

# The Modelling between Force & Torque and Crank Angle on Crank Linkage of Engine in Vehicle by Lagrange Formula I

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## Abstract

The torque will be big according to the differential to crank in crank and linkage mechanism by Lagrange formula. That may be fitted to the stroke ratio well. The bigger the one is the bigger the torque is. The crank force is the biggest in this study. The linkage force may incline as the crank mass and linkage length is big. The torque will be big when the stroke ratio is big and the crank mass is inclining on the crank linkage mechanism in vehicle. The force and torque effective turn is Crank angle  $\theta_3 >$  linkage angle  $\theta_2 >$  piston angle  $\theta_1 >$  piston displacement  $s$ . The torque turn in one status is piston  $M_1 >$  crank  $M_3 >$  linkage  $M_2$ .

**Keywords:** property; simulation; crank; crank linkage; stroke ratio; vehicle; engine; kinematic.

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## 1. INTRODUCTION

The crank linkage in engine of vehicle is important mechanism so the simulation may be done to search its properties and their effective value further and details. Because its important role in engine the optimum parameters have been solved to find the intrinsic relations among them is very significant currently [1-5]. However up to now the Kinematic equation of Lagrange formula has been a little status so it is necessary to establish the simulation to analyze the Kinematics of it. Although some searched the simulation to multibody system it is confined at three ordinations profile. They didn't search more detail to every parameter so in this study the role of parameters with movement, linkage angle and acceleration has been investigated with Lagrange formula and try to find the intrinsic relationship among them.

In this study the parameters including properties of engine crank linkage have been investigated in detail with Lagrange formula in multibody system. It is found how the acceleration and angular speed play a role and with their other parameters link. Those properties may be clarified through drawing in order to compare with the torque. It

is hoped that the effect factor which affect the torque further through comparing their value with torque.

In short the torque may be inclined has been our destination and other parameters can promote the torque through regulating the parameter value. We try to find that the parameter deeds to improve the general torque value. The stroke ratio  $\lambda$  is an important parameter in crank linkage mechanism which may affect the engine properties bigly so it is noticed that the reasonable crank lenth  $R$  and linkage  $L$  has been chosen for good combinative properties in engine of vehicle.

## 2. Simulation for kinematic equation in multibody system of crank linkage mechanism

According to Figure 1 which is kinematic graphs on the crank linkage in engine in vehicle. It is supposes that crank  $R=45\text{mm}\sim 60\text{mm}$ , crank linkage  $L=160\text{mm}\sim 210\text{mm}$ . This is the engine driving crank linkage. A is sliding piston and cylinder wall;  $\theta_1, \theta_2$  and  $\theta_3$  is sliding piston, crank linkage and crank angle respectively;  $v$  is its speed;  $n$  is shaft rotation.

The Lagrange equation is

$$\frac{d}{dt} \left( \frac{\partial E_k}{\partial \dot{q}_i} \right) - \frac{\partial E_k}{\partial q_i} + \frac{\partial E_p}{\partial q_i} = F_i, (i=1,2,...,n) \quad (1)$$

Here  $E_k$  is kinetic of system;  $E_p$  is potential energy of system;  $q_i$  is generalized coordinate, it is a group of independent parameters that can define mechanical system movement;  $F_i$  is generalized force,

$q_i$  is a angular displacement or linear displacement;  $n$  is system generalized coordinate.

According to Figure 1, Lagrange formula and kinetic energy and potential energy it has

$$E_k = \frac{1}{2} m_1 v^2 + \frac{1}{2} I_2 \omega_1^2 + \frac{1}{2} I_2 \omega_2^2 + \frac{1}{2} I_3 \omega_3^2 - I_2 \omega_2 \omega_3 \quad (2)$$

$$E_p = m_1 g l \quad (3)$$

The result is as below

$$F_s = m_1 \dot{v} - m_1 v \dot{v} + m_1 g \quad (4)$$

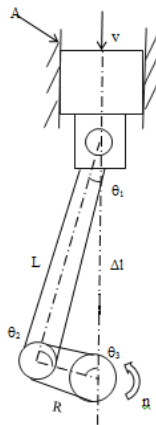
$$F_1 = I_2 \dot{\omega}_1 \omega_1 - I_2 \dot{\omega}_1 + I_2 \omega_1 + m_1 g \quad (5)$$

$$F_2 = I_2 \dot{\omega}_2 \omega_2 - I_2 \dot{\omega}_3 + I_2 \omega_3 + m_1 g \quad (6)$$

$$F_3 = I_3 \dot{\omega}_3 \omega_3 - I_2 \dot{\omega}_2 + I_2 \omega_2 + m_1 g \quad (7)$$

Here rotary inertia is

$$I_i = m_i l_i^2, (i=1,2,\dots,n) \quad (8)$$



**Fig-1: The kinematic of crank linkage linkage length in the engine of vehicle**

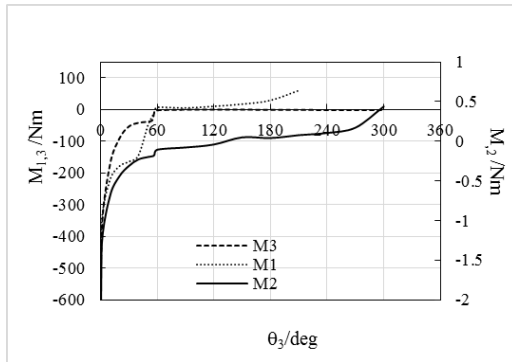
Here  $\omega_1$ ,  $\omega_2$  and  $\omega_3$  is the angular speed for piston, linkage and crank;  $\theta_1$ ,  $\theta_2$  and  $\theta_3$  is the angle for them respectively;  $v$  is piston speed;  $m_1$  is its mass;  $R$  is crank length;  $L$  is linkage length;  $\Delta l$  is displacement  $O_1$  and  $O_3$ .

## 2. DISCUSSIONS

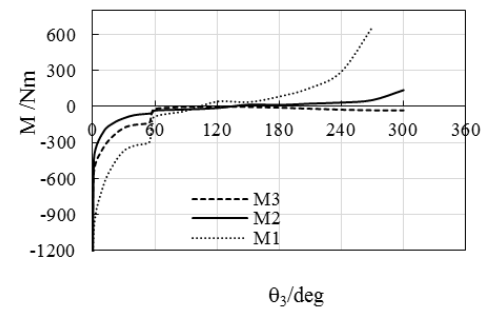
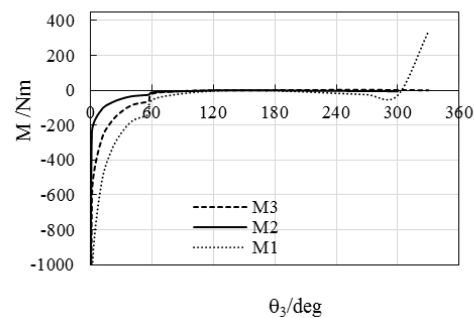
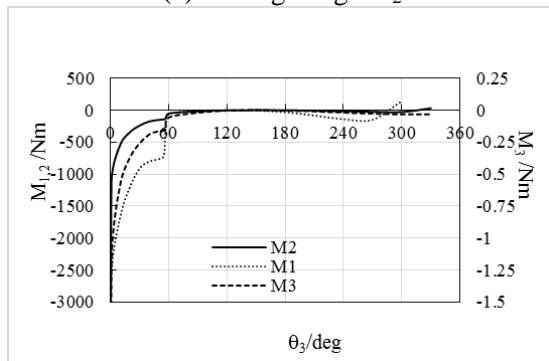
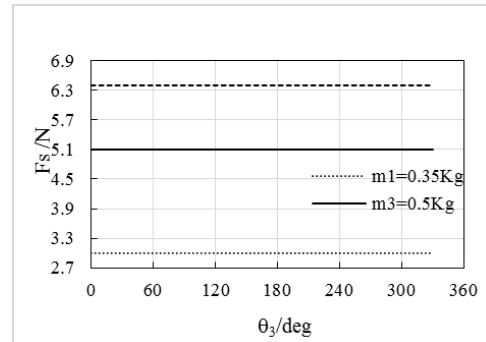
The torque will be in proportion to the piston mass, crank and linkage length in Figure 2(a~d). The torque may increase from 350Nm to 2500Nm with crank linkage changing from piston displacement, piston angle, linkage angle to crank angle. Meantime the troque of crank linkage increases from 350Nm to 600Nm with the the same conditions whilst the torque of crank is the least with 1Nm~0.25Nm which can be neglected as seen in Figure2 (a,d). As seen in Figure 3(a~c) the force will decrease and maintain a parallel line when the crank angle increases. The biggest one is 2800Nm with the crank mass of 0.5Kg as the differential to crank angle. Meantime the one is 850Nm as the diferential to piston angle. In this paper it is supposed that the crank rotation is 780r/m and crank radius is 53mm while linkage length is 195mm and crank mass is 0.35Kg. Meantime the linkage length is supposed to be 160mm and 120mm with crank mass 0.5Kg and 0.64Kg respectively. After Lagrange transformation the force and torque may be solved it is concluded as above and their relation has been drawn as below in this multibody system of engine as seen in Table 1.

**Table-1: The conditions of crank linkage mechanism in engine of vehicle**

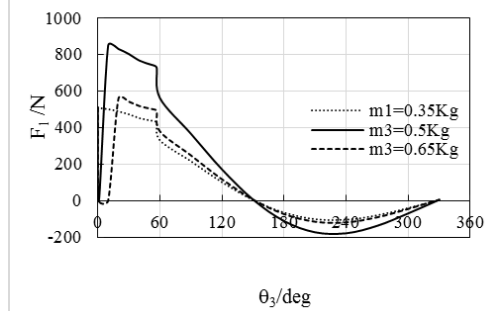
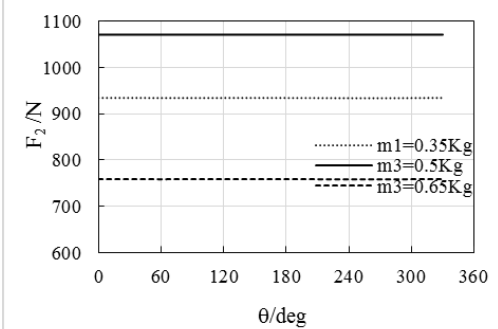
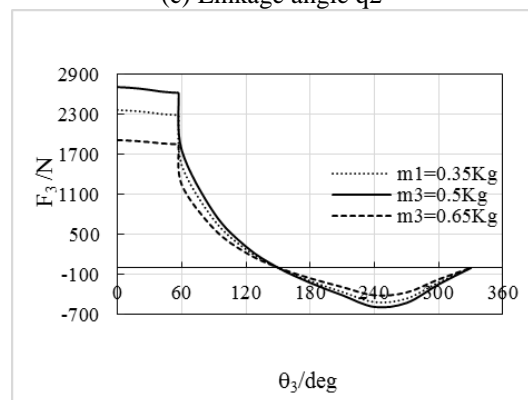
Table 1: The conditions of crank linkage mechanism in engine of vehicle						
Conditions	items	n r/m	R mm	L mm	m3 Kg	Stroke ratio
1	780	53	195	0.35	0.27	
2			160	0.5	0.33	
3			120	0.65	0.44	
4		63	295	0.3	0.21	
5		72	320		0.23	
6		83	420		0.20	

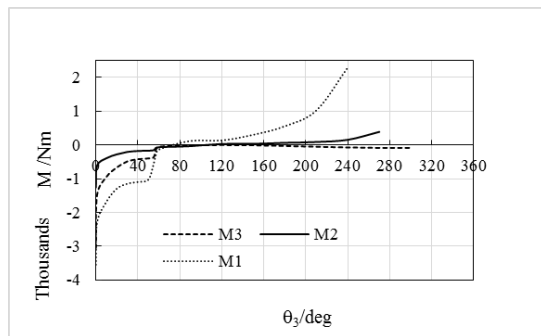


(a) Piston displacements

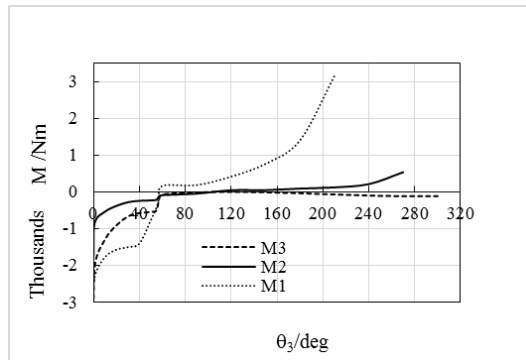
(b) Piston angle  $\theta_1$ (c) Linkage angle  $\theta_2$ (d) Crank angle  $\theta_3$ **Fig-2: The relationship between torque and crank angle with different variable**

(a) Piston displacements

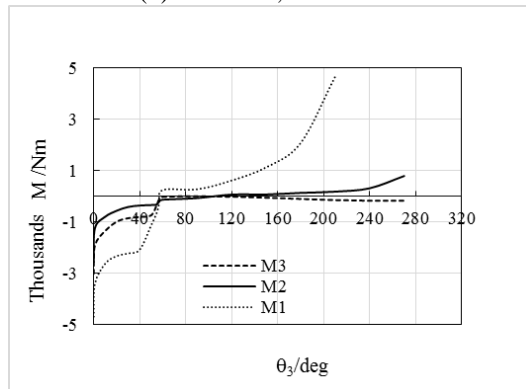
(b) Piston angle  $q_1$ (c) Linkage angle  $q_2$ (d) Crank angle  $q_3$ **Fig-3: The relationship between torque and crank angle with different variable**



(a) R=63mm, L=295mm



(b) R=72mm, L=320mm



(c) R=83mm, L=420mm

**Fig-4: The relationship between torque and crank angle on crank mass of 0.35Kg with variable piston angle  $\theta_1$ .**

In Figure 3 the force will be big when the differential approaches the crank on the crank linkage mechanism in vehicle from Figure 3(b,c). In Figure 3(a,b) the one increases from 6.5N to 1080N whilst the linkage condition has some in terms of Table 1.

It is known that the force will be big when the crank mass is big and the linkage is big too on the crank linkage mechanism in vehicle from Figure 3(a~d) whilst the force of crank angle has big onw. As seen in Figure 3(b) the piston force may decline from 850m/s to

500m/s respectively when the piston mass and linkage length declines. Here the force is bigger not to be neglected like Figure 3(a). The bigger force is caused by big stroke ratio. In order to gain the better force including mass of piston and the linkage length like smaller than 0.44=53/120 in optimum design according to Figure 3(a~d). In Figure 4(a~c) the three torques may increase from R=63mm and L=295mm to R=83mm and L=420mm under piston force of differential piston angle with the stroke ratio decreasing from 0.213 to 0.197.  $M_1$  is the biggest one among them from 2KNm to 3KNm at initial small angle and from 2KNm to 5KNm at 210° whilst the others somewhat increases.

In general the force and torque effective turn is Crank angle  $\theta_3 >$  linkage angle  $\theta_2 >$  piston angle  $\theta_1 >$  piston displacement  $s$ . The torque turn in one status is piston  $M_1 >$  crank  $M_3 >$  linkage  $M_2$ .

### 3. CONCLUSIONS

The torque will be big as the ratio of crank length and linkage is high by Lagrange formula. The crank force and torque will be big as the ratio is big. These are the conclusions searching in this paper. The force will be big when the crank mass is big on the crank linkage mechanism in vehicle. The force and torque effective turn is Crank angle  $\theta_3 >$  linkage angle  $\theta_2 >$  piston angle  $\theta_1 >$  piston displacement  $s$ .

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