect the partial factors and γ_c -related quantities to differ accordingly. The comparison code is BS 8110-1:1997 [BSI, 1997]. BS 8110 was withdrawn in 2010 and is included for one specific reason: it remains in routine use for building design in Nigeria and much of West Africa, where all-in-laterite concrete would be built, so the margin it delivers is of direct practical interest. It is reported as a comparison only, and no conclusion of this paper rests on it.

All quantities are in SI units. Throughout, f is in MPa (N/mm²), lengths in mm except spans in m, forces in kN, moments in kNm, distributed loads in kN/m and density in kg/m³. Self-weight is obtained as $w = \rho_c b h g$ with ρ_c in kg/m³, b and h in mm and $g = 9.81$ m/s², the product being converted to kN/m by the factor $10^{-6} \times 10^{-3}$ implicit in those units. In the deflection expression, a load in kN/m is numerically equal to the same load in N/mm, so Equation (10) requires no further conversion.

A methodological point deserves emphasis. The true resistance of a section does not depend on which code the designer consulted; the code enters only through

the amount of reinforcement provided. Therefore, compare the two codes by designing the section twice and assessing both designs against a single probabilistic resistance model.

Under EN 1992-1-1, with the rectangular stress block ($\lambda = 0.8$, $\eta = 1.0$), $\alpha_{cc} = 0.85$, $\gamma_c = 1.5$, $\gamma_s = 1.15$ and the combination $1.35G_k + 1.5Q_k$, the neutral-axis depth ratio follows from $K = M_{Ed}/(bd^2f_{cd})$ as $\xi = (1 - \sqrt{1 - 2.5K})/0.8$, the lever arm is $z = d(1 - 0.4\xi)$, and $A_s = M_{Ed}/(f_{yd}z)$, subject to $\xi \leq 0.45$ for a singly reinforced section and to the minimum area $\max(0.26f_{ctm}/f_{yk}, 0.0013)bd$. Under BS 8110, with $1.4G_k + 1.6Q_k$, $K = M_{Ed}/(bd^2f_{cu})$, $z = d(0.5 + \sqrt{(0.25 - K/0.9)})$ and $A_s = M_{Ed}/(0.87f_yz)$, subject to $K \leq 0.156$ and a minimum of $0.0013bd$.

Shear reinforcement is designed by the variable-strut-inclination truss of EN 1992-1-1 clause 6.2.3 with $\cot \theta = 2.5$, giving a required $A_{sw}/s = V_{Ed}/(z f_{ywd} \cot \theta)$ with $z = 0.9d$, and is then made compliant with the minimum shear reinforcement ratio $\rho_{w,\min} = 0.08\sqrt{f_{ck}/f_{yk}}$ of clause 9.2.2(5) and the maximum longitudinal spacing $s_{\max} = \min(0.75d, 300 \text{ mm})$ of clause 9.2.2(6). The reinforcement actually assessed is what those detailing rules deliver, not the strength-based requirement alone, because that is what a compliant beam contains. Verification that the resulting bar and link arrangement fits within the section at the nominal cover is reported in Section 3.3.

2.5 Limit-State Functions

Four limit states are evaluated. All are written at actual rather than design material strengths, with f_c denoting the in-place strength of Equation (1).

Flexure, ultimate:

$$g_M = \theta_R A_s f_y \left(d - 0.5 \frac{A_s f_y}{f_c b} \right) - \theta_E (w_{sw} + G + Q) \frac{L^2}{8}$$

The λ factor cancels between the compression-block depth and the lever arm, so Equation (5) is free of the stress-block shape parameter.

Shear of the compliant member, ultimate, as the lesser of tie yield and strut crushing:

$$g_V = \theta_{Vs} \min \left[k_{Asw} \left(\frac{A_{sw}}{s} \right) z f_y \cot \theta, \frac{b z v_1 f_c}{\cot \theta + \tan \theta} \right] - \theta_E (w_{sw} + G + Q) \frac{L}{2} \quad (6)$$

with $z = 0.9d$ and $v_1 = 0.6(1 - f_c/250)$. Link and main reinforcement are from the same supply and are therefore assigned the same yield variable.

Shear of a member **without** links, evaluated as a screening diagnostic only:

$$g_{Vc} = \theta_{Vc} \max[0.18k(100\rho_1 f_c)^{1/3}, 0.035k^{3/2}\sqrt{f_c}] bd - \theta_E (w_{sw} + G + Q) \frac{L}{2} \quad (7)$$

with $k = \min(1 + \sqrt{(200/d)}, 2.0)$ and $\rho_1 = \min(A_s/(bd), 0.02)$. Equation (7) does **not** describe the

designed beam, which contains links; it is retained because it isolates the mechanism in which concrete strength enters the resistance most directly, and so serves to locate where concrete quality matters. It is reported as such throughout and is never used to characterize the compliant member.

Deflection, quasi-permanent serviceability. The secant modulus and the creep-adjusted effective modulus are

$$E_{cm} = 22000 \left(\frac{f_{c+8}}{10} \right)^{0.3}, \quad E_{c,eff} = \frac{E_{cm}}{1+\varphi} \quad (8)$$

and with I_I the gross and I_{II} the fully cracked transformed second moment of area, EN 1992-1-1 clause 7.4.3 interpolation gives

$$\zeta = 1 - 0.5 \left(\frac{M_{cr}}{M_{qp}} \right)^2, \quad \delta = \zeta \delta_{II} + (1 - \zeta) \delta_I, \quad \delta_i =$$

$$\frac{5w_{qp}L^4}{384E_{c,eff}I_i} \quad (9)$$

$$g_\delta = \frac{L}{250} - \theta_\delta \delta \quad (10)$$

where $M_{cr} = f_{ctm} b h^2/6$, $w_{qp} = w_{sw} + G + Q_{sus}$ with Q_{sus} the quasi-permanent imposed load, and the coefficient 0.5 in ζ is the EN 1992-1-1 value for sustained loading. The creep coefficient φ is treated as a random variable with mean 2.5, reflecting an interior member of this strength class, and is bounded to the physically admissible interval $[0, 12]$. The creep behaviour of an aggregate with 8 to 13.5 per cent absorption has not been measured, so the deflection result is presented as a screening analysis and is swept over φ in Section 3.10.

2.6 Probabilistic Models and the Dependency Model

Fifteen basic variables enter the four limit states. Their marginals follow the JCSS Probabilistic Model Code [JCSS, 2001], the fib Model Code [fib, 2013] and the EN 1990 calibration literature [Holický, 2009; Nowak & Collins, 2013], with concrete strength and density taken from the source study. Table 1 gives the full specification, including for each variable whether it is estimated from the source data, transferred from published models, or assumed.

Table 1: Probabilistic models of the fifteen basic variables, with the basis of each stated explicitly. "Estimated" means derived from the source dataset; "transferred" means taken from published conventional-concrete models; "assumed" means assigned by the author. The four resistance model uncertainties are mutually exclusive, one entering each limit state. Values shown are those of the reference beam ($\chi = 0.30$, good control)

Variable	Description	Distribution	Mean	Std. dev.	CoV	Basis
fc,cube	Standard-cured 28-d cube strength, MPa	Lognormal	23.18	4.10	0.177	estimated (source regression) + assigned tier
kis	Standard-cured to in-place conversion	Lognormal	0.90	0.05	0.060	assumed (EN 13791 / ACI 214.4R range)
fy	Reinforcement yield strength, MPa	Lognormal	520.00	30.00	0.058	transferred (JCSS)
As	Steel area, multiplier on nominal	Normal	1.00	0.02	0.020	transferred (JCSS)
b	Section width, mm	Normal	230.00	5.38	0.023	transferred (JCSS tolerance)
d	Effective depth, mm	Normal	410.00	10.00	0.024	transferred (cover/placement)
h	Overall depth, mm	Normal	450.00	6.70	0.015	transferred (JCSS tolerance)
pc	Hardened density, kg/m ³	Normal	2443.00	97.72	0.040	estimated (source study)
G	Superimposed permanent load, kN/m	Normal	5.92	0.59	0.100	transferred (JCSS)
Q	Imposed load, 50-yr maximum, kN/m	Gumbel	2.16	0.76	0.350	transferred (EN 1990 calibration)
θ_E	Load-effect model uncertainty	Lognormal	1.00	0.05	0.050	transferred (JCSS)
kAsw	Link area/spacing, multiplier on nominal	Normal	1.00	0.05	0.050	assumed (placement tolerance)
φ	Creep coefficient	Lognormal	2.50	0.62	0.250	assumed (not measured for laterite)
Qsus	Quasi-permanent imposed load, kN/m	Lognormal	1.08	0.38	0.350	transferred (JCSS)
θ_R , flexure	Resistance model uncertainty (per limit state)	Lognormal	1.00	0.07	0.070	transferred (JCSS / fib)
θ_{Vs} , truss shear	Resistance model uncertainty (per limit state)	Lognormal	1.00	0.15	0.150	transferred (JCSS / fib)

Variable	Description	Distribution	Mean	Std. dev.	CoV	Basis
θV_c , concrete shear	Resistance model uncertainty (per limit state)	Lognormal	1.00	0.20	0.200	transferred (JCSS / fib)
$\theta \delta$, deflection	Resistance model uncertainty (per limit state)	Lognormal	1.00	0.25	0.250	transferred (JCSS / fib)

Dependence is declared rather than left implicit. Correlations are specified directly in standard normal space as a Gaussian copula (the Nataf model): with R the copula correlation matrix and L its Cholesky factor, $z = Lu$ for u of independent standard normal components, and each variable is recovered as $x_i = F_i^{-1}(\Phi(z_i))$. Four cases are evaluated. The independent case sets $R = I$ and is the reference. The plausible case applies positive correlation between concrete strength and hardened density (0.60), both being driven by water content and compaction, between section width and overall depth (0.50) and between effective and overall depth (0.50), both being consequences of shared formwork and placement tolerance, between density and superimposed permanent load (0.30), and between concrete strength and the in-place conversion factor (0.30). The strong case raises those coefficients to 0.85, 0.75, 0.75, 0.50 and 0.50. The adverse case reverses the signs of the strength–density, depth, and strength–conversion correlations, which is not physically expected but bounds the effect of an incorrect sign. Where R is indefinite, it is projected onto the nearest positive-definite correlation matrix by eigenvalue truncation.

When $R = I$, the first-order sensitivity factors α^2 are the per-variable shares of limit-state variance and are reported as such. When $R \neq I$, the decomposition is basis-dependent, so only β is reported for those cases.

2.7 Reliability Methods and Reproducibility

The reliability index was computed by the first-order reliability method using the Hasofer–Lind–Rackwitz–Fiessler algorithm [Hasofer & Lind, 1974; Rackwitz & Fiessler, 1978], with gradients evaluated by central differences of step 10^{-4} in standard normal space and iteration terminated when the change in the design-point norm fell below 10^{-9} , subject to a cap of 300

iterations. Convergence was reached in every case reported; the iteration count is recorded in the supplementary output.

Two independent checks were applied. Crude Monte Carlo simulation verifies the limit-state implementation and marginal sampling. Because the failure probabilities of interest are small, crude simulation cannot resolve them at feasible sample sizes and is reported only as an implementation check, with the failure count stated so the reader can see how many failures it captured. Importance sampling centred on the first-order design point provides the quantitative verification [Melchers & Beck, 2018]: the sampling density is a unit-covariance Gaussian in $\mathbf{u}$ -space shifted to $\mathbf{u}^*$, and the estimator is $P_f = E[1\{g \leq 0\} w]$ with weights $w = \exp(-\frac{1}{2}\|\mathbf{u}\|^2 + \frac{1}{2}\|\mathbf{u} - \mathbf{u}^*\|^2)$. The coefficient of variation of the estimated failure probability and the number of sample points falling in the failure domain are reported for every case, so that an estimate resting on too few hits can be identified as such. Reliability indices are obtained as $\beta = -\Phi^{-1}(P_f)$ rather than $\Phi^{-1}(1 - P_f)$, the latter saturating in double precision for the small probabilities returned by the shear-reinforced member.

Computations used Python 3.11.16 with NumPy 2.4.6 and SciPy 1.17.1. Crude Monte Carlo used 10^6 samples with seed 12345 and importance sampling used 4×10^5 samples with seed 2024; all random draws come from the NumPy PCG64 generator seeded as stated, so every number in this paper is reproducible from the supplied code. The reliability engine, the result driver, the reference database and the figure scripts are provided as supplementary material, together with the reconstructed dataset and every result table in machine-readable form.

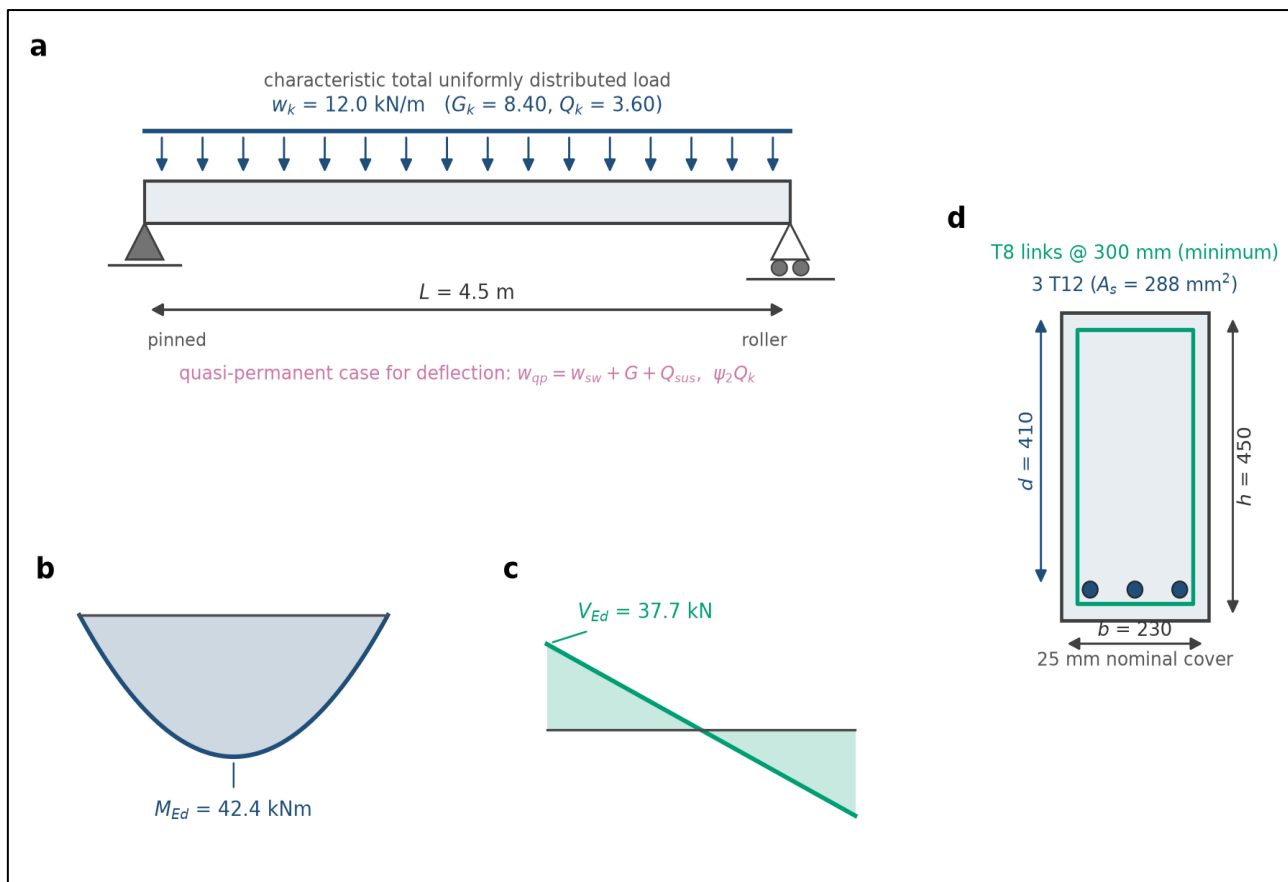


Figure 1: Definition of the analysed member. (a) Simply supported span and loading arrangement under the characteristic uniformly distributed load, with the quasi-permanent case used for the deflection check, as noted below. (b) Design bending-moment and (c) design shear-force diagrams for the ultimate combination $1.35G_k + 1.5Q_k$. (d) Midspan cross-section showing the tension reinforcement and the code-compliant links. Dimensions in millimetres

3. RESULTS AND DISCUSSION

3.1 Verification of the Deterministic Base

Refitting the reconstructed forty-eight-observation dataset returned coefficients of 25.962 for the intercept, -26.878 for w/c , 1.130 for a/c , and 3.390 for $\ln t$, with standard errors of 1.243, 1.578, 0.246, and 0.231, identical to the published values to the printed precision. The coefficient of determination was 0.9362, the Shapiro–Wilk statistic on the residuals was 0.960 with $p = 0.097$, the leave-one-mix-out cross-validated error was 1.471 MPa, and the cross-validated coefficient of determination was 0.913, all matching the source study. The residual sum of squares, 75.775, reproduces the published 75.774. The published root-mean-square error of 1.256 MPa is the maximum-likelihood estimate $\sqrt{(SSE/n)}$; the unbiased estimate $\sqrt{(SSE/(n-p))}$ is 1.312 MPa. The deterministic base is confirmed, and the

probabilistic model therefore inherits a verified regression.

3.2 Strength Characterisation and the Choice of Marginal

Table 2 gives the bounding variance model of Section 2.3. For the richest mix, 1:1:2 at w/c 0.65, the model predicts a mean 28-day standard-cured cube strength of 23.18 MPa; predictive error contributes a standard deviation of 1.53 MPa and assumed batching scatter of 0.88 MPa, giving an irreducible floor of 1.77 MPa, or a coefficient of variation of 7.6 per cent. Across the four candidate mixes, this floor ranges from 7.6 to 10.9 per cent. The ACI 214R-11 control tiers imply coefficients of variation of 12.1, 14.7, 17.7, and 20.7 per cent for the richest mix under excellent, very good, good, and fair control, respectively.

Table 2: Assumed bounding variance model and characteristic strengths of the four candidate mixes. The σ floor is the irreducible model-plus-batching standard deviation of Equations (2) and (3); characteristic values shown are for good control ($\sigma = 4.1$ MPa). Strengths are standard-cured values; the in-place transfer of Equation (1) is applied in the limit states, not here

Mix	Mean cube fc (MPa)	σ model (MPa)	σ batching (MPa)	σ floor (MPa)	V floor	fck,cube good (MPa)	fck,cyl good (MPa)	V good
A 1:1:2, w/c 0.65	23.18	1.53	0.88	1.77	0.076	16.43	13.15	0.177
B 1:1½:3, w/c 0.75	22.20	1.52	1.03	1.84	0.083	15.45	12.36	0.185
C 1:1:2, w/c 0.75	20.50	1.52	1.02	1.83	0.089	13.75	11.00	0.200
D 1:2:4, w/c 0.95	18.53	1.54	1.31	2.02	0.109	11.78	9.42	0.221

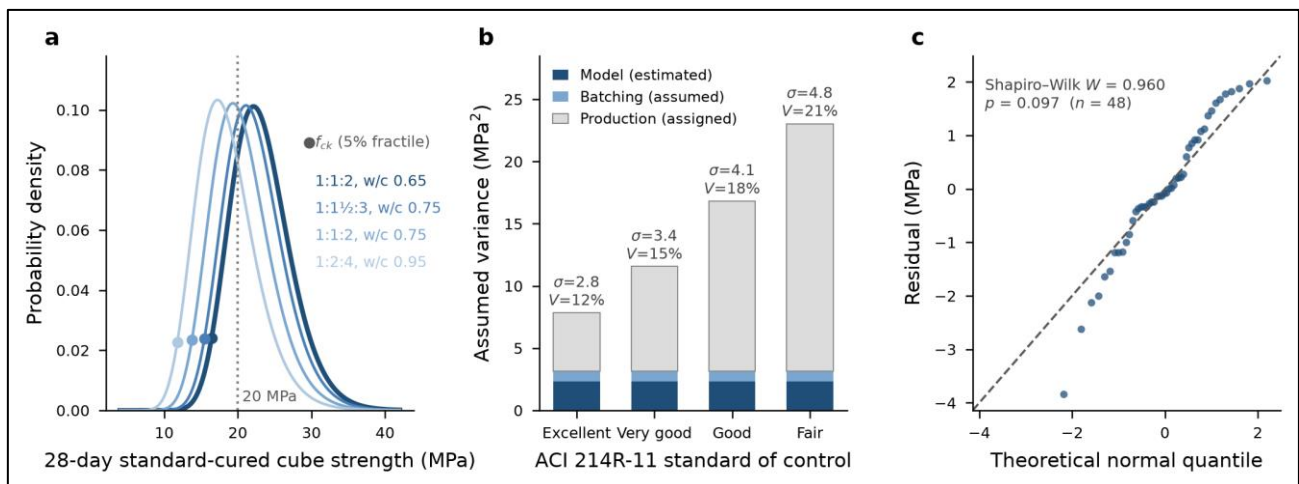


Figure 2: Strength characterisation. (a) Lognormal densities of 28-day standard-cured cube strength for the four candidate mixes under good control, with the 5 per cent fractile marked and the indicative 20 MPa threshold shown. (b) The assumed bounding variance model across the four ACI 214R-11 standards of control for the 1:1:2 mix at w/c 0.65; only the darkest component is estimated from the source data, with the other two assumed and assigned accordingly. (c) Normal quantile–quantile plot of the regression residuals, supporting the additive-normal error assumption from which the lognormal strength marginal follows

Within this bounding model, the mix-design model is the smallest of the three components: under good control it contributes 14 per cent of the total variance and assumed batching a further 5 per cent, the remainder being the assigned production and testing term. This is a statement about the assumed model, not a measured decomposition, and it should be read as indicating where the assumed uncertainty budget sits rather than as an empirical apportionment. Its practical implication is nonetheless worth stating: if the assigned production bands are approximately right for site-produced all-in-laterite concrete, then further refinement of the strength model would change little, whereas site control would change much. Testing that implication requires a production study, which this paper does not provide.

The consequence for specification is direct. Characteristic standard-cured cube strengths for the richest mix are 18.57, 17.58, 16.43 and 15.28 MPa across the four tiers, giving characteristic cylinder strengths of

14.86, 14.07, 13.15 and 12.23 MPa. All-in-laterite concrete from this source, under the assumed control bands, therefore supports strength class C12/15 and no higher, this being the lowest class for which EN 1992-1-1 provides design rules. The leanest mix, 1:2:4 at w/c 0.95, falls short of C12/15 under good or fair control and is unsuitable for reinforced work.

The lognormal marginal for concrete strength is adopted for three reasons: it is non-negative, which a normal marginal is not at the coefficients of variation considered here; the regression residuals are consistent with an additive-normal error on strength, as the quantile–quantile plot of Figure 2(c) shows and the Shapiro–Wilk result of Section 3.1 supports, which combined with a multiplicative production term gives an approximately lognormal in-place strength; and it is the JCSS Probabilistic Model Code convention for concrete compressive strength [JCSS, 2001], which keeps the present results comparable with published calibrations. Because the choice is a modelling decision rather than a

fitted result, Section 3.10 reports what happens when it is replaced by a normal marginal. Forty-eight observations from twelve mixes at one source cannot discriminate sharply between candidate distributions in the lower tail, which governs reliability; the distributional assumption is therefore a genuine limitation, not a validated finding.

3.3 Reference Design and Detailing Compliance

Designed for the reference loading, the beam requires 287.9 mm² of tension steel under EN 1992-1-1 and 295.3 mm² under BS 8110, corresponding to steel ratios of 0.305 and 0.313 per cent, both above the minimum of 122.6 mm². The design shear force is 37.7 kN against a concrete-only design resistance of 30.5 kN, so shear reinforcement is required.

The strength-based requirement is $A_{sw}/s = 0.1021$ mm²/mm, but the minimum shear reinforcement ratio of clause 9.2.2(5) demands 0.1450 mm²/mm, so the minimum-reinforcement rule governs. Using 8 mm two-leg links at the maximum permitted spacing of 300 mm

provides 0.3351 mm²/mm, which is 3.3 times the strength-based requirement. The corresponding design resistances are $VR_{d,s} = 123.7$ kN for tie yield and $VR_{d,max} = 145.8$ kN for strut crushing, against the design shear of 37.7 kN. The compliant member is thus substantially over-provided in shear, entirely as a consequence of detailing rules rather than of the strength calculation. This fact governs the interpretation of every shear result below.

Detailing feasibility was checked: three 12 mm bars (287.9 mm²) are furnished in one layer (339 mm²), requiring 152 mm width at 25 mm nominal cover with 8 mm links and 25 mm clear bar spacing, against the 230 mm provided. The arrangement fits, and the assessed section is therefore buildable rather than merely arithmetic.

3.4 Reliability of the Reference Beam

Table 3 gives the reference beam reliability across the four limit states, each verified by importance sampling.

Table 3: Reliability of the reference beam across the four limit states (4.5 m span, 230 × 450 mm, $d = 410$ mm, Grade 460 reinforcement, 1:1:2 mix at w/c 0.65 under good control, characteristic total load 12 kN/m, $\chi = 0.30$, independent case). Importance sampling used 4×10^5 samples at seed 2024; crude Monte Carlo used 10^6 samples at seed 12345 and is reported as an implementation check, with its failure count shown so its resolution is visible.

The concrete-only shear row is a screening diagnostic for a member without links and does not describe the designed beam

Limit state / design	β FORM	Pf FORM	β IS	Pf IS	CoV(Pf)	IS hits	Crude MC
Flexure, EC2 design	5.018	2.61e-07	4.995	2.94e-07	0.0039	202,832	1 in 10^6 , $\beta=4.753$
Flexure, BS 8110 design	5.149	1.31e-07	5.127	1.48e-07	0.0039	202,690	1 in 10^6 , $\beta=4.753$
Shear, concrete-only screening	3.103	9.56e-04	3.056	1.12e-03	0.0031	206,155	1086 in 10^6 , $\beta=3.066$
Shear, compliant linked beam	8.763	9.53e-19	8.749	1.08e-18	0.0151	201,128	0 in 10^6
Deflection, quasi-permanent SLS	3.297	4.88e-04	3.273	5.32e-04	0.0031	203,195	561 in 10^6 , $\beta=3.258$

In flexure, the EN 1992-1-1 design attains $\beta = 5.018$ with a failure probability of 2.61×10^{-7} , verified by importance sampling at 4.995 with a coefficient of variation on the failure probability of 0.0039 from 202,832 points in the failure domain. The BS 8110 design attains 5.149, verified at 5.127. Agreement between the first-order method and importance sampling within 0.047 across all limit states indicates that the limit-state surfaces are nearly linear at their design points, justifying the use of the first-order method alone for the parametric studies that follow.

The compliant shear-reinforced member attains $\beta = 8.763$, far above the flexural value, which follows directly from the detailing-governed over-provision established in Section 3.3. The quasi-permanent deflection state attains $\beta = 3.297$, to be compared with the EN 1990 indicative value of 1.5 for irreversible serviceability rather than with 3.8. The concrete-only

shear mechanism of Equation (7) returns $\beta = 3.103$; as stated in Section 2.5, this does not describe the designed beam and is reported as a screening diagnostic, discussed in Section 3.9.

Both flexural designs exceed the EN 1990 target of 3.8 under the assumed inputs. BS 8110 delivers the higher index, by 0.130 on average across load ratios, because its permanent-load factor of 1.4 exceeds 1.35 and its imposed-load factor of 1.6 exceeds 1.5, with identical material factors on the resistance side.

3.5 Variance Decomposition in Flexure

Table 4 and Figure 3(a) give the first-order sensitivity factors at the reference design for the independent case. In flexure, imposed load dominates with $\alpha^2 = 0.6332$. Resistance model uncertainty follows at 0.1406, reinforcement yield strength at 0.0862, load-effect model uncertainty at 0.0718, superimposed

permanent load at 0.0342, effective depth at 0.0196 and steel area at 0.0106.

Table 4: First-order sensitivity factors α^2 for the four limit states at the reference beam, independent case, ordered by flexural sensitivity, with the flexural design-point values. α^2 is the share of limit-state variance carried by each variable. Zero entries indicate variables that do not enter that limit state

Variable	α^2 flexure	α^2 shear (links)	α^2 shear (no links)	α^2 deflection	x^* flexure
Q	0.6332	0.4322	0.1381	0.0000	8.17
θ	0.1406	0.3883	0.6980	0.8052	0.87
f_y	0.0862	0.0580	0.0000	0.0000	475.81
θE	0.0718	0.0436	0.0444	0.0000	1.07
G	0.0342	0.0124	0.0448	0.0640	6.48
d	0.0196	0.0109	0.0023	0.0355	402.80
As	0.0106	0.0000	0.0008	0.0026	0.99
$f_{c,cube}$	0.0022	0.0000	0.0610	0.0143	21.86
ρ_c	0.0010	0.0004	0.0013	0.0018	2458.64
k _{is}	0.0003	0.0000	0.0071	0.0017	0.89
b	0.0001	0.0001	0.0020	0.0003	230.33
h	0.0001	0.0000	0.0002	0.0003	450.40
k _{Asw}	0.0000	0.0541	0.0000	0.0000	1.00
φ	0.0000	0.0000	0.0000	0.0414	2.43
Q _{sus}	0.0000	0.0000	0.0000	0.0329	1.02

Concrete compressive strength contributes $\alpha^2 = 0.0022$, and the in-place conversion factor a further 0.0003, despite the strength carrying a coefficient of variation of 17.7 per cent, the largest of any variable in the analysis. At the design point, the in-place strength is drawn down only from a mean of 16.69 MPa to 15.64 MPa, whereas imposed load is driven from a mean of 2.16 kN/m to 8.17 kN/m.

The mechanism is geometric. In an under-reinforced section, concrete strength enters the resistance only through the depth of the compression block, and hence only through the lever arm. Differentiating Equation (5),

$$\frac{\partial \ln M_R}{\partial \ln f_c} = \frac{0.4x}{d-0.4x} \quad (11)$$

which at the reference steel ratio evaluates to 0.050. This elasticity is bounded within the admissible design space, because the maximum steel ratio for a singly reinforced section of this class is

$$\rho_{\max} = \frac{0.36\alpha_{cc}f_{ck}\gamma_s}{\gamma_c f_{yk}} = 0.670 \text{ per cent} \quad (12)$$

at which the elasticity reaches only 0.117. This is the paper's central mechanism and is stated here once: for this member, in this material, under these inputs, the flexural limit state is close to indifferent to concrete

compressive strength, because the material's own low strength caps $p_{\max}$ through Equation (12) and so caps the elasticity through Equation (11). Later sections refer back to this result rather than restating it.

Two qualifications belong with it. It is a property of under-reinforced flexure, not of the material; the same concrete in a column or a deep beam would behave quite differently, as Section 3.9 shows. It also depends on the assumed model uncertainty θ_R , which carries $\alpha^2 = 0.1406$ and is transferred rather than measured; Section 3.10 quantifies that exposure.

3.6 Load Ratio and Code Comparison

Figure 4(a) traces the flexural reliability index against load ratio from 0.05 to 0.75. For the richest mix under EN 1992-1-1, the index falls from a maximum of 5.398 to 3.927 at $\chi = 0.75$; under BS 8110, it ranges from 5.549 to 4.048. The non-monotonic behaviour at low load ratio arises because the minimum-reinforcement rule governs the design when the imposed load is very small, conferring surplus capacity. Across all four mixes and both codes, the index spans 3.927 to 5.635 and remains above 3.8 throughout the range examined, with the minimum at the highest load ratio, which corresponds to storage or plant loading rather than the residential and light-commercial occupancies for which the material is proposed.

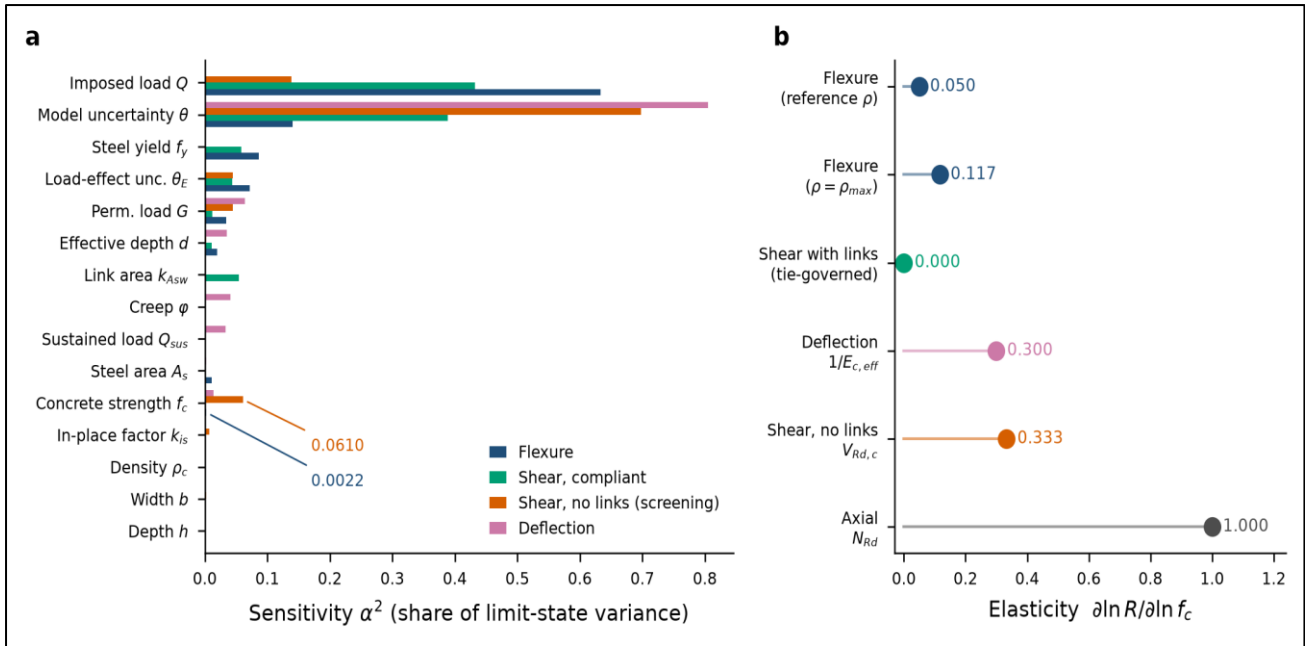


Figure 3: Where concrete strength enters. (a) First-order sensitivity factors α^2 for the four limit states at the reference beam, independent case. Reinforcement yield strength does not enter the concrete-only shear resistance of Equation (7). (b) Elasticity of resistance to in-place concrete strength, $\partial \ln R / \partial \ln f_c$, across limit states, with the flexural value bounded between the reference steel ratio and ρ_{max}

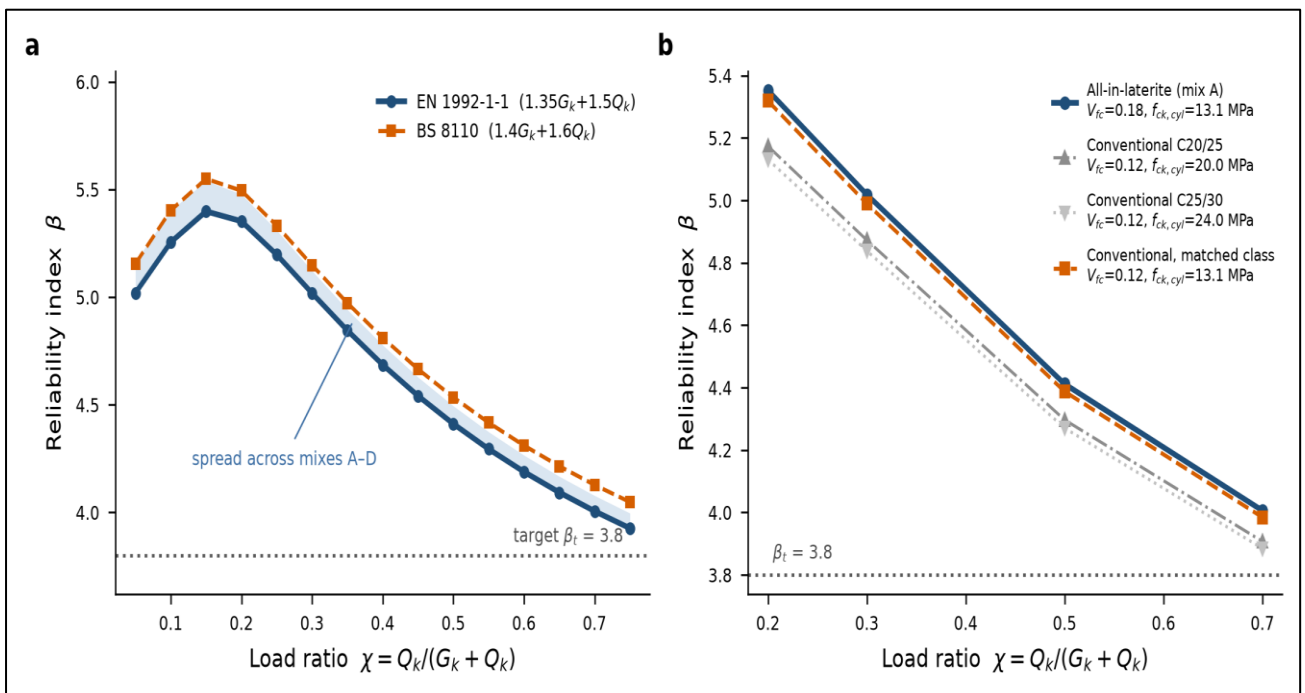


Figure 4: Flexural reliability against load ratio. (a) Reliability index for the 1:1:2 mix at w/c 0.65 designed to EN 1992-1-1 and to BS 8110, with the shaded band showing the spread across the four candidate mixes. (b) Comparison with conventional concrete, including a benchmark matched to the identical strength class but with the lower strength variability typical of conventional production

3.7 Effect of Control Standard

Figure 5 examines the consequence of control standard across all four limit states. As the assumed strength coefficient of variation rises from 0.08 to 0.26,

the flexural index increases from 4.908 to 5.172, the compliant shear index remains at 8.763, the deflection index increases from 3.269 to 3.344, and the concrete-only screening index falls from 3.166 to 3.023.

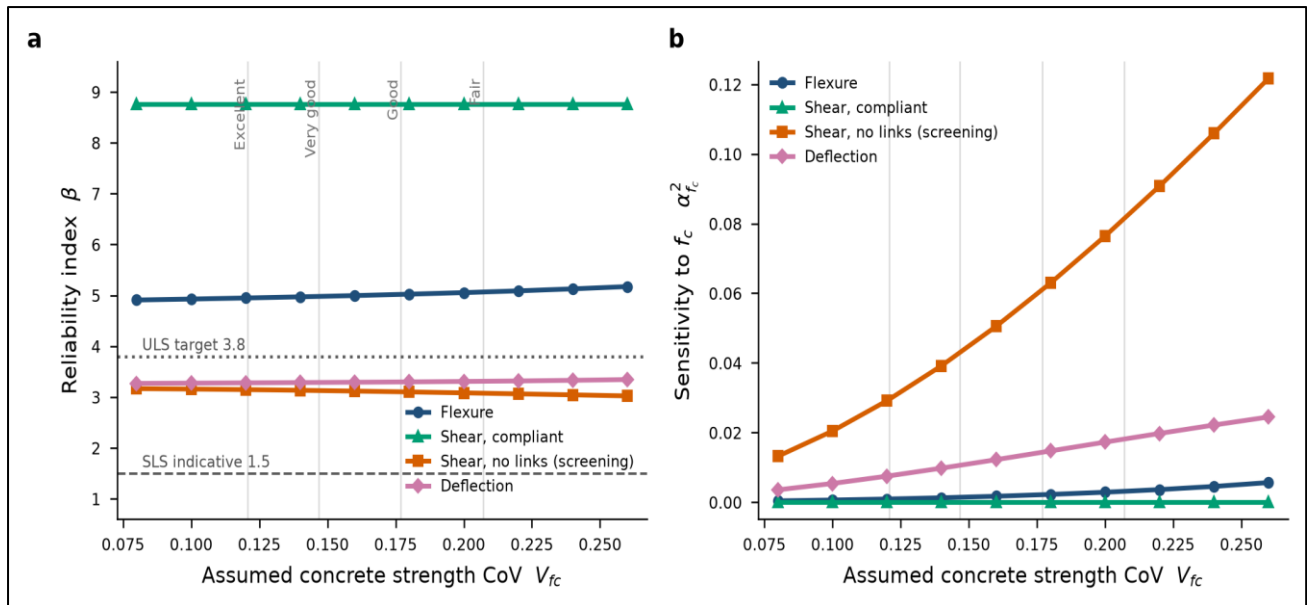


Figure 5: Effect of the assumed strength coefficient of variation on all four limit states. (a) Reliability index, with vertical rules marking the coefficients of variation implied by the four ACI 214R-11 standards of control and horizontal rules at the ultimate and serviceability targets. (b) Sensitivity of each limit state to concrete strength across the same range

The rise in the flexural index with increasing variability has a sound mechanism. Greater assumed variability depresses the 5 per cent fractile, so the designer is handed a lower characteristic strength; the design procedure responds by increasing the reinforcement area, and because reinforcement governs flexural resistance far more strongly than concrete does, the net effect on mean resistance outweighs the added concrete uncertainty. Within the range examined, the partial-factor format therefore over-compensates for concrete variability in flexure. The effect is small, 0.264 in the index across a threefold range of variability, but its sign is instructive: for this limit state, tightening concrete quality control does not buy flexural safety. The opposite sign on the concrete-only mechanism is equally instructive and is taken up in Section 3.9.

3.8 Comparison with Conventional Concrete of Equal Class

Testing whether laterite variability penalizes structural performance requires a matched comparison. A conventional concrete was specified with the same characteristic cylinder strength as the laterite mix, 13.15 MPa, but with a coefficient of variation of 12 per cent typical of conventional production, requiring a mean cube strength of 20.47 MPa against the laterite mix's 23.18 MPa. Both concretes yield an identical design, with the same 287.9 mm² of steel, so any difference in the index is attributable purely to differences in strength variability.

At $\chi = 0.30$, the laterite beam attains 5.018 and the matched conventional beam 4.989, a difference of 0.029. Across load ratios of 0.20 to 0.70, the difference never exceeds 0.034, and in every case the laterite beam is marginally the more reliable, as Figure 4(b) shows. Conventional concretes of higher class return slightly lower indices, 4.872 for C20/25 and 4.838 for C25/30 at $\chi = 0.30$, because a higher characteristic strength permits marginally less steel, and steel is the variable that matters. Under the assumed inputs, the elevated strength variability of all-in-laterite concrete therefore does not measurably reduce flexural reliability relative to conventional concrete of equal strength class. This answers research question 4 for the configuration examined; it is not a general result for laterite concretes, for other sources, or for other members.

3.9 Where Concrete Strength Governs

Section 3.5 established that flexure is nearly indifferent to concrete strength. The complementary question is where it is not, and this is the part of the argument that the limit-state comparison is for. Table 5 gives the elasticity of resistance to in-place concrete strength across limit states. Flexure is the least dependent, at 0.050 for the reference beam and 0.117 at $\rho_{\max}$. Deflection follows at 0.300, since the secant modulus scales as $f_{cm}^{0.3}$; concrete-only shear at 0.333, the resistance scaling as $f_c^{1/3}$; and axial compression at 1.000, where concrete carries the load directly.

Table 5: Elasticity of resistance to in-place concrete compressive strength across limit states. Flexural values follow from Equation (11) at the reference steel ratio and at $\rho_{\max}$. The shear-with-links value is zero because the truss is tie-governed at $\cot \theta = 2.5$; strut crushing, which does depend on f_c , is not the governing mechanism for this member

Limit state	$\partial \ln R / \partial \ln f_c$
Flexure, M Rd (reference, $\rho = 0.305\%$)	0.050
Flexure, M Rd ($\rho = \rho_{\max} = 0.670\%$)	0.117
Shear with links, V Rd,s (tie-governed)	0.000
Deflection, $1/E_{c,eff}$	0.300
Shear without links, V Rd,c	0.333
Axial compression, N Rd	1.000

The reliability results sharpen this into a practical statement, and the distinction between the designed member and the screening mechanism is essential.

For the compliant member, which contains the links required by clause 9.2.2, shear is not the governing limit state: $\beta = 8.763$ against 5.018 in flexure. Concrete strength carries $\alpha^2 = 0.0000$ in this state because, with $\cot \theta = 2.5$, the truss is controlled by tie yield rather than strut crushing, and the tie is steel. The compliant beam is thus insensitive to concrete strength in shear as well as in flexure, though for a different reason: in flexure because the lever arm barely moves, in shear because the governing mechanism does not involve concrete at all.

For a member without links, the concrete-only mechanism of Equation (7) returns $\beta = 3.103$, below the 3.8 target, with concrete strength carrying $\alpha^2 = 0.0610$, some 27 times its flexural share, rising to 0.0815 under fair control. This mechanism alone shows reliability *decreasing* with strength variability, from 3.166 to 3.023 across the range of Figure 5, the opposite of the flexural trend. Model uncertainty nonetheless dominates, with θV_c carrying $\alpha^2 = 0.6980$, reflecting the empirical

character of the EN 1992-1-1 concrete shear expression and its calibration on conventional aggregates rather than gap-graded laterite.

The correct reading is therefore not that shear governs the designed beam. Rather, concrete strength sensitivity in this material concentrates in the concrete-only shear mechanism, and the code's minimum-link detailing rules already neutralise that mechanism by a wide margin. The safeguard is the detailing rule, not the material factor. The practical corollary is specific and checkable: an all-in-laterite beam that is detailed with the code minimum links has ample shear reliability, whereas one that relies on concrete-only shear resistance, as a lightly loaded member might if links were omitted on the basis of $V_{Ed} \leq V_{Rd,c}$, would be the one case in this study where concrete quality genuinely governs. Links should therefore not be omitted in this material on that basis.

3.10 Sensitivity to Transferred Assumptions and to Dependence

Every conclusion above is conditional on inputs that were transferred rather than measured. Figure 6 and Table 6 report how far each conclusion moves when those inputs are varied over defensible ranges.

Table 6: Sensitivity of the reliability index to each transferred or assumed input, varied over the stated range with all other inputs held at their reference values

Assumption	Basis	Range examined	Limit state	β range	Span of β
Flexural resistance model uncertainty, CoV	transferred	0.05 to 0.20	Flexure	3.312 to 5.206	1.894
Flexural resistance model uncertainty, mean	transferred	0.85 to 1.10	Flexure	4.123 to 5.527	1.405
Truss (shear) model uncertainty, CoV	transferred	0.15 to 0.30	Shear, compliant	5.812 to 8.763	2.950
Truss (shear) model uncertainty, mean	transferred	0.85 to 1.10	Shear, compliant	8.083 to 9.161	1.078
Concrete-shear model uncertainty, CoV	transferred	0.15 to 0.35	Shear, no links (screening)	1.902 to 3.735	1.833
Concrete-shear model uncertainty, mean	transferred	0.85 to 1.10	Shear, no links (screening)	2.414 to 3.504	1.090
Deflection model uncertainty, CoV	transferred	0.15 to 0.45	Deflection	2.169 to 4.610	2.442
In-place conversion factor k_{is} , mean	assumed	0.80 to 1.00	Flexure	4.985 to 5.045	0.059
In-place conversion factor k_{is} , mean	assumed	0.80 to 1.00	Shear, no links (screening)	2.938 to 3.251	0.314

Assumption	Basis	Range examined	Limit state	β range	Span of β
Creep coefficient	assumed	1.50 to 3.50	Deflection	2.988 to 3.655	0.666
Dependence (four declared copula cases)	declared	independent to adverse	Flexure	5.001 to 5.018	0.017
Dependence (four declared copula cases)	declared	independent to adverse	Shear, no links (screening)	3.088 to 3.103	0.015
Dependence (four declared copula cases)	declared	independent to adverse	Shear, compliant	8.754 to 8.763	0.009
Dependence (four declared copula cases)	declared	independent to adverse	Deflection	3.280 to 3.297	0.018
Concrete strength marginal (lognormal vs normal)	assumed	lognormal / normal	Flexure	5.018 to 5.022	0.004
Concrete strength marginal (lognormal vs normal)	assumed	lognormal / normal	Shear, no links (screening)	3.103 to 3.106	0.003

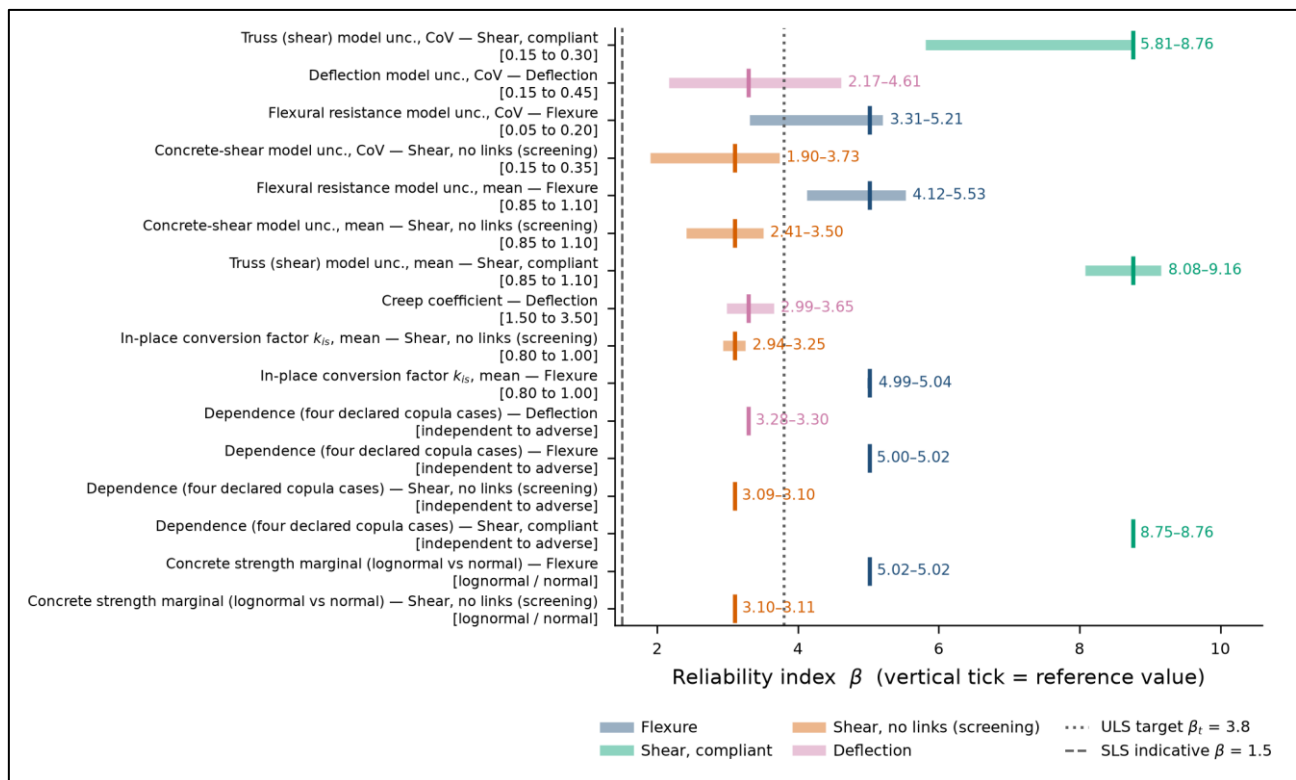


Figure 6: Sensitivity of the reliability index to the transferred and assumed inputs, each varied over the range given in Table 6 with all others held at their reference values. Bars show the resulting span of β for each limit state; the reference value is marked. The dashed rules are the ultimate and serviceability targets

The flexural index is most exposed to the resistance model uncertainty θ_R , as its $\alpha^2 = 0.1406$ implies, and this is the one assumption capable of overturning the flexural conclusion. Raising its coefficient of variation from 0.05 to 0.15 moves β from 5.206 to 3.974, which is still above target; raising it to 0.20 moves β to 3.312, below the target of 3.8. A downward bias in its mean is less damaging: means of 0.90 and 0.85 give 4.443 and 4.123, both above target. The flexural conclusion of this paper therefore holds provided the resistance model uncertainty of all-in-laterite beams in bending is no worse than a coefficient of variation of about 0.15, which is the conventional-concrete value inflated by roughly a factor of two.

Whether it is that good is unknown because no all-in-laterite beams have been tested in bending; this is why Section 3.13 identifies member testing as the single most valuable next step, and it is a real qualification on the headline result rather than a formality. The in-place conversion factor k_{is} matters less than might be expected: varying its mean from 0.80 to 1.00 moves the flexural index only from 4.985 to 5.045, which is a direct corollary of the low elasticity of Equation (11), but moves the concrete-only screening index from 2.938 to 3.251, a far larger swing, again locating the sensitivity in the concrete-governed mechanism. Replacing the lognormal strength marginal with a normal one changes the flexural index from 5.018 to 5.022 and the screening

index from 3.103 to 3.106, so the distributional choice is not what carries the flexural conclusion. The deflection index is most exposed to the creep coefficient, moving from 2.988 to 3.655 as ϕ varies from 1.5 to 3.5, and to θ_{δ} , moving from 2.169 to 4.610; since laterite creep is unmeasured, the deflection result should be read as a screening estimate with that range attached.

Dependence was tested over the four cases of Section 2.6. The flexural index moves from 5.018 in the independent case to 5.016 under plausible correlation, 5.014 under strong correlation and 5.001 under the sign-reversed adverse case, a total span of 0.017. The compliant shear and deflection states move by 0.009 and

0.018, respectively. Dependence among the basic variables therefore does not determine these results; the useful outcome of declaring independence is that it is an adequate working assumption here, and it is now a tested statement rather than an unexamined one.

3.11 Design Envelope

Figure 7 maps the flexural index over spans of 3 to 7 m and characteristic loads of 5 to 30 kN/m for the reference section in the richest mix designed to EN 1992-1-1. Across the feasible domain, the index ranges from 4.823 to 8.557, and 49 per cent of the grid is infeasible because the required steel ratio exceeds $\rho_{\max}$. The 3.8 contour does not appear anywhere in the domain.

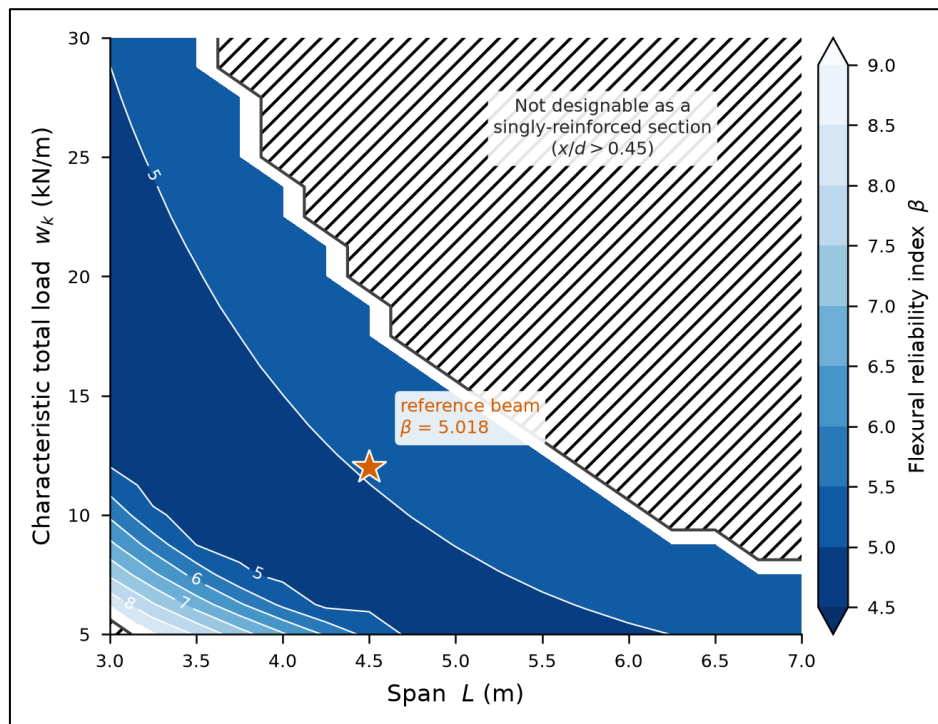


Figure 7: Flexural reliability envelope for a 230 × 450 mm all-in-laterite section designed to EN 1992-1-1 across the span and characteristic-load domain. The hatched region cannot be designed as a singly reinforced section because x/d exceeds 0.45. Across the whole designable domain, β lies between 4.823 and 8.557, so the target of 3.8 is met everywhere the section is buildable

Within the region examined, the binding constraint on an all-in-laterite beam is therefore the capacity of a low-strength singly reinforced section, reached when the compression block can no longer balance the required tension force, and not the reliability of that section. The designer's difficulty with this material, for this member type and load range, is fitting the moment into an economic section rather than achieving an acceptable failure probability once it fits.

3.12 Implications for Partial Factors and Quality Control

A natural regulatory response to a variable material is to raise its material partial factor. For this member's flexural limit state, the results indicate this would be ineffective. Because concrete strength carries

$\alpha^2 = 0.0022$, γ_c has almost no leverage on the flexural index; increasing it would reduce the design strength, marginally shorten the lever arm, add a little reinforcement, and leave the index essentially unchanged, at the cost of a less economical section. The safeguards that carry flexural reliability here are γ_s and the action factors, and these are material-independent. The BS 8110 comparison quantifies the action-factor effect: its heavier load factors buy 0.130 in the index, an order of magnitude more than a plausible adjustment to γ_c would deliver.

Three recommendations follow, each scoped to what was actually analysed. First, all-in-laterite beams should be detailed with the code minimum links as a matter of course, and links should not be omitted on the

basis of $V_{Ed} \leq V_{Rd,c}$, because the concrete-only mechanism is the one case examined here in which concrete quality governs. Second, quality-control expenditure, including pre-soaking the aggregate towards a saturated-surface-dry condition, controlling effective rather than nominal water content, and maintaining consistent curing, is justified by the concrete-governed mechanisms and by serviceability rather than by flexural safety. Third, the material should not be used in axially loaded members without a dedicated study, since axial resistance is fully proportional to concrete strength and the elasticity is more than twenty times the flexural value; no column was analysed here, and that statement is an extrapolation from Equation (11) and Table 5, not a result.

Placed against the laterite literature, the finding offers a possible reason why reported strengths vary so widely between studies [Balogun & Adepegba, 1982; Ettu et al., 2013; Udoeyo et al., 2006; Ukpatu et al., 2024] while structural performance, where assessed, has generally been satisfactory: if flexural capacity is governed by reinforcement, mixes differing by several megapascals will perform comparably in beams. That reading is consistent with the present analysis but is not established by it, since this study examined one source and one configuration.

3.13 Limitations

The following limitations bound every conclusion above, and several are material enough to qualify the paper's contribution rather than merely to refine it.

Single source and single configuration. The strength model derives from one deposit at Idah, Kogi State, and is valid only within its calibration envelope, w/c of 0.65 to 1.25, a/c of 3.0 to 6.0 and ages of 7 to 64 days. The author analyzed one member, one section, one reinforcement grade, and one load pattern in detail. Nothing here establishes behaviour for other laterite sources, other member types, continuous or two-way members, or concentrated loading.

Model-based, not experimental. No laterite beams were tested. The study demonstrates what follows from a published strength model combined with transferred probabilistic models; it does not validate that combination against member tests. The single most valuable next step is a programme of all-in-laterite beam tests to establish θ_R and θ_{Vc} for this material, since θ_R carries the second-largest share of flexural variance and is currently borrowed from conventional-concrete calibrations on gap-graded, highly absorptive aggregate for which it may not hold.

Assumed rather than measured uncertainty inputs. The production-variance component, the batching tolerances, the in-place conversion k_{is} , the creep coefficient, and all model uncertainties are assumed or

transferred. Section 3.10 reports the sensitivity to each, and the flexural conclusion survives the ranges examined, but a conclusion that survives an assumed range is not the same as one resting on measured inputs.

In-place strength not verified. Equation (1) is an assumption. No cores were taken from all-in-laterite members, and the in-place-to-standard ratio for a highly absorptive gap-graded aggregate may differ from conventional values in either direction. Core testing to establish k_{is} for this material is required before the numerical indices reported here should be relied upon for design.

Serviceability is treated as screening. The deflection analysis uses assumed creep and a transferred deflection model uncertainty, and cracking was not analysed at all. Crack width is likely the more onerous serviceability criterion for low-modulus concrete and is not addressed.

Distributional and dependence assumptions. Forty-eight observations cannot discriminate lower-tail behaviour, and the dependency structure was declared and tested rather than estimated. Both were shown to be non-governing over the ranges examined, which is reassuring but not equivalent to having measured them.

Time effects not exploited. The source study reported continued strength gain from 28 to 64 days, which the analysis did not use, so it is conservative in that respect. Durability, which for a clay-bearing aggregate in a tropical environment may well be the binding practical constraint, was not considered.

4. CONCLUSION

This paper developed a model-based stochastic analysis of a simply supported all-in-laterite concrete beam, using a verified regression model for the material's compressive strength from one Nigerian source and probabilistic models transferred from conventional-concrete practice. The conclusions below hold for that source, that member configuration, and the stated inputs; Section 3.10 reports their sensitivity to the transferred assumptions, and Section 3.13 the limits of their scope.

1. Under the assumed bounding variance model, the mix-design model and batching scatter contribute a strength coefficient of variation of 7.6 to 10.9 per cent against 12.1 to 20.7 per cent implied by ACI 214R-11 standards of control, so the assigned production term dominates the assumed uncertainty budget. Whether it dominates the real one requires a production study.
2. All-in-laterite concrete from this source supports EN 1992-1-1 strength class C12/15 and no higher, with characteristic cylinder strengths of 14.86, 14.07, 13.15 and 12.23 MPa across the four control tiers; the leanest mix

examined does not reach C12/15 under good control.

3. A simply supported beam designed to EN 1992-1-1 attains a flexural index of 5.018, and to BS 8110 5.149, against the EN 1990 target of 3.8; across load ratios of 0.05 to 0.75 and four mixes the index spans 3.927 to 5.635. Importance sampling confirmed the first-order values to within 0.047.
4. Flexural reliability of this member is close to indifferent to concrete strength: the concrete term carries $\alpha^2 = 0.0022$ against 0.6332 for imposed load, and the elasticity of flexural resistance to concrete strength is bounded at 0.117 throughout the admissible range of steel ratios. The index rises slightly as assumed variability grows, because a depressed characteristic value forces additional reinforcement.
5. Matched against conventional concrete of identical strength class, the flexural index differs by 0.029, despite one-and-a-half times the strength variability. For this limit state and configuration, the material's perceived unsuitability on grounds of strength scatter is not supported.
6. The code-compliant shear-reinforced member attains 8.763, well above the flexural value, because its shear reinforcement is governed by the minimum-ratio and maximum-spacing rules rather than by the strength calculation. Shear does **not** govern the compliant beam.
7. Concrete strength sensitivity concentrates instead in the concrete-only shear mechanism, which as a screening diagnostic for a member without links returns 3.103 with $\alpha^2(f_c) = 0.0610$. The practical safeguard is the minimum-link detailing rule, not a material partial factor; links should not be omitted in this material on the basis of $V_{Ed} \leq V_{Rd,c}$.
8. The quasi-permanent deflection state attains 3.297 against the EN 1990 indicative serviceability value of 1.5, but is sensitive to the assumed creep coefficient over the range 2.988 to 3.655 and should be treated as a screening result until laterite creep is measured.
9. Dependence among the basic variables, tested over four declared correlation cases, moves the flexural index by 0.017, so independence is an adequate working assumption for this problem.

For practice, and within the scope stated above, all-in-laterite concrete may be used in lightly loaded reinforced flexural members without its strength variability compromising flexural safety, provided it is proportioned on an effective-water basis, designed as class C12/15, and detailed with the code minimum links. A material-specific increase in the concrete partial factor would be an ineffective safeguard for flexure. Further work should prioritize member tests to quantify

resistance-model uncertainty for this material, core testing to establish in-place strength transfer, a production study to replace the assigned variance bands, and an investigation of cracking and durability.

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Data reuse and publication overlap.

The laboratory dataset analysed here was previously published by the author [Rowland-Lato, 2026], from which we reconstructed the twelve-mix pooled table; this paper reports no new laboratory data. The present contribution is the probabilistic and structural reliability analysis, which does not appear in the earlier work. The earlier paper reported strength prediction only and drew no structural or reliability conclusions. This manuscript is not under consideration elsewhere.

Data availability. The reconstructed dataset, the reliability engine, the result driver, the figure scripts and all result tables are provided as supplementary material. The underlying laboratory data are those of Rowland-Lato (2026).

Use of AI and AI-assisted technologies.

An AI language model (Claude, Anthropic) assisted with implementing the reliability computations, preparing figures, and editing the language. The author formulated the problem, made probabilistic modelling decisions, and interpreted the results and conclusions; the author also reviewed and takes full responsibility for all AI-assisted work.

Conflict of Interest. The author declares no conflict of interest.

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