DOI: 10.21276/sjbms.2017.2.3.1

Saudi Journal of Business and Management Studies
Scholars Middle East Publishers
Dubai, United Arab Emirates
Website: http://scholarsmepub.com/

ISSN 2415-6663 (Print)
ISSN 2415-6671 (Online)

Comparative Analysis of Fuzzy Entropic Order Quantity (FEnOQ) Model

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Abstract: This paper focuses on comparative analysis of fuzzy entropic economic order quantity model in presence of flexible ordering cost in finite planning horizon. The objective of this fuzzy model is to optimize the net profit so as to determine the entropic order quantity, the number of replenishment cycle and flexible ordering cost. After testing optimality and concavity of the net profit function it is compared with other different models through suitable numerical examples which show that an appropriate optimal fuzzy entropic paradigm may benefit the manufacturer. Hence, this paper shows a numerical example which elucidates the impact of entropy, flexible ordering cost and fuzzy. Finally, sensitivity analyses and ANOVA testing of the present model are presented to validate the effectiveness and sensitivity of the different parameters with respect to the fuzzy net profit of the proposed new fuzzy model. The proposed mathematical model has demonstrated how this fuzzy approach to decision making can achieve a global optimum and outperform the traditional models.

Keywords: Fuzzy, Entropy, FEnOQ, Flexible ordering cost, Profit, ANOVA

INTRODUCTION

Most of the literature on inventory control and production planning has dealt with the assumption that the demand for a product will continue infinitely in the future either in a deterministic or in a stochastic fashion. This assumption does not always hold true. Inventory management plays a crucial role in businesses since it can help companies reach the goal of ensuring prompt delivery, avoiding shortages, helping sales at competitive prices and so forth. The mathematical modeling of real-world inventory problems necessitates the simplification of assumptions to make the mathematics flexible. However, excessive simplification of assumptions results in mathematical models that do not represent the inventory situation to be analyzed. In the whole production system production function is the mid between the procurement function and physical distribution function. Other two functions are not processing in terms of production only they are facilitating for the smooth functioning and cost effecting of the production system in competitive advantage but production function processes to produce the finished products. So inventory plays a significant role in smooth functioning of the production function in a supply chain management. The physical characteristics of stocked items dictate the nature of inventory policies implemented to manage and control in production system. The question is how reliable are the EOQ models when items stocked deteriorate one time.

Many models have been proposed to deal with a variety of inventory problems. The classical analysis of inventory control considers three costs for holding inventories. These costs are the procurement cost, carrying cost and shortage cost. The classical analysis builds a model of an inventory system and calculates the EOQ which minimize these three costs so that their sum is satisfying minimization criterion. One of the unrealistic assumptions is that items stocked preserve their physical characteristics during their stay in inventory. Items in stock are subject to many possible risks, e.g. damage, spoilage, dryness; vaporization etc., those results decrease of usefulness of the original one and a cost is incurred to account for such risks. This model considers a continuous review, using fuzzy arithmetic approach to the system cost for instantaneous production process. In traditional inventory models it has been common to apply fuzzy on demand rate, production rate and deterioration rate, whereas applying fuzzy arithmetic in system cost usually ignored in Salameh *et al.* [1]. From practical experience, it has been found that uncertainty occurs not only due to lack of information but also as a result of ambiguity concerning the description of the semantic meaning of declaration of statements relating to an economic world. The fuzzy set theory was developed on the basis of non-random uncertainties. Vujosevic *et al.* [2] introduced the EOQ model where inventory system cost is fuzzy. Mahata and Goswami [3] then presented production lot size model with fuzzy production rate and fuzzy demand rate for deteriorating items where permissible delay in payments are allowed. Roy and Maiti [4] presented fuzzy EOQ model with demand dependent unit cost under limited storage capacity. The model provides an approach for quantifying the benefits of nonrandom uncertainty which can be substantial, and should be reflected in fuzzy arithmetic system cost.

The predominant criterion in traditional inventory models is minimization of long-run average cost per unit time. The costs considered are usually fixed ordering cost and holding cost. Costs associated with disorder in a system tied up in inventory are accounted for by including an entropy cost in the total costs. Entropy is frequently defined as the amount of disorder in a system. Jaber *et al*. [15] proposed an analogy between the behavior of production system and the behaviour of physical system. This paper introduced the concept of entropy cost to account for hidden cost such as the additional managerial cost that is needed to control the improvement process.

Product perishability is an important aspect of inventory control. Deterioration in general, may be considered as the result of various effects on stock, some of which are damage, decay, decreasing usefulness and many more. While kept in store fruits, vegetables, food stuffs etc. suffer from depletion by decent spoilage. Decaying products are of two types. Product which deteriorate from the very beginning and the products which start to deteriorate after a certain time. Lot of articles is available in inventory literature considering deterioration. Interested readers may consult the survey model of Pattnaik [5] investigated an entropic order quantity model for perishable items with pre and post deterioration discounts under two component demands in finite horizon. Raafat [6] surveyed for perishable items to optimize the EOQ model. The EOQ inventory control model was introduced in the earliest decades of this century and is still widely accepted by many industries today. Pattnaik [7, 8] derived different types of typical deterministic EOQ models in crisp and fuzzy decision space.

In previous deterministic inventory models, many are developed under the assumption that demand is either constant or stock dependent for deteriorated items. Jain and Silver [9] developed a stochastic dynamic programming model presented for determining the optimal ordering policy for a perishable or potentially obsolete product so as to satisfy known time-varying demand over a specified planning horizon. They assumed a random lifetime perishability, where, at the end of each discrete period, the total remaining inventory either becomes worthless or remains usable for at least the next period. Hariga [10] considered the effects of inflation and the time-value of money with the assumption of two inflation rates rather than one, i.e. the internal (company) inflation rate and the external (general economy) inflation rate. Hariga [11] proposed the correct theory for the problem supplied with numerical examples. The most recent work found in the literature is that of Hariga [12] who extended his earlier work by assuming a time-varying demand over a finite planning horizon. Goyal *et al*. [13] and Shah [14] explored the inventory models for deteriorating items.

This model establishes and analyzes the fuzzy inventory model under profit maximization which extends the classical economic order quantity (EOQ) model. An efficient FEOQ does more than just reduce cost. It also creates revenue for the retailer and the manufacturer. The evolution of the FEOQ model concept tends toward revenue and demand focused strategic formation and decision making in business operations. Evidence can be found in the increasingly prosperous revenue and yield management practices and the continuous shift away from supply-side cost control to demand-side revenue stimulus.

All mentioned above inventory literatures with deterioration has the basic assumption that the retailer owns a storage room with optimal order quantity. In recent years, companies have started to recognize that a tradeoff exists between product varieties in terms of quality of the product for running in the market smoothly. In the absence of a proper quantitative model to measure the effect of product quality of the product, these companies have mainly relied on qualitative judgment. The problem consists of the optimization of fuzzy EOQ model, taking into account the conflicting payoffs of the different decision makers involved in the process. Numerical experiment is carried out to analyze the magnitude of the approximation error. A policy iteration algorithm is designed and optimum solution is obtained through LINGO 13.0 version software. Finally, sensitivity analyses of the optimal solution with respect to the major parameters are also studied to draw the managerial insights. In order to make the comparisons equitable a particular evaluation function based dynamic ordering cost is suggested. In this model, replenishment decision under none wasting the percentage of on-hand inventory due to deterioration are adjusted arbitrarily upward or downward for fuzzy profit maximization model in response to the change in market demand within the finite planning horizon with dynamic ordering cost. The objective of this model is to determine optimal replenishment quantities in an instantaneous fuzzy profit maximization model. However, adding of dynamic ordering cost in fuzzy model might lead to super gain for the retailer.

In the present paper, ordering cost is a variable cost, entropy cost is considered and time is the decision variable and the goal is to determine the optimal profit as well as the lot size using fuzzy optimization procedure. The major assumptions used in the above research articles are summarized in Table1.

Table-1: Summary of the Related Researches

Author (s) and published Year	Structure of the model	Demand	Demand patterns	Deterioration	Units Lost due Deterioration	Ordering Cost	Entropy Cost	Planning	Model
Hariga (1994)	Crisp (EOQ)	Time	Non-stationary	Yes	No	Constant	No	Finite	Cost
Tsao *et al.* (2008)	Crisp (EOQ)	Time and Price	Linear and Decreasing	Yes	No	Constant	No	Finite	Profit
Pattnaik (2009)	Crisp (EnOQ)	Constant (Deterministic)	Constant	Yes (Instant)	No	Constant	No	Finite	Profit
Present model (2017)	Fuzzy (FEnOQ)	Constant (Deterministic)	Constant	Yes	No	Flexible	Yes	Finite	Profit

The remainder of this paper is organized as follows: in Section 2 assumptions and notations are provided for the development of the model. The mathematical model is formulated in Section 3 and then fuzzy mathematical model is developed in Section 4. The solution procedure is derived in Section 5. In Section 6, numerical example is presented to illustrate the expansion of the model. The comparative analysis is carried out to validate the model with present assumptions in Section 7. The sensitivity analyses and ANOVA testing are studied in Section 8 to observe the sensitivity of the major parameters with respect to the fuzzy net profit. Finally in Section 9 the summary and the concluding remarks are explained through proper interpretations and managerial implications.

Assumptions and Notations

r Consumption rate,
t_c Cycle length,
h Holding cost of one unit for one unit of time,
HC (q) Holding cost per cycle,
c Purchasing cost per unit,
P_s Selling Price per unit,
α Percentage of on-hand inventory that is lost due to deterioration,
q Entropic Order quantity,
$K \times (q^{\gamma-1})$ Ordering cost per cycle where, $0 < \gamma < 1$,
q* Traditional economic ordering quantity (EOQ),

EC Entropy Cost, Entropy generation rate must satisfy $S = \frac{d\sigma(t_c)}{dt_c}$ where, $\sigma(t_c)$ is the total entropy generated by time t_c and S is the rate at which entropy is generated. The entropy cost is computed by dividing the total commodity flow in a cycle of duration t_c. The total entropy generated over time t_c as $\sigma(t_c) = \int_0^{t_c} S dt$, $S = \frac{r}{P_s}$, Entropy cost per cycle is EC = $(EC)_{\text{Without deterioration}} = \frac{r}{\sigma(t_c)}$

$(EC)_{\text{With deterioration}} = \frac{q_{\text{With deterioration}}}{\sigma(t_c)}$. EC is measured in an appropriate price unit with no deterioration and with deterioration respectively for two different profit models.

φ(t) On-hand inventory level at time t,
$\pi_1(q)$ Net profit per unit of producing q units per cycle in crisp strategy,
π (q) Average profit per unit of producing q units per cycle in crisp strategy,
$\tilde{\pi}_1(q, \rho)$ The net profit per unit per cycle in fuzzy decision space,
$\tilde{\pi}$ (q, ρ) The average profit per unit per cycle in fuzzy decision space,
$\tilde{h}$ Fuzzy holding cost per unit,

$\widetilde{K} \times (q^{\gamma-1})$ Fuzzy setup cost per cycle.

Mathematical Model

Denote φ(t) as the on-hand inventory level at time t. During a change in time from point t to t+dt, where t + dt > t, the on-hand inventory drops from φ(t) to φ(t+dt). Then φ(t+dt) is given as Salameh *et al.* [1]:

$\varphi(t+dt) = \varphi(t) - r\,dt - \alpha\,\varphi(t)\,dt$

Equation φ(t+dt) can be re-written as: $\frac{\varphi(t+dt)-\varphi(t)}{dt} = -r - \alpha\varphi(t)$

and dt → 0, equation $\frac{\varphi(t+dt)-\varphi(t)}{dt}$ reduces to: $\frac{d\varphi(t)}{dt} + \alpha\varphi(t) + r = 0$.

It is a differential equation, solution is $\varphi(t) = \frac{-r}{\alpha} + \left(q + \frac{r}{\alpha}\right) \times e^{-\alpha t}$

Where q is the order quantity which is instantaneously replenished at the beginning of each cycle of length t_c units of time. The stock is replenished by q units each time these units are totally depleted as a result of outside demand and deterioration. Behavior of the inventory level for the above model is illustrated in Figure 1. The cycle length, t_c, is determined by first substituting t_c into equation φ(t) and then setting it equal to zero to get: $t_c = \frac{1}{\alpha}\ln\left(\frac{\alpha q + r}{r}\right)$

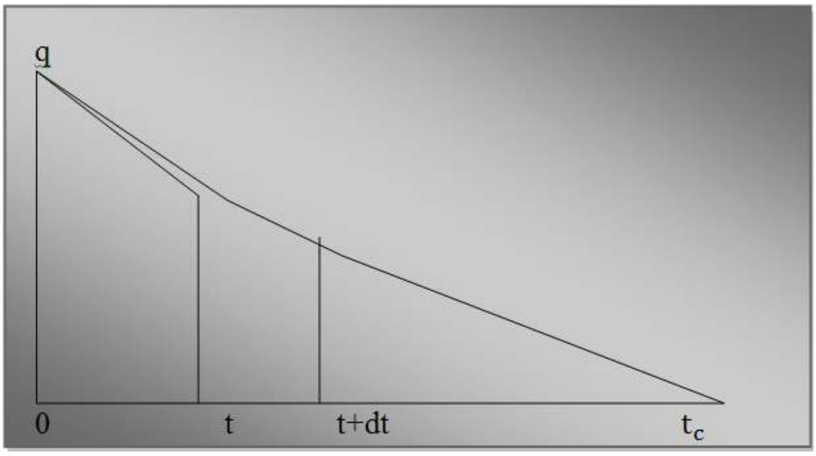

Fig-1: Behavior of the Inventory over a Cycle for a Deteriorating Item

Equation φ(t) and t_c are used to develop the mathematical model. It is worthy to mention that as α approaches to zero, t_c approaches to $\frac{q}{r}$.

The total cost per cycle, TC (q), is the sum of the ordering cost per order OC, procurement cost per cycle PC, the holding cost per unit per cycle HC and entropy cost per cycle EC.

$OC = K \times q^{\gamma-1}, PC = cq$ and HC (q) is obtained from the equation φ(t) as:

$$HC(q) = \int_0^{tc} h\varphi(t)dt = h\int_0^{\frac{1}{\alpha}ln\left(\frac{\alpha q+r}{r}\right)} \left[-\frac{r}{\alpha} + \left(q + \frac{r}{\alpha}\right) \times e^{-\alpha t}\right] dt$$

$$= h \times \left[\frac{q}{\alpha} - \frac{r}{\alpha^2} ln\left(\frac{\alpha q + r}{r}\right)\right]$$

Entropy cost in the cycle is $EC = (EC)_{\text{without deterioration}} + (EC)_{\text{with deterioration}}$

$$= \frac{r}{\sigma(t_c)} + \frac{q_{\text{ with Deterioration}}}{\sigma\ (t_c)}$$. Where $\sigma(t_c) = \int_0^{t_c} S dt = \int_0^{t_c} \frac{r}{P_s} dt = \frac{rt_c}{P_s}$

So, $EC = \frac{P_s}{t_c} + \frac{qP_s}{rt_c}$

$$TC(q) = OC + PC + HC + EC = \left(K \times q^{(\gamma-1)}\right) + cq + h \times \left[\frac{q}{\alpha} - \frac{r}{\alpha^2} \times ln\left(\frac{\alpha q + r}{r}\right)\right] + \frac{P_s}{t_c} + \frac{qP_s}{rt_c}$$

The total cost per unit of time, TCU (q), is given by dividing equation TC(q) by equation t_c to give: $TCU(q) = \frac{TC(q)}{t_c} = \left[\left(K \times q^{(\gamma-1)}\right) + cq + h \times \left[\frac{q}{\alpha} - \frac{r}{\alpha^2} ln\left(\frac{\alpha q + r}{r}\right)\right] + \frac{P_s}{t_c} + \frac{qP_s}{rt_c}\right] \times \left[\frac{1}{\alpha}\ln\left(\frac{\alpha q + r}{r}\right)\right]^{-1}$

As α approaches zero in equation TC (q) and TCU (q) reduces to TC (q) without deterioration and TC (q) with deterioration. $TC(q) without\ deterioration = \left(K \times q^{(\gamma-1)}\right) + cq + \frac{hq^2}{2r} + \frac{P_s}{t_c}$ and $\mathrm{TCU(q)without\ deterioration} = Krq^{(\gamma-2)} + cr + \frac{hq}{2} + \frac{P_s}{t_c^2}$ respectively, and $TC(q) with\ deterioration = \left(K \times q^{(\gamma-1)}\right) + cq + \frac{hq^2}{2r} + \frac{qP_s}{rt_c}$ and $TCU\ \mathrm{(q)with\ deterioration} = Krq^{(\gamma-2)} + cr + \frac{hq}{2} + \frac{qP_s}{rt_c^2}$ respectively. The total profit per cycle without deterioration is π_1(q).

$$\pi_1(q) = q \times P_s - TC(q) = (q \times P_s) - \left(K \times q^{(\gamma-1)}\right) - cq - \frac{hq^2}{2r} - \frac{P_s}{t_c}$$

Where, TC (q) the total cost per cycle without deterioration. The average profit π(q) per unit time without deterioration is obtained by dividing t_c in π_1(q). The total profit per cycle with deterioration is π_1(q).

$$\pi_1(q) = q \times P_s - TC(q) = (q \times P_s) - \left(K \times q^{(\gamma-1)}\right) - cq - \frac{hq^2}{2r} - \frac{qP_s}{rt_c}$$

Where, TC (q) the total cost per cycle with deterioration. The average profit π(q) per unit time with deterioration is obtained by dividing t_c in π_1(q).

Hence the profit maximization problem without deterioration is:
Maximize π_1 (q)
$\forall\, q \geq 0.$
$\pi_1(q) = F_1(q) + F_2(q)h + F_3(q)K$
Where,
$F_1(q) = (q \times P_s) - cq - \frac{P_s}{t_c}$, $F_2(q) = -\left[\frac{q^2}{2r}\right]$, and $F_3(q) = -q^{(\gamma-1)}$

Fuzzy Mathematical Models

The holding cost and constant of ordering cost are replaced by fuzzy numbers $\tilde{h}$ and $\tilde{K}$ respectively. By expressing $\tilde{h}$ and $\tilde{K}$ as the normal triangular fuzzy numbers (h_1, h_0, h_2) and (K_1, K_0, K_2) where, $h_1 = h - \Delta_1$, $h_0 = h$, $h_2 = h + \Delta_2$, $K_1 = K - \Delta_3$, $K_0 = K, K_2 = K + \Delta_4$, such that $0 < \Delta_1 < h, 0 < \Delta_2, 0 < \Delta_3 < K, 0 < \Delta_4; \Delta_1, \Delta_2, \Delta_3\ and\ \Delta_4$ are determined by the decision maker based on the uncertainty of the problem.

The membership function of fuzzy holding cost and fuzzy ordering cost constant are considered as:

$$\mu_{\tilde{h}}(h) = \begin{cases} \dfrac{h-h_1}{h_0-h_1}, & h_1 \leq h \leq h_0 \\ \dfrac{h_2-h}{h_2-h_0}, & h_0 \leq h \leq h_2 \\ 0 & \text{, otherwise} \end{cases}$$

$$\mu_{\tilde{k}}(K) = \begin{cases} \dfrac{K-K_1}{K_0-K_1}, & K_1 \leq K \leq K_0 \\ \dfrac{K_2-K}{K_2-K_0}, & K_0 \leq K \leq K_2 \\ 0 & \text{, otherwise} \end{cases}$$

Then the centroid for $\tilde{h}$ and $\tilde{K}$ are given by

$$M_{\tilde{h}} = \frac{h_1 + h_o + h_2}{3} = h + \frac{\Delta_2 - \Delta_1}{3} \text{ and } M_{\tilde{k}} = \frac{K_1 + K_o + K_2}{3} = K + \frac{\Delta_4 - \Delta_3}{3} \text{ respectively.}$$

For fixed value of q, let $\pi_1(h, K) = F_1(q) + F_2(q)h + F_3(q)K = y$.

Let $h=\dfrac{y-F_1-F_3K}{F_2}$, $\dfrac{\Delta_2-\Delta_1}{3}=\psi_1$ and $\dfrac{\Delta_4-\Delta_3}{3}=\psi_2$

By extension principle the membership function of the fuzzy profit function is given by

$$\mu_{\pi_1(\tilde{h},\tilde{k})}(y)=\sup_{(h,k)\in\pi_1^{-1}(y)}\left\{\mu_{\tilde{h}}(h)\wedge\mu_{\tilde{k}}(K)\right\}$$

$$=\sup_{k_1\le k\le k_2}\left\{\mu_{\tilde{h}}\left(\frac{y-F_1-F_3K}{F_2}\right)\wedge\mu_{\tilde{k}}(K)\right\}$$

Now,

$$\mu_{\tilde{h}}\left(\frac{y-F_1-F_3K}{F_2}\right)=\begin{cases}\dfrac{y-F_1-F_2h_1-F_3K}{F_2(h_0-h_1)} & , \quad u_2\le K\le u_1\\ \dfrac{F_1+F_2h_2+F_3K-y}{F_2(h_2-h_0)} & , \quad u_3\le K\le u_2\\ 0 & , \quad \text{otherwise}\end{cases}$$

where, $u_1=\dfrac{y-F_1-F_2h_1}{F_3}$, $u_2=\dfrac{y-F_1-F_2h_0}{F_3}$ and $u_3=\dfrac{y-F_1-F_2h_2}{F_3}$

It is clear that for every $y\in[F_1+F_2h_1+F_3K_1,\ F_1+F_2h_0+F_3K_0]$, *there exists* $\mu_y(y)=PP'$. From $\mu_{\tilde{h}}(h)$ and $\mu_{\tilde{h}}(\frac{y-F_1-F_3K}{F_2})$ the value of PP' may be found by solving the following equation:

$$\frac{K-K_1}{K_0-K_1}=\frac{y-F_1-F_2h_1-F_3K}{F_2(h_0-h_1)} \text{ or } \quad K=\frac{(y-F_1-F_2h_1)(K_0-K_1)+F_2K_1(h_0-h_1)}{F_2(h_0-h_1)+F_3(K_0-K_1)}$$

Therefore, $PP'=\dfrac{K-K_1}{K_0-K_1}=\dfrac{y-F_1-F_2h_1-F_3K}{F_2(h_0-h_1)+F_3(K_0-K_1)}=\mu_1(y)$, (say).

It is clear that for every $y\in[F_1+F_2h_2+F_3K_2,\ F_1+F_2h_0+F_3K_0]$, *there exists* $\mu_y(y)=PP''$. From $\mu_{\tilde{h}}(h)$ and $\mu_{\tilde{h}}(\frac{y-F_1-F_3K}{F_2})$ the value of PP'' may be found by solving the following equation:

$$\frac{K_2-K}{K_2-K_0}=\frac{F_1+F_2h_2+F_3K_2-y}{F_2(h_2-h_0)} \text{ or } \quad K=\frac{F_2K_2(h_2-h_0)-(F_1+F_2h_2-y)(K_2-K_0)}{F_2(h_2-h_0)+F_3(K_2-K_0)}$$

Therefore, $PP''=\dfrac{K_2-K}{K_2-K_0}=\dfrac{F_1+F_2h_2+F_3K_2-y}{F_2(h_2-h_0)+F_3(K_2-K_0)}=\mu_2(y)$, (say).

Thus the membership function for fuzzy total profit is given by

$$\mu_{\pi_1(\tilde{h},K)}(y)=\begin{cases}\mu_1(y); & F_1+F_2h_1+F_3K_1\le y\le F_1+F_2h_0+F_3K_0\\ \mu_2(y); & F_1+F_2h_0+F_3K_0\le y\le F_1+F_2h_2+F_3K_2\\ 0\ ; & \text{otherwise}\end{cases}$$

Now, let $P_1=\int_{-\infty}^{\infty}\mu_{\pi_1(\tilde{h},\tilde{K})}(y)\,dy$ and $R_1=\int_{-\infty}^{\infty}y\,\mu_{\pi_1(\tilde{h},\tilde{K})}(y)\,dy$

Hence, the centroid for fuzzy total profit is given by $\tilde{\pi}_1(q)=M_{\widetilde{TP}}(q)=\frac{R_1}{P_1}=F_1(q)+F_2(q)h+F_3(q)K+\Psi_1F_2(q)+\Psi_2F_2(q)=F_1+(h+\Psi_1)F_2+(K+\Psi_2)F_3$

Where, $F_1(q), F_2(q)$ *and* $F_3(q)$ are given by the equations.
Hence the profit maximization problem is

Maximize $\tilde{\pi}_1(q) = M_{\widetilde{TP}}(q) \;\; \forall q \geq 0.$

Optimization

The optimal ordering quantity q per cycle can be determined by differentiating equation $\tilde{\pi}_1(q)$ with respect to q, then setting these to zero.

In order to show the uniqueness of the solution in, it is sufficient to show that the net profit function throughout the cycle is concave in terms of ordering quantity q. The second order derivates of the equation $\tilde{\pi}_1(q)$ with respect to q are strictly negative. Consider the following proposition.

Proposition 1 The net profit $\tilde{\pi}_1(q)$ per cycle is concave downward or convex upward in q. Conditions for optimal q is $\frac{d\tilde{\pi}_1(q)}{dq} = (P_s) - \left((K + \Psi_2)(\gamma - 1)q^{\gamma-2} + \frac{c(h+\Psi_1)q}{r}\right) = 0$

The second order derivative of the net profit per cycle with respect to q can be expressed as: $\frac{d^2\tilde{\pi}_1(q)}{dq^2} = -(K + \Psi_2)(\gamma - 1)(\gamma - 2)q^{\gamma-3} - \frac{(h+\Psi_1)c}{r}$ Since, r > 0, $(h + \Psi_1) > 0, (K + \Psi_2) > 0, q > 0, c > 0$ and $0 < \gamma < 1$ so the equation $\frac{d^2\tilde{\pi}_1(q)}{dq^2}$ is negative.

Proposition 1 shows that the second order derivative of equation $\tilde{\pi}_1(q)$ with respect to q are strictly negative.

The objective is to determine the optimal values of q to maximize the unit profit function of$\tilde{\pi}_1(q)$. It is very difficult to derive the optimal values of q, hence unit profit function. There are several methods to cope with constraints optimization problem numerically. But here LINGO 13.0 software is used to derive the optimal values of the decision variable.

Numerical Examples

Consider an inventory situation where K is Rs. 200 per order, h is Rs. 5 per unit per unit of time, r is 8 units per unit of time, c is Rs. 100 per unit, the selling price per unit P_s is Rs. 125, γ is 0.5, $\Delta_1 = 0.002$, $\Delta_2 = 0.02$, $\Delta_3 = 0.002$ and Δ_4=0.2 and α is 0%, . Figure 2 represents the relationship between the order quantity q and dynamic setup cost OC. Figures 3a and 3b show the concavity of the net profit per cycle function with respect to different EnOQ and t_c for fuzzy and crisp, without deterioration and variable ordering cost. Figures 4a and 4b show the concavity of the net profit per cycle function with respect to different EnOQ and t_c for fuzzy and crisp, with deterioration and variable ordering cost. Figures 5a and 5b show the concavity of the net profit per cycle function with respect to different EnOQ and t_c for fuzzy and crisp, without deterioration and fixed ordering cost. Figures 6a and 6b show the concavity of the net profit per cycle function with respect to different EOQ and t_c for fuzzy and crisp, without deterioration and variable ordering cost. Figures 7a and 7b show the concavity of the net profit per cycle function with respect to different EOQ and t_c for fuzzy and crisp, with deterioration and variable ordering cost. The optimal solution that maximizes equation $\tilde{\pi}_1(q), t_c^*$ and q^* for with and without deterioration are determined by using LINGO 13.0 version software and the results are tabulated in Table 2.

Table-2: Optimal Values of the Proposed Model

Model	Iteration	$t_c{}^*$	Entropy Cost	EnOQ (q)	Flexible OC	$\tilde{\pi}_1(q)$	$\tilde{\pi}(q)$
Fuzzy, without Deterioration	52	5.184912	24.10841	41.47929	31.06402	443.4985	85.53637
Fuzzy, with Deterioration	40	5.071335	125	40.57068	31.40994	342.8710	67.60963
% Change	-	2.190528981	418.4912651	2.190514833	1.113571263	22.68947922	20.95803224

Comparative Analyses

The present model of fuzzy entropic order quantity model without deterioration and with variable ordering cost is compared with other different models with fuzzy or crisp, with or without deterioration, fixed or variable ordering cost and traditional EOQ or EnOQ cases. All the results of different models are compared with the present model and respective fuzzy and crisp models are also compared In Table 3. It is observed that the average profit per cycle for the present model is approximately good for this flexible model. The present model is more flexible as the holding cost per unit per cycle and the constant of variable ordering cost are fuzzy in nature; the hidden entropy cost is taking account without deterioration to frame the model. For the third and fourth case there is a 20.43% and 26% decrease from the present model respectively. The average profit per cycle in the fifth and sixth case 37.22% and 37.12% decrease from the average profit per cycle of the present model respectively. But for the case of seventh, eighth, ninth and tenth the average profit per cycle 7.87% approximately increase in percentage from the average profit per cycle of the present model respectively where the hidden cost that is entropy cost is not taken into account to frame the total inventory cost.

Table-3: Comparative Analysis of Different Models

Sl. No.	Model	Iteration	t_c^*	Entropy Cost	EnOQ (q)	Flexible OC	$\tilde{\pi}_1(q)$	$\tilde{\pi}(q)$
1	Fuzzy, without Deterioration	52	5.184912	24.10841	41.47929	31.06402	443.4985	85.53637
2	Crisp, without Deterioration	52	5.190723	24.08142	41.52578	31.03638	444.1547	85.56702
	% Change (1)	-	-0.11208	0.111953	-0.112080	0.088978	-0.147951	-0.035833
3	Fuzzy, with Deterioration	40	5.071335	125	40.57068	31.40994	342.8710	67.60963
	% Change (1)	-	2.190529	-418.491265	2.190515	-1.11357	22.68948	20.958032
4	Crisp, with Deterioration	40	5.077259	125	40.61807	31.38126	343.4994	67.65448
	% Change	-	- 0.1168134	0	-0.116808	0.091308	-0.183276	-0.066337
	% Change (1)	-	2.0762744	-418.491265	2.07626505	-1.02125	22.547788	26.431199
5	Fuzzy, without Deterioration	57	5.113382	24.44566	40.90706	Fixed OC, 200.066	274.6037	53.70295
	% Change (1)	-	1.3795798	-1.3988894	1.3795559	-544.044	38.082384	37.21624
6	Crisp, without Deterioration	54	5.119244	24.41767	40.95396	Fixed OC, 200	275.2979	53.77707
	% Change	-	-0.114640	0.114499	0.8223030	0.032989	-0.252800	-0.138018
	% Change (1)	-	1.266521	-1.282788	1.2664874	-543.832	37.92586	37.12959
7	Fuzzy, without Deterioration	35	5.071335	-	EOQ, 40.57068	31.40994	467.8710	92.25797
	% Change (1)	-	2.1905281	-	2.1905148	-1.11357	-5.49551	- 7.8581778
8	Crisp, without Deterioration	36	5.077259	-	EOQ, 40.61807	31.38126	468.4994	92.27407
	% Change	-	-0.116813	-	-0.1168084	0.091309	-0.134311	-0.017451
	% Change (1)	-	2.076274	-	2.0762650	-1.02125	-5.637201	- 7.8770001
9	Fuzzy, with Deterioration	35	5.071335	-	EOQ, 40.57068	31.40994	467.8710	92.25797
	% Change	-	2.190529	-	2.190515	-1,11357	-5.49551	- 7.8581777
10	Crisp, with Deterioration	36	5.077259	-	EOQ, 40.61807	31.38126	468.4994	92.27407
	% Change	-	-0.116813	-	0.0913087	0.091309	-0.134311	-0.017451
	% Change (1)	-	2.0762743	-	2.0762650	-1.02125	-5.637201	- 7.8770001

Sensitivity Analyses

It is interesting to investigate the influence of the major parameters$\widetilde{K}$, $\tilde{h}$, r, c, P_s and γ on retailer's behavior. The computational results shown in Table 4 indicate the following managerial phenomena:

- $t_c$the replenishment cycle length, q the optimal replenishment quantity, EC the entropy cost , $\tilde{\pi}_1$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are insensitive to the parameter $\widetilde{K}$ but OC variable setup cost is sensitive to the parameter $\widetilde{K}$.
- q the optimal replenishment quantity, EC the entropy cost, OC variable setup cost and $\tilde{\pi}_1$ the optimal net profit per unit per cycle are sensitive to the parameter $\tilde{h}$ but $t_c$the replenishment cycle length and $\tilde{\pi}$ the optimal average profit per unit per cycle are moderately sensitive to the parameter $\tilde{h}$.
- q the optimal replenishment quantity, OC variable setup cost, EC the entropy cost, $\tilde{\pi}_1$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are sensitive to the parameter r but $t_c$the replenishment cycle length is insensitive to the parameter r.
- q the optimal replenishment quantity, $\tilde{\pi}_1$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are sensitive to the parameter c but EC the entropy cost and OC variable setup cost are insensitive and $t_c$the replenishment cycle length is moderately sensitive to the parameter c.
- t_cThe replenishment cycle length and q the optimal replenishment quantity are insensitive to the parameter P_s. OC variable setup cost and EC the entropy cost are moderately sensitive to the parameter P_s but $\tilde{\pi}_1$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are sensitive to the parameterP_s.
- t_cThe replenishment cycle length , q the optimal replenishment quantity and EC the entropy cost are insensitive to the parameter γ.$\tilde{\pi}_1$ the optimal net profit per unit per cycle, $\tilde{\pi}$ the optimal average profit per unit per cycle and OC variable setup cost is sensitive to the parameter γ.

The computational results shown in Table 5 indicate the following managerial phenomena:

- $t_c$the replenishment cycle length, q the optimal replenishment quantity, EC the entropy cost are insensitive to the parameter $\widetilde{K}$. $\tilde{\pi}_1$ the optimal net profit per unit per cycle, $\tilde{\pi}$ the optimal average profit per unit per cycle and OC variable setup cost are sensitive to the parameter $\widetilde{K}$.
- $t_c$the replenishment cycle length and EC the entropy cost are insensitive to the parameter $\tilde{h}$ but, q the optimal replenishment quantity, OC variable setup cost, $\tilde{\pi}_1$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are sensitive to the parameter $\tilde{h}$.
- q the optimal replenishment quantity, OC variable setup cost, $\tilde{\pi}_1$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are sensitive to the parameter r but $t_c$the replenishment cycle length and EC the entropy cost are insensitive to the parameter r.
- q the optimal replenishment quantity, $\tilde{\pi}$ the optimal average profit per unit per cycle, EC the entropy cost, OC variable setup cost and t_c the replenishment cycle length are insensitive to the parameter c but $\tilde{\pi}_1$ the optimal net profit per unit per cycle is moderately sensitive to the parameter c.
- $t_c$the replenishment cycle length, q the optimal replenishment quantity, OC variable setup cost, $\tilde{\pi}_1$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are sensitive to the parameter P_s but EC the entropy cost is insensitive to the parameter P_s.
- $t_c$the replenishment cycle length , q the optimal replenishment quantity and EC the entropy cost are insensitive to the parameter γ.$\tilde{\pi}_1$ the optimal net profit per unit per cycle, $\tilde{\pi}$ the optimal average profit per unit per cycle and OC variable setup cost is sensitive to the parameter γ.

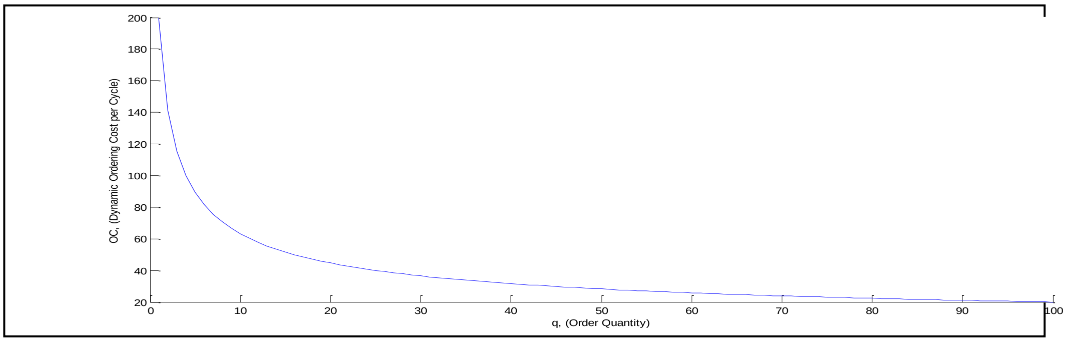

Fig-2: Two Dimensional Plot of Order Quantity, q and Flexible Ordering Cost, OC

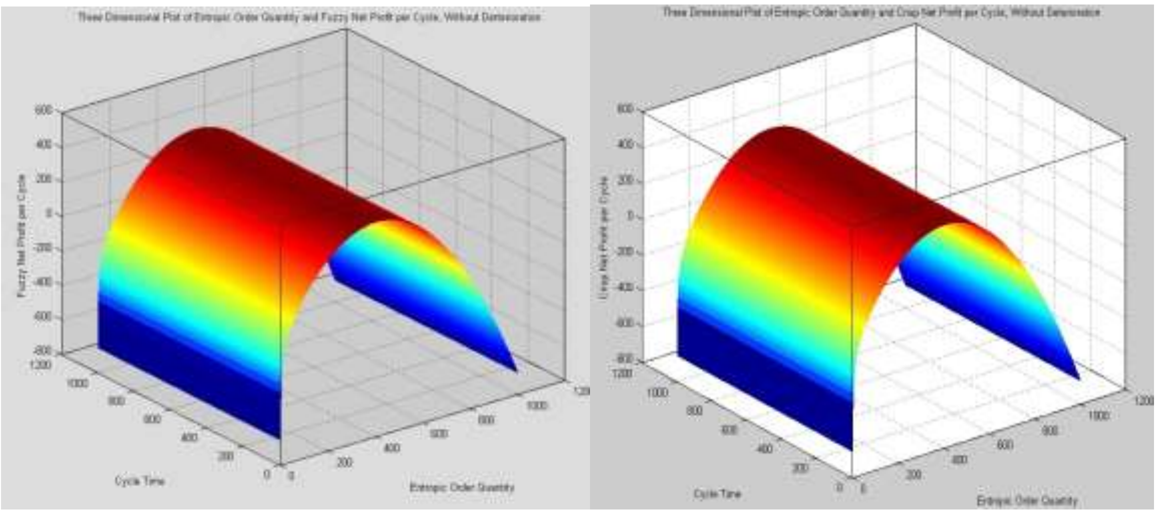

Fig-3a, 3b: Three Dimensional Mesh Plot of Entropic Order Quantity, Cycle Time and (Fuzzy & Crisp) Concavity of Net Profit per Cycle, Without Deterioration & Variable Ordering Cost

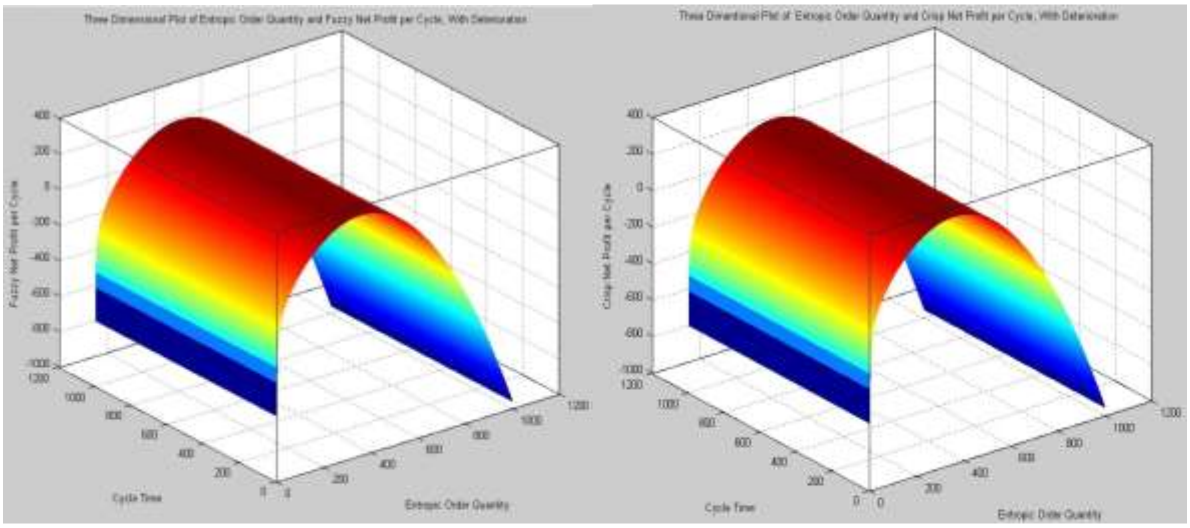

Fig-4a, 4b: Three Dimensional Mesh Plot of Entropic Order Quantity, Cycle Time and (Fuzzy & Crisp) Concavity of Net Profit per Cycle, With Deterioration & Variable Ordering Cost

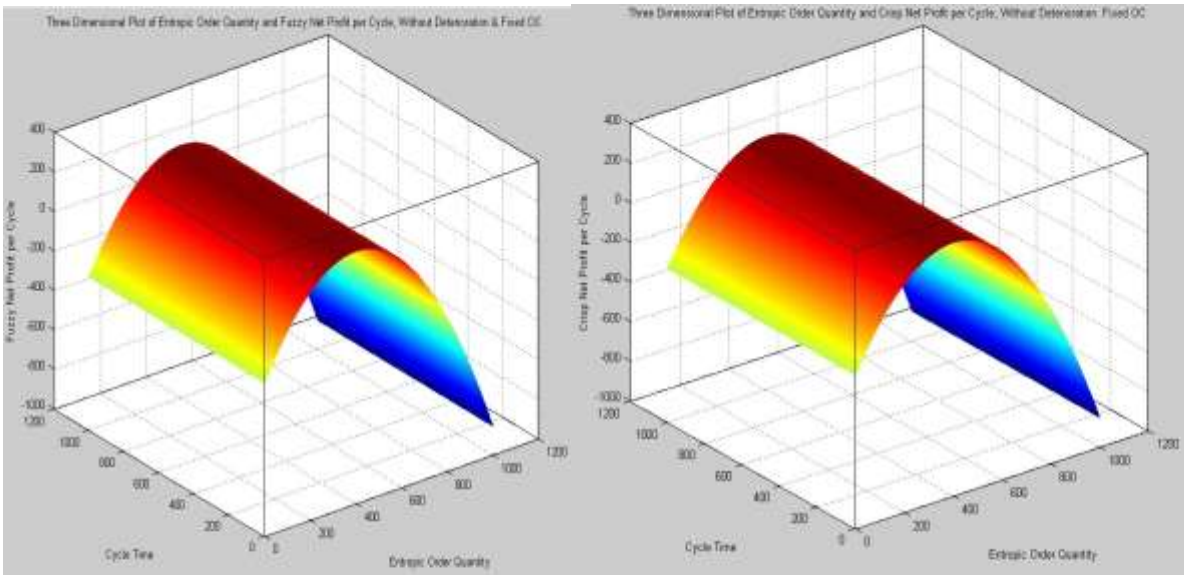

Fig-5a, 5b: Three Dimensional Mesh Plot of Entropic Order Quantity, Cycle Time and (Fuzzy & Crisp) Concavity of Net Profit per Cycle, Without Deterioration & Fixed Ordering Cost

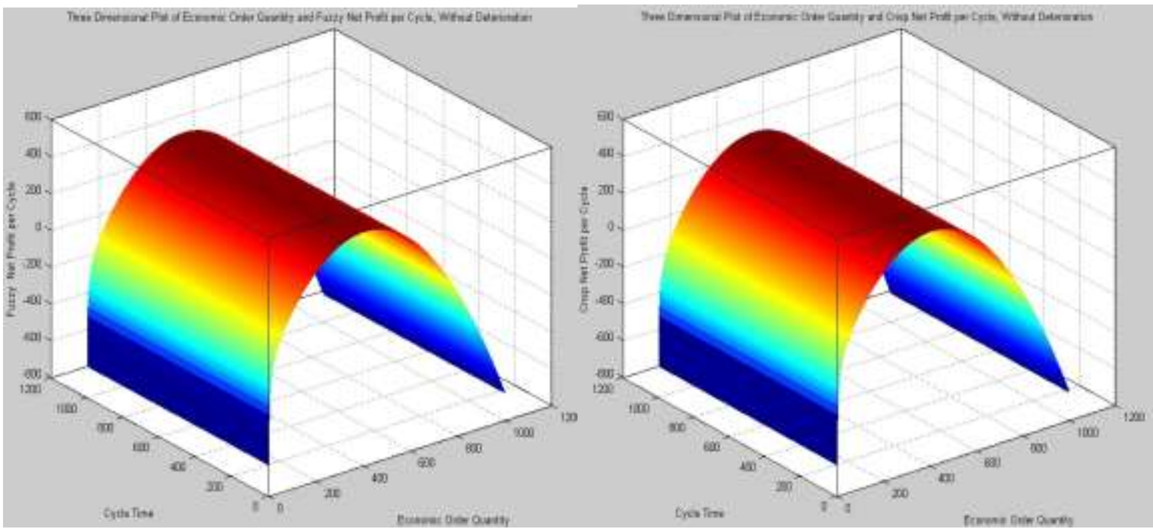

Fig-6a, 6b: Three Dimensional Mesh Plot of Economic Order Quantity, Cycle Time and (Fuzzy & Crisp) Concavity of Net Profit per Cycle, Without Deterioration & Variable Ordering Cost

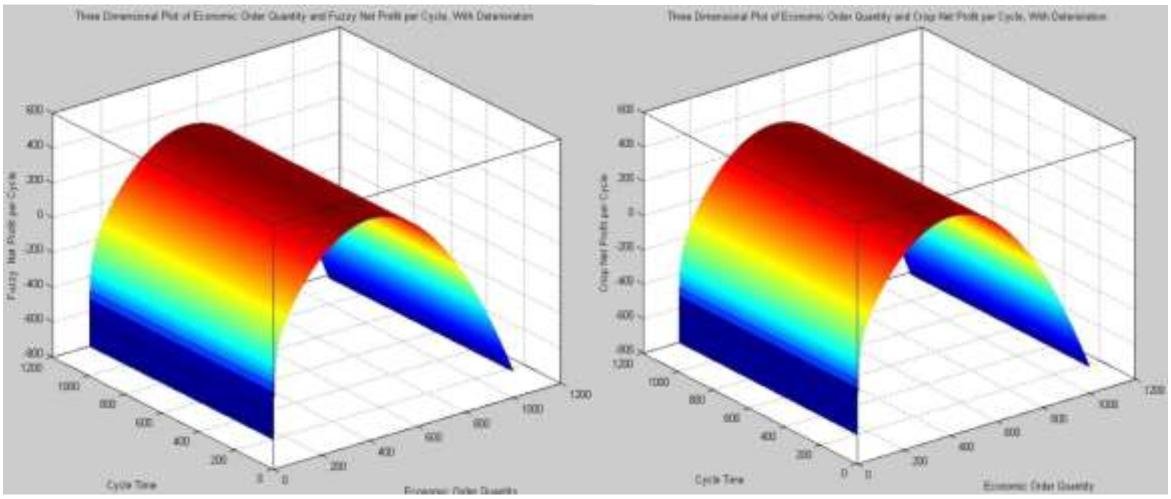

Fig-7a, 7b: Three Dimensional Mesh Plot of Economic Order Quantity, Cycle Time and (Fuzzy & Crisp) Concavity of Net Profit per Cycle, With Deterioration & Variable Ordering Cost

Table-4: Sensitivity Analyses of the Parameters $\tilde{K}$, $\tilde{h}$, r, c, P_s and γ (without Deterioration)

Parameter	Value	Iteration	t^*	q	EC	OC	$\tilde{\pi}_1(q)$	$\tilde{\pi}(q)$
$\tilde{K}$	150	45	5.167274	41.33819	24.19070	23.33004	451.2788	87.33402
	250	42	5.202335	41.61868	24.02767	38.75215	435.7518	83.76081
	500	41	5.287203	42.29763	23.64199	76.87976	397.1576	75.11677
$\tilde{h}$	4.5	50	5.732780	45.86224	21.80443	29.54239	503.6434	87.85325
	5.5	47	4.749820	37.99266	26.32088	32.45813	394.8542	83.14327
	6	39	4.382561	35.06049	28.52213	33.78814	353.2377	80.60075
r	6	40	5.257366	31.54419	23.77617	35.62161	314.1111	59.74687
	7	39	5.161276	36.51001	23.96603	33.11062	379.0377	72.67225
	9	49	5.215715	46.45148	24.21882	29.35442	507.6223	98.35209
c	100.5	49	5.091405	40.73124	24.55118	31.34797	422.9460	83.7058
	101	42	4.998217	39.98573	25.00892	31.63885	402.7669	80.58211
	102	41	4.812873	38.50299	25.97201	32.24230	363.5235	75.53148
P_s	125.5	45	5.279138	42.23311	23.77282	30.78554	464.3310	87.95582
	126	49	5.37362	42.98896	23.44788	30.51370	485.5425	90.35669
	200	47	5.753816	46.03053	22.24611	29.48834	574.1978	99.79436
γ	0.2	46	5.151241	41.20992	24.266	10.21344	464.4262	90.15814
	0.3	56	5.161277	41.29022	24.21881	14.79382	459.8278	89.09187
	0.6	42	5.196195	41.56956	24.05606	45.04671	429.4794	82.65266

Table-5: Sensitivity Analyses of the Parameters $\widetilde{K}$, $\tilde{h}$, r, c, P_s and γ (with Deterioration)

Parameter	Value	Iteration	t^*	Q	EC	OC	$\tilde{\pi}_1(q)$	$\tilde{\pi}(q)$
$\widetilde{K}$	160	52	5.056128	40.44902	125	25.15741	349.1660	69.05799
	250	41	5.090101	40.72081	125	40.8800	335.0388	65.82164
	500	44	5.181151	41.44921	125	27.0098	296.0368	57.13727
$\tilde{h}$	4.5	39	5.629115	45.03292	125	29.81318	400.6450	71.17371
	4.8	41	5.284157	42.27325	125	28.99078	364.9520	69.06533
	6	36	4.886401	39.09121	125	31.9988	323.6416	66.23312
r	6.5	38	5.104677	33.18040	125	34.73222	439.8085	48.25765
	7.5	38	5.084948	38.13711	125	32.3966	246.3397	61.2038
	9	39	5.060029	48.07027	125	28.85592	311.2181	86.91818
c	100.05	40	5.061571	40.49256	125	31.44023	339.4350	67.33966
	100.1	40	5.057733	40.46186	125	31.45215	335.3966	67.11209
	100.2	38	5.038186	40.30549	125	31.51311	340.8445	66.57092
P_s	125.5	40	5.169033	41.35226	125	31.11169	362.8517	70.19722
	126	40	5.26683	42.13464	125	30.82149	383.2234	72.76169
	200	41	19.9591	159.8873	125	15.82218	7774.587	389.0034
γ	0.4	42	5.058346	40.46677	125	21.72272	352.5951	69.70561
	0.6	49	5.083299	40.66639	125	45.44426	328.7968	64.68177
	0.8	47	5.087589	40.70071	125	95.33511	278.8902	54.81777

Sensitivity Analysis through ANOVA Testing

The sensitivity of the fuzzy entropic total profit per cycle with respect to the important parameters like h and K has been presented using the Analysis of Variance (ANOVA) method. The main conclusions drawn from the sensitivity analysis are as follows:

H_{01}: Null Hypothesis: the optimal fuzzy entropic total profit is insignificant for different values of h and K.

H_{11}: Alternative Hypothesis: the optimal fuzzy entropic total profit differs significantly for different values of h and K.

The ANOVA in Table 7 is constructed for the data values of Table 6, it is seen that the calculated values of F for different values of h and K are greater than the tabulated values of $F_{0.05;5,40}$and $F_{0.05;8,40}$(i.e. F-distribution at 5% level), respectively. Hence the optimal fuzzy entropic total profit per cycle differs significantly for different values of h and K.

Table-6: Effect of K and h on TP

K \ h	3.006	4.006	5.006	6.006	7.006	8.006
198.066	682.4315	471.1095	343.1850	257.0989	194.9982	147.9381
199.066	682.3094	470.9688	343.0280	256.9273	194.8134	147.7410
200.066	682.1872	470.8280	342.8710	256.7557	194.6285	147.5439
201.066	682.0651	470.6873	342.7140	256.5842	194.4437	147.3469
202.066	681.9429	470.5466	342.5571	256.4126	194.2588	147.1498
203.066	681.8208	470.4058	342.4001	256.2411	194.0740	146.9528
204.066	681.6986	470.2651	342.2431	256.0695	193.8892	146.7557
205.066	681.5765	470.1244	342.0861	255.8980	193.7044	146.5587
210.066	680.9658	469.4208	341.3014	255.0405	192.7806	145.5738

Table-7: ANOVA table for the results of Table 6

Sources of Variation	Sum of Square (SS)		Degrees of Freedom (df)	Mean Sum of Square (MS)	The Calculated Value of F	The Value of F at 5% Level from Table
h	Sum of squares due to h effects	1790557.537	5	358111.507	34542655.023	2.45
k	Sum of squares due to K effects	16.768	8	2.096	202.17	2.18
Error	Sum of squares due to errors	0.415	40	0.010	-	-
Total	Total sum of squares	1790574.72	53	-	-	-

CONCLUSIONS

In this present paper, a new instantaneous FEnOQ model is introduced which investigates the optimal entropic order quantity with variable ordering cost and assumes that 0% of the on-hand inventory is wasted due to deterioration. These are the significant features and the inventory conditions which regulate the item stocked in an inventory. In this study, the effect of variable ordering cost and entropy cost has been studied. This model provides a useful mathematical property for finding the optimal profit, cycle time and ordering quantity for deteriorated items. The present fuzzy net profit per unit per cycle is more than that of the traditional net profit per unit per cycle for fixed ordering cost traditional model. Hence the utilization of variable setup cost makes the scope of the application broader in fuzzy space. Further, a numerical example is presented to illustrate the present model, and typical observations are identified through sensitivity analysis and comparative analysis. Further investigation of the present model reveals that accounting for entropy cost may be more relevant for the low demand and expensive items, than for high demand and low-priced items. In the future study, it is hoped to further incorporate the proposed model into several situations such as shortages, inflation, are allowed and the consideration of multi-item and multi-objective problems with fuzzy space. Furthermore, it may also take partial backlogging into account for determining the optimal replenishment policy.

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