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Non-Monotone Wedge Trust-Region Method for Derivative-Free Unconstrained Optimization

Fenghua Liu[1*], Qinghua Zhou[2]

[1]College of Computer Science and Engineering, Cangzhou Normal University, 060001, Hebei Province, China

[2]School of Applied Mathmatics, Beijing Normal University, Zhuhai, China

***Corresponding author**
Fenghua Liu

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Abstract: We propose a non-monotone wedge trust region method for derivative-free optimization. Wedge trust region method based on traditional trust region is designed for derivative-free problems, and the non-monotone strategy is efficient to solve the trust region method. This paper combined the non-monotone strategy into wedge trust region methods, and the computational results proved the efficiency of the new composite strategy.

Keywords: wedge trust region, non-monotone method, unconstrained optimization, derivative free optimization.

INTRODUCTION

In this paper, we consider the unconstrained optimization problem:

$$\min f(x), \quad x \in R^n, \qquad (1)$$

Where, the objective function $f(x)$ is continuous and its derivatives can not be explicitly computed [1, 2].

Considering the class of derivative-free trust-region methods, many algorithms can be found in the literature.

One know that the trust region methods which are famous for having global convergence quality [3, 4], and the traditional trust-region methods obtain a trial step by solving the quadric model m_k,

$$m_k(x_k+s) = f(x_k) + g_k^T s + \frac{1}{2} s^T G_k s, \quad (2)$$

Where, the $g_k \in R^n$ and the $n \times n$ symmetric matrix G_k are unknown variables and determined by the model interpolates f at a set of sample points, as the following

$$m(x_k) = f(x_k), m_k(y^l) = f(y^l), l = 1, 2, \cdots, m, \qquad (3)$$

Where, $Y_k = \{y^1, y^2, \cdots, y^m\} \cup \{x_k\}$ is the interpolation point set.

The parameter m should be chosen as $m = (n+1)(n+2)/2 - 1$ and the interpolation points set must be poised with the purpose of ensuring the uniqueness and existence of the quadratic model [5-8]. When the model m_k is determined by the above conditions, the interpolation set Y_k is nonsingular.

We can set out the current iteration with a nonsingular set of sample points Y_k firstly. Before computing a new trial point using the model m_k, let us figure out $y^{l_{out}}$ which is the farthest satellite from current iterate x_k, and it can ensure the virtue of the models. Actually, the wedge trust region method is to compute a trial step s_k by approximately solving

$$\min_s m_k(x_k + s) \qquad (4)$$

$$s.t. \|s\| \le \Delta_k \qquad (5)$$

$$s \notin W_k , \qquad (6)$$

Where, W_k is a set which contains the "taboo region" area [9-11], and its purpose is to avoid the new point falling into it. The trail step s_k is calculated by the method which is introduced in [8]. This method is very ingenious and the computational results are promising. Unfortunately, we can not find out the optimal point rapidly. We must choose the relatively good point in the iteration for the next iteration point.

In 1982, Chamberlain *et al*., in [12] came up with the watchdog technique for constrained optimization to conquer the Maratos effect. Motivated by this idea, Grippo*et al*., introduced a non-monotone line search technique for Newton's method in [13]. Due to the high efficiency of the non-monotone techniques, a lot of authors are interested in working on the combination of non-monotone techniques and trust region methods [14]. Let

$$f_{l(k)} = f(x_{l(k)}) = \max_{0 \le j \le m(k)} \{f_{k-j}\}, \quad k = 0, 1, 2, \ldots \qquad (7)$$

Where, $m(k) = \min\{M, k\}$ and $M \ge 0$ is an integer constant. Actually, the most common non-monotone ratio is defined as follows:

$$\bar{r}_k = \frac{f_l(x_k) - f(x_k + s_k)}{m(0) - m_k(s_k)}.$$

The rest of this paper is organized as follow. In section 2, the new non-monotone wedge trust region algorithm will be established, and the algorithm analysis is interpreted. Numerical results are proved in section 3 which is indicated that the new method is very efficient for unconstrained optimization problems. Some conclusions are given in section 4.

A non-monotone wedge trust region algorithm

Step 0 Set the trial parameters, an initial trust region radius $\Delta_k > 0$, and an initial guess x_0. The interpolation set $Y_K = x_k \cup Y$, $Y = \{y^1, y^2, \cdots, y^m\}$, and it such that $f(x_k) \le f(y) \forall y \in Y$.

Step 1 According to the current iteration point x_k, compute

$$y^{l_{out}} = \arg\max_{y \in Y} \|y - x_k\|.$$

Step 2 Construction quadratic model m_k and define the wedge W_k.

Step 3 Solve the sub-problem (2) and compute the trial step s_k, and calculate

$$r_k = \frac{Ared(d_k)}{Pred(d_k)} = \frac{f(x_k) - f(x_k + s_k)}{m(0) - m_k(s_k)}, \quad \bar{r}_k = \frac{f_l(x_k) - f(x_k + s_k)}{m(0) - m_k(s_k)}.$$

Step 4 Update the trust region radius Δ_k with the following Algorithm analysis.

Step 5 Update the interpolation set and the iteration point, if it is a successful iteration, that is $\bar{r}_k > \alpha_1$, then $x_{k+1} = x_k + s$, $Y = \{x_k\} \cup y / \{y^{l_{out}}\}$.

Else it is a unsuccessful iteration, that is $\bar{r}_k < \alpha_1$, then $x_{k+1} = x_k$,

$$Y = \begin{cases} \{x_k + s\} \cup y / \{y^{l_{out}}\}, if \left\| y^{l_{out}} - x_k \right\| \geq \left\| (x_k + s) - x_k \right\| \\ Y, \text{otherwise} \end{cases}.$$

Step 6 $k = k + 1$, go to step 1

Algorithm analysis: In the above algorithm, the trust region radius must be reduced when the function value rises. However, the non-monotone method is different from the iteration point which makes the function value rise, so we can set some different parameters. Thenew rule for updating the trust region radius is constructed as follows,

$$\Delta_{k+1} = \begin{cases} \gamma_1 \|s_k\|_k \, \bar{r}_k < \alpha_1; \\ \gamma_2 \|s_k\|, \bar{r}_k > \alpha_2 且 \|s_k\| = \Delta_k \\ \Delta_k, otherwise \end{cases}.$$

This strategy still reduces the trust region radius when the function value decreases. In the numerical experiment, the parameters are constructed as the follows, $\alpha_1 = 0.25$, $\alpha_2 = 0.75$, $\gamma_1 = 0.5$, $\gamma_2 = 2$. We choose

$$\bar{r}_k = \frac{f_l(x_k) - f(x_k + s_k)}{m(0) - m_k(s_k)},$$

$$f_{l(k)} = f(x_{l(k)}) = \max_{0 \le j \le m(k)} \{f_{k-j}\}, \quad k = 0, 1, 2, \ldots$$

Where, $m(k) = \min\{M, k\}$ and $M \geq 0$ is an integer constant

Numerical results

In this section, we compare the quadratic version of the new with the $a\lg$. We set the rotating is $\pi / 600$ used in quadratic model. The initial value $\gamma = 0.4$ is allowed to change over the iteration. The MATLAB source code for wedge trust region algorithm is in [15]. Specifically, we select 45 trial problems, which come from the CUTE. In the following table, the name of 45 test questions and results are given. We define n is the dimension of the objective function, and nf is the calculative times of an experimental function value. f is the optimal point and the $wed\ act$ represents the number of wedge constraints play a role. The final value of parameter γ which is a parameter used to control the space of "taboo region" is given in the last column when the algorithms stop.

Table-1: Comparison non-monotone wedge trust region algorithm with wedge trust region

n	p	nf		f		wed act		final γ	
		new	alg	new	alg	new	alg	new	alg
2	POWELL-E	18	17	0.00E+00	0.00E+00	12	12	1.83E-13	7.82E-12
2	CLIFF	55	65	2.90E-01	2.00E-01	4	16	2.75E-05	1.57E-11
2	DENSCHNA	19	47	2.49E-33	2.74E-35	9	16	1.30E-11	3.85E-14
3	GROWTHLS	59	1365	2.58E+03	1.00E+00	4	29	8.94E-04	4.51E-16
4	WOODS	44	359	4.45E-30	6.87E-30	13	25	3.24E-14	8.95E-17
5	OSBORNEA	70	1357	1.74E-01	5.46E-05	11	30	3.45E-06	2.57E-15

6	EDENSCH	77	120	1.08E+02	1.03E+02	10	22	4.66E-05	1.57E-14
6	HEART6LS	77	8000	2.21E+01	4.21E-01	11	36	1.34E-05	4.40E-10
10	BRYBND	115	322	1.58E-29	1.58E-29	19	31	2.23E-13	2.93E-14
2	BROWNBS	55	8000	1.00E+12	6.77E+11	4	6	2.00E-08	2.73E-07
2	HIMMELBB	53	51	0.00E+00	0.00E+00	14	15	4.89E-18	9.70E-20
2	HIMMELBH	55	45	-1.00E+00	-1.00E+00	1	25	8.17E-02	2.87E-16
4	ALLINITU	64	73	5.82E+00	5.74E+00	3	21	2.70E-03	9.19E-15
10	BDQRTIC	343	113	1.83E+01	1.83E+01	31	19	5.95E-14	3.68E-13
2	BEALE	23	39	3.94E-31	3.94E-31	14	16	9.01E-13	1.77E-15
3	BOX3	33	34	3.03E-33	3.03E-33	13	14	8.27E-15	1.96E-16
2	BRKMCC	46	51	1.69E-01	1.69E-01	3	23	3.28E-05	2.32E-15
4	BROWNDEN	63	116	8.58E+04	8.58E+04	6	23	3.30E-03	8.77E-15
10	BROWNAL	115	267	1.14E-28	1.14E-28	25	34	1.61E-13	8.07E-17
10	CRAGGLVY	115	858	2.52E+00	1.89E+00	3	28	3.76E-04	6.55E-04
2	CUBE	26	27	0.00E+00	0.00E+00	17	15	2.35E-17	2.95E-17
3	DENCHNE	31	121	2.44E-34	2.44E-34	10	16	1.79E-12	1.91E-15
2	DENSCHNF	18	18	0.00E+00	0.00E+00	11	11	6.84E-11	6.96E-11
2	ENGVAL1	21	36	0.00E+00	0.00E+00	11	16	7.78E-10	2.97E-10
2	EXPFIT	40	68	2.41E-01	2.41E-01	18	16	1.77E-15	3.61E-15
3	GULF	41	393	6.37E-31	7.06E-31	11	28	1.41E-16	2.17E-17
3	HATFIDD	51	140	6.62E-08	6.62E-08	18	28	2.87E-16	1.33E-16
3	HATFLDE	55	152	4.43E-07	4.43E-07	18	32	2.57E-16	6.12E-15
4	HIMMELBF	65	400	3.19E-02	3.19E-02	19	27	9.09E-15	3.90E-16
2	HIMMELBG	22	59	2.63E-163	2.63E-163	7	37	1.34e-11	1.42e-77
2	JENSMP	40	40	1.24E+02	1.24E+02	21	22	6.37E-15	4.69E-15
2	SINEVAL	21	318	0.00E+00	0.00E+00	12	83	1.21E-15	6.98E-159
15	BOX2	21	21	3.03E-33	3.03E-33	10	12	1.68E-12	1.75E-15
2	HAIRY	18	20	2.00E+01	2.00E+01	8	12	3.11E-11	7.04E-12
8	PALMER8C	94	1150	6.65E-01	1.60E-01	6	46	2.59E-04	1.69E-15
10	DQDRTIC	67	144	2.23E-42	2.23E-42	0	13	4.00E-01	1.96E-13
3	ENGVAL2	35	84	2.86E-30	2.86E-30	18	14	1.73E-13	1.47E-15
2	SISSER	54	218	4.38E-58	7.38E-56	38	30	3.32E-15	3.95E-16
2	ROSENBR	22	118	0.00E+00	4.93E-30	12	18	6.83E-15	1.78E-17
4	KOWOSB	63	140	3.08E-04	3.08E-04	22	20	6.30E-15	4.19E-16
2	MEXHAT	43	77	-4.01E-02	-4.01E-02	14	17	1.53E-12	2.05E-14
10	MOREBV	115	207	2.51E-32	2.51E-32	19	41	8.40E-15	1.55E-18
2	NASTY	7	7	5.00E-41	5.00E-41	0	0	4.00E-01	4.00E-01
10	POWER	71	77	3.16E-30	5.21E-27	3	6	2.34E-02	8.72E-16
2	ZANGWIL2	20	51	-1.82E+01	-1.82E+01	13	24	9.69E-15	9.06E-15

Comparing “nf” between the new and $a\lg$, we can see that our rule is better then former one. The numbers of win of two algorithms are 37 and 4, In the midst of 9 questions whose optimal solution is uniform, all of them reduced the time of calculations. For example, to the problem “ROSENBR”, new is called 22 function evaluations while $a\lg$ is called 118 function evaluations. For example, to the text problem “BOX2”, the wedge constraint of new is more active than the wedge constraint of $a\lg$, although the numbers of function evaluations are the same.

CONCLUSIONS

In this paper, we investigate a non-monotone wedge trust region method for unconstrained optimization without derivatives. The performance of the non-monotone wedge trust region method is improved. The results of numerical texts show that the number of function evaluations is reduced for a majority of text problems. Our improvement may be active and efficient in practice.

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