

**Parametric Analysis of Vertical Deflections of Bridges under Moving Loads**Lezin Seba Minsili<sup>1\*</sup>, Gilbert Tchemou<sup>2</sup>, Ayina Ohandja Louis Max<sup>1</sup>, Mandegue Lotin Josette A<sup>1</sup><sup>1</sup>LMM GC Department of Civil Engineering, ENSP, The University of Yaoundé 1, Yaoundé, Cameroon<sup>2</sup>Department of Civil Engineering, ENSET, The University of Douala, Douala, Cameroon**\*Corresponding author***Lezin Seba Minsili***Article History***Received: 04.12.2017**Accepted: 09.12.2017**Published: 30.12.2017***DOI:**

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**Abstract:** In order to develop the sustainability of bridge engineering design in many design offices of underdeveloped countries, this work outlines design guidelines by examining the behavior of bridges vertical displacement in relation to several parameters such as the moving speed, the track irregularities, the span-length, the elasticity modulus, the cross-section geometry and the material damping. A simply supported Euler-Bernoulli bridge-beam model coupled to a multiple moving point loading under different conditions is developed based on the Hooke's law, the Navier's hypothesis and the Saint-Venant's principle. Obtained results give a deeper understanding on the influence of these parameters in the initial design, and show that the use of a single dynamic magnification factor alone, for different bridge parameters, is not technically optimal for an improved dynamic analysis of existing and under-design bridges, and that local Design Bureau should implement a more sustainable design methodology to tackle uncertainties in bridge behavioral responses for its wider understanding.

**Keywords:** Mid span deflection, bridge, single load, loaded train, parameter.

**INTRODUCTION**

The African Union, through the second aspiration of its Agenda 2063 [1], has designed a new integrated Pan-African multimodal transport scheme and several foreign stakeholders have become very active in promoting, lobbying government and even investing in various national and regional infrastructures.

From any viewpoint, the requirement is a clear understanding of the fundamentals of the design, the construction and the exploitation of planned and existing complex structures such as dams and bridges for a sustainable transport system. This is obtained through an unbiased assessment, especially at the design stage of these important projects. Simulation and application of structural design principles are very important in order to show the efforts for calibrating parameters that can significantly reduce the consistency between all stages of the lifespan of a structure from design to its decommissioning [2]. Unfortunately, in many African countries, this strategy is hindered by a shortage of skilled engineers and the familiarity with modern construction and management principles of transport infrastructures.

The dynamic behavior of bridges under the action of moving loads is a complicated phenomenon that is yet to be well understood by many African engineers. Bridges are complex structures consisting of different materials and various structural components, with different structural properties, that individually affect its overall behavior. The influence of each of these parameters, as well those depending on the moving load, is to be well controlled and implemented in the bridge lifespan. The tendency, in the industry in general and particularly in Cameroon is to increase the velocity and the axle load without a proper attention to the bridge bearing capacity required by designed codes. Of course efforts are made in many countries like Cameroon to revitalize the bridges network by conducting appropriate technical studies prior to construction or rehabilitation [3], but the reality is that, in the same country, many bridges are in dilapidated stage because of extreme overloading without adequate management schemes as seen in Figure 1.



**Fig-1: The changing state of bridges in Cameroon**

An important engineering aspect associated to the design of structural components of bridges can be found in the dynamic effect due to moving loads. Its relevance at early stages of project initiation has been developed in classical solutions by many researchers [4, 5]. Nowadays most Design Bureau in Africa are using the dynamic response through an impact factor, which represents the increase in the dynamic response with respect to the static one for a single moving load [5-7]. The sustainability of a bridge structure in the case of scarce and limited funds is greatly affected by several uncertainties like changes in loading, material properties, improper maintenance plans, design models and damage occurrence [8-10]. The Eurocodes have adopted a more elaborated design to tackle these uncertainties through partial safety factors. A proper consideration of all internal and external factors affecting the loading of the bridge during its life-cycle can lead to significant uplifts to its structural performances and social benefits.

Zibdeh *et al.* [11] studied the random dynamic response of a simply supported laminated-composite coated beam, with different orientations in the coats under the action of a point load moving with acceleration. Recently Tekili S. *et al.* [12] investigated free and forced vibration of simple-supported beams strengthened by composite coats subjected to moving loads under different boundary conditions and found, through different configurations of the beams and the superposition method of individual moving loads, the optimal dynamic behavior of aluminum beams. There is no firm agreement between National design codes on how the dynamic factor should be evaluated. The dynamic magnification factor is usually a function of the natural frequency and span length of the bridge, and state how many times the static effects have to be magnified in order to cover the additional dynamic loads. Thus, improved dynamic analysis that consider all the important parameters that influence the dynamic response are required, in order to take account of the complex structural response in terms of its span length, its structural weight, its stiffness and damping, its individual axle loads and speed.

The aim of the present work is to outline a parametric analysis of the free and forced vibrations of bridges modeled as a simply supported Euler-Bernoulli beam subjected to multiple point forces moving at different speeds. The dynamic response of the given beam is presented in terms of the vertical displacement as a function of bridge parameters such as the moving speed, the track irregularities, the span-length, the elasticity modulus, the cross-section geometry and the material damping. The effect of these parameters led to facts that the deflection of a bridge roughly increases with the speed increment, the track irregularities have no significant influence on the bridge deflection even, and that higher values of material properties reduce the bridge vibration. In the same manner the use of single dynamic magnification factor, for different bridge parameters, is not technically optimal for an improved dynamic analysis of existing and under-designed bridges, and Design Bureaus should implement more elaborated design practices to tackle uncertainties in bridge behavioral responses.

### **THE PROBLEM MODEL**

The technology development today allows the formulation of three-dimensional models with related equations that will include the calculation and visualization of the behavior of a complex bridge structure [13-15]. In general the mathematical model of such models induces lengthy and time-consuming computational efforts that might not be affordable by African engineers dealing with simplified structures in a technical sustainable environment. Taking into account the dynamic effect of the moving load on the bridge, a reduced model of the train-bridge system is used.

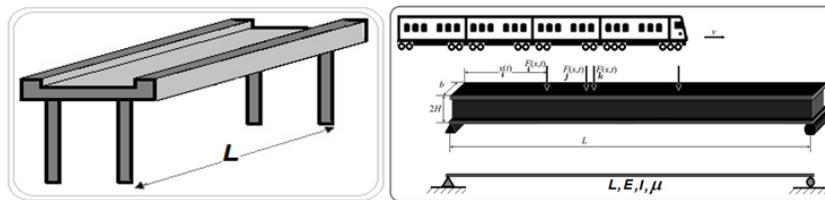


Fig-2: The simply supported beam model

Since we are dealing with vertical vibrations in this work and parameters that have great influence on their vibration amplitude during motion, the model is based on the 2-dimensional Euler-Bernoulli beam with the schematic illustration as shown in figure 2, with corresponding assumptions and limitations that can be found in adequate structural engineering books. Hence, simplified models are suggested and usually used [16–18] which take into account only certain aspects. However a key assumption in this work is that vertical modes of vibration contributes to the vertical acceleration of the bridge, which implies that accurate results may be achieved even though torsional and horizontal bending modes are neglected. Even if all bridges have torsional bending modes in particular when the bridge is eccentrically excited, such a situation is excluded in this work.

### The Bridge Model

We study simple supported concrete bridges of span length  $L$  over 30 m. This implies that the span length of the bridge is greatly higher than the height of its cross section and the effect of shear deformation can be neglected. The self-weight of these structures are high enough to neglect shear and rotational effects. The equation of motion is deduced on the assumption of the theory of small deformations  $v(x,t)$  at point  $x$  and time  $t$ , using the Hooke's law, the Navier's hypothesis and the Saint-Venant's principle. In addition, damping is considered proportional to the velocity of vibration. The equation of such a bridge is then given by the Euler-Bernoulli theory [4, 19] as:

$$EI \frac{\partial^4 v(x,t)}{\partial t^4} + \mu \frac{\partial^2 v(x,t)}{\partial t^2} + 2\mu\omega_b \frac{\partial v(x,t)}{\partial t} = f(x,t) \quad (1)$$

Where:

- $E$  is the elasticity modulus and  $I$  the cross section inertia;
- $v(x,t)$  is the vertical deflection of the beam at point  $x$  and time  $t$ ;
- $\mu$  is the mass per unit length of the beam ;
- $\omega_b$  is the circular frequency of viscous damping ;
- $f(x,t)$  is the load per unit length of the beam at point  $x$  and time  $t$ .

### Loads model

It is a very complex problem to give an exact representation of the moving load model  $f(x,t)$ , and simplifications are made with a reliable approximation which depends on the purpose and level of the analysis. For the dynamic analysis presented in this study, it is necessary to consider in details all contact forces between the wheel and the rail or the road. The model adopted is the moving vertical force of the train with a series of axle loads moving at a constant speed over the bridge. This method considers the components of forced and free vibrations. The shortcoming of this method is that it does not take in to account the inertia effect of the train mass and dynamic interaction between the train and the track. This can be permitted in the case of medium span bridges, and just as Broquet [20] and Xia [4], we assume that the vehicle weight in form of axle forces moves along the bridge, while the vehicle mass is located at its center of gravity.

The motion of the vehicle along the bridge is often expressed by the means of the Dirac function  $\delta(x)$  which in mechanics characterizes the action of a unit force concentrated at point  $x = 0$ . The Dirac function has the following properties:

$$\int_{-\infty}^{+\infty} \delta(x) dx = 1$$

$$\int_{-\infty}^{+\infty} \delta(x-a)f(x) dx = f(a) \quad (2)$$

$$\int_a^b \delta(x - \xi) f(x) dx = \begin{cases} 0 & \text{for } \xi < a < b \\ f(\xi) & \text{for } a < \xi < b \\ 0 & \text{for } a < b < \xi \end{cases}$$

Where  $f(x)$  is a continuous function within the interval  $(a, b)$  and  $a, b, \xi$  are constants.

The Heaviside function [21] describes the moment when the force  $F_n$  arrives on the beam and the moment when this force leaves the beam.

$$h(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t \geq 0 \end{cases} \quad (3)$$

If a concentrated constant force  $F$  moves along a beam structure with a constant velocity  $c$ , the load per unit length, which appears on the right hand side of equation (1), has the form :

$$f(x, t) = \delta(x - ct)F \quad (4)$$

For a set of  $N$  moving concentrated forces  $F_n$  [22], the Heaviside unit function is also used to obtain the corresponding load model as:

$$f(x, t) = \sum_{n=1}^N [h(t - t_n) - h(t - T_n)] \delta(x - ct - d_n) F_n \quad (5)$$

Where:  $t_n = d_n/c$  is the time when the  $n^{th}$  force enters the beam after traveling a distance  $d_n$  ;  
 $T_n$  is the time when the  $n^{th}$  force leaves the beam.

### Computation of the vertical deflection

For the simplest case of a constant force moving along the bridge, the deflection is obtained by solving equations (1) and (4) with the following assumptions: the mass of the moving load is small compared with the mass of the beam; the load moves at constant speed from left to right ; the beam damping is proportional to the velocity of the moving load ; the beam is simply supported, this leads to zero deflection and zero bending moment at both ends ; and at the instant of the arrival of the load, the beam has neither an initial deflection nor velocity. The boundary and initial conditions are thus given as:

$$\begin{aligned} v(0, t) = 0; \quad v(L, t) = 0; \\ v(x, 0) = 0; \quad \left(\frac{\partial v(x, t)}{\partial t}\right)_{t=0} = 0 \\ \left(\frac{\partial^2 v(x, t)}{\partial x^2}\right)_{x=0} = 0; \quad \left(\frac{\partial^2 v(x, t)}{\partial x^2}\right)_{x=L} = 0; \end{aligned} \quad (6)$$

Solving Equation (6) with the method of integral transformations [19], each term is multiplied by  $\sin \frac{j\pi x}{L}$  and then integrated with respect to  $x$  between 0 and  $L$ . The solution is then given by:

$$v(x, t) = \sum_{j=1}^{\infty} V(j, t) \sin \frac{j\pi x}{L} \quad (7)$$

Where

$$V(j, t) = \int_0^L v(x, t) \sin \frac{j\pi x}{L} dx \quad , j = 1, 2, 3 \dots$$

$V(j, t)$  is the transform of the original  $v(x, t)$ .

Equation (1) then yields to:

$$\frac{j^4 \pi^4}{L^4} EI V(j, t) + 2\mu\omega_b \dot{V}(j, t) + \mu \ddot{V}(j, t) = F \sin \frac{j\pi ct}{L} \quad (8)$$

Using the expression of the circular frequency at the  $j^{th}$  mode of vibration for a simply supported beam  $\omega_j$  [23] and of the force circular frequency, and then rearranging the terms, we obtain an expression that is solved using the method of Laplace-Carson integral transformation [19], and after several manipulations the basic relation of equation (7) becomes:

$$V^*(j, p) = p \int_0^\infty V(j, t) e^{-pt} dt$$

$$V_j(j, t) = \frac{1}{2\pi i} \int_{a_0 - i\infty}^{a_0 + i\infty} e^{pt} \frac{V^*(j, p)}{p} dp \tag{9}$$

Where  $a_0$  signifies that the integration is carried out along a straight line parallel to the imaginary axis lying to the right of all the singularities of the function of the complex variable  $e^{pt} \frac{V^*(j, p)}{p}$  (the real argument of all the singularities is therefore less than  $a_0$ ). Transforming now equation (8) using equation (9) gives:

$$p^2 V^*(j, p) + 2\omega_b p V^*(j, p) + \omega_j^2 V^*(j, p) = \frac{Fj\omega}{\mu} \frac{p}{p^2 + j^2 \omega^2} \tag{10}$$

Where  $V^*(j, p) = \frac{Fj\omega}{\mu} \frac{p}{p^2 + j^2 \omega^2} \frac{1}{p^2 + 2\omega_b p + \omega_j^2}$

Depending on the position of the poles of complex variable function  $V^*(j, p)$ , distinction is made between several cases of analysis. After computations, the general expression of the bridge deflection under one moving constant force  $F$  is obtained as:

$$v(x, t) = v_0 \frac{1}{[(1-\alpha^2)^2 + 4\alpha^2\beta^2]} \times \left\{ \begin{aligned} &(1 - \alpha^2) \sin \omega t - \frac{\alpha[(1-\alpha^2) - 2\beta^2] e^{-\omega_b t} \sin \omega_1' t}{\sqrt{1-\beta^2}} \\ &- 2\alpha\beta (\cos \omega t - e^{-\omega_b t} \cos \omega_1' t) \end{aligned} \right\} \sin \frac{\pi x}{L} \tag{11}$$

$v_0$  is the deflection at mid span of the beam loaded with static force  $F$  at point  $x = L/2$ .

$\alpha, \beta$  are the velocity and damping parameters respectively  $\alpha = \frac{\omega}{\omega_1}$  and  $\beta = \frac{\omega_b}{\omega_1}$ ;

$\omega_1'$  is the circular frequency of the damped beam.

For a more complex case when several constant forces are moving along the bridge, the problem is still to solve the differential equations (1) and (5) in the form :

$$EI \frac{\partial^4 v(x, t)}{\partial t^4} + \mu \frac{\partial^2 v(x, t)}{\partial t^2} + 2\mu\omega_b \frac{\partial v(x, t)}{\partial t} = \sum_{n=1}^N [h(t - t_n) - h(t - T_n)] \delta(x - ct - d_n) F_n \tag{12}$$

Once again, the method of integral transformation is applied [22] using the procedure as in the case of a single constant force moving along the bridge to obtain the corresponding  $v(x, t)$  solution in the following form :

$$v(x, t) = \sum_{n=1}^N v_0 \frac{F_n}{F} \omega \omega_1^2 [f(t - t_n) h(t - t_n) - (t - T_n) h(t - T_n)] \sin \frac{\pi x}{L} \tag{13}$$

where the Laplace-Carson inverse transformation is given as :

$$f(t) = \frac{1}{\omega_1' D} \left[ \frac{\omega_1'}{\omega} \sin(\omega t + \lambda_1) + e^{-\omega_b t} \sin(\omega_1' t + \lambda_2) \right]$$

and the following notations are adopted :

$$D^2 = \omega_1^2 - \omega^2 + 4\omega^2 \omega_d^2$$

$$\lambda_1 = \arctan \left( -\frac{2\omega\omega_d}{\omega_1^2 - \omega^2} \right)$$

$$\lambda_2 = \arctan \left( -\frac{2\omega_d\omega_1'}{\omega_d^2 - \omega_1' + \omega^2} \right)$$

$v_0$  is the deflection of the beam center due to the predominant force  $F = \max F_i$  applied at the same point.  $v_0$  is given by equation (\*).

The first part of the Laplace-Carson inverse  $f(t)$  expresses the forced vibration due to the moving forces while its second term denotes the free damped vibration

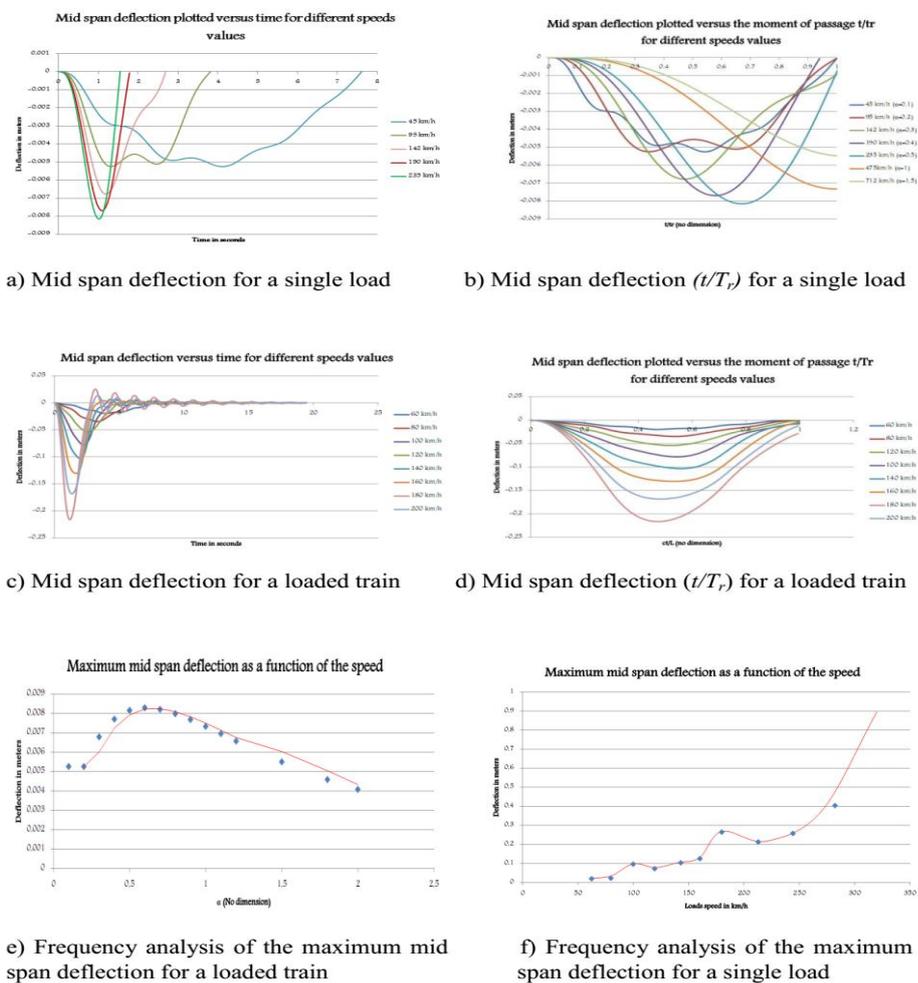
### THE PARAMETRIC ANALYSIS

The bridge considered in this study has the following characteristics: span length  $L = 100m$ ; concrete elasticity modulus  $E_c = 20000Mpa$ ; the bridge density  $\rho = 2500kg/m^3$  that yields a linear density  $\mu = 225000kg/m$  or a bridge

weight  $G = 225MN$ . The bridge is made up of girders with variable sections, and using the on-piles and bay section characteristics the mean value, obtained with the Hermitian interpolation, is  $S = 90m^2$ . Similarly, the mean inertia is  $= 200m^4$  and the damping ratio is computed according to the Eurocode 2 prescriptions [18] to get  $\beta = 5\%$ . Two types of vehicles are passing on the bridge: an idealized point load vehicle that weights  $F = 0,97MN$  corresponding to a loaded truck; and a set of 22 axles loads 2.0 m spaced, corresponding to a train representation with axles loads taken as follows:  $F_i = 0.21 MN$  for the locomotive axles and  $F_j = 0.17 MN$  for the wagons.

**The Speed Parameter**

For speeds ranging from 45 km/h to 235 km/h, for all types of vehicles, the mid span deflection is computed and plotted on figures 3 with corresponding descriptions. Taking  $T_r=L/c$  as the time spent on the bridge by the load, an additional overview of the effect of the train on the bridge is considered, with the normalized time abscissa  $t/T_r$ . In order to homogenize the results, take into account the time difference spent on the bridge for different speeds. Thus if the position of the vehicle is  $x = ct$ , the maximum mid span deflection is plotted against  $t/T_r = ct/L$ .



**Fig-3: Effect of speed on vertical deflections**

At time  $t = 0$  and  $t = T_r$ , the vehicle is on the first and the last piers of the bridge respectively, modeled as simple supports, respectively and we have a zero deflection at these instants. For different velocities, it is easy to notice that the dynamic deflection increases with increased velocity. For low values of the speed, the maximum deflection is obtained approximately at  $t = T_r/2$ . When the speed increases, the maximum deflection is shifts slowly to later instants as the vertical energy due to the loading is building up in the beam to increase the vertical deflection (figures 3.b and 3.d). An in-depth study of the effect of the vehicle speed on the deflection shows that it is necessary to compute the deflection for higher speeds than the ones computed in the previous plots. By making use of the dimensionless speed's

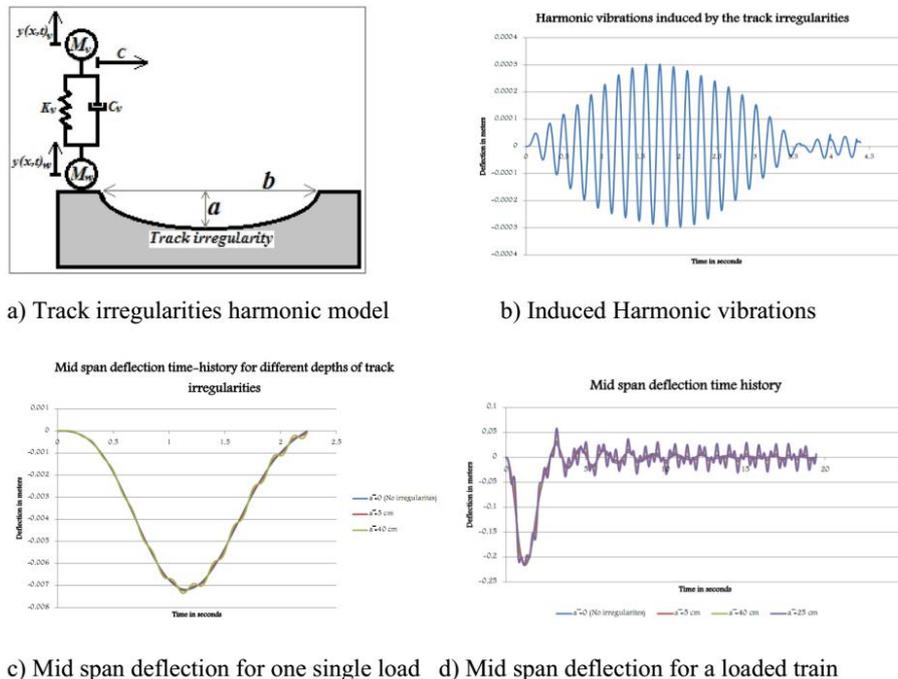
parameter  $\alpha = \omega/\omega_1$ ,  $\omega_1$  being the first natural frequency of the bridge, we plot the maximum dynamic deflections as a function of  $\alpha$  for the single load model and for the loaded train model.

The dynamic effect of the single load (figure 3.e) on the bridge increases with increasing speeds when  $\alpha$  is about 0.7, and for greater values of  $\alpha$  (or for greater values of speeds) the mid span deflection drops. In practice these values of  $\alpha$  for which the maximum deflection drops are not reachable since  $\alpha$  approaches and crosses unity. The work done by the moving crossing load in the vertical direction remains lower than the spring energy required for bending the bridge in the first mode  $\omega_1$ . For contemporary bridges, we can conclude that the vertical response will always increase with increasing speed.

With the moving train load (figure 3.f) along the bridge, we notice an increasing tendency of the dynamic deflection with the speed increment. Some observed local peaks in that figure highlight the complexity of the bridge-train interaction that takes its roots in dangerous phenomenon of local resonances in bridges. A situation that can also be explained theoretically with the factor  $D$  in the denominator of the term  $f(t)$  of equation (13) where we can find a local mode critical speed  $c_{cr}$ .

### The Track Irregularities Parameter

The track irregularities are modeled as harmonic variable functions having the maximum track unevenness equal to  $\bar{a}$ , and the mean length of the irregularity equal to  $l_a$  [19, 24] as seen in figure 4.a. When the vehicle is moving along a beam subjected to these track irregularities, harmonic loadings caused by the unevenness is of the form  $F(t)=F_o \sin \Omega t$ , with the force amplitude  $F_o=1/(2ka)$  and the circular frequency  $\Omega=2\pi c/b$ , where  $a, b$  are the depth and length of sinusoidal irregularities, respectively, and  $k$  is the vehicle spring stiffness. Considering the same bridge with characteristics of track irregularities  $\bar{a} = 10 \text{ cm}$ ,  $l_a = 15 \text{ cm}$ , with a vehicle moving at a constant speed of  $100 \text{ km/h}$ , figure 4 are displaying the mid span deflections time history for a single load and a train load.

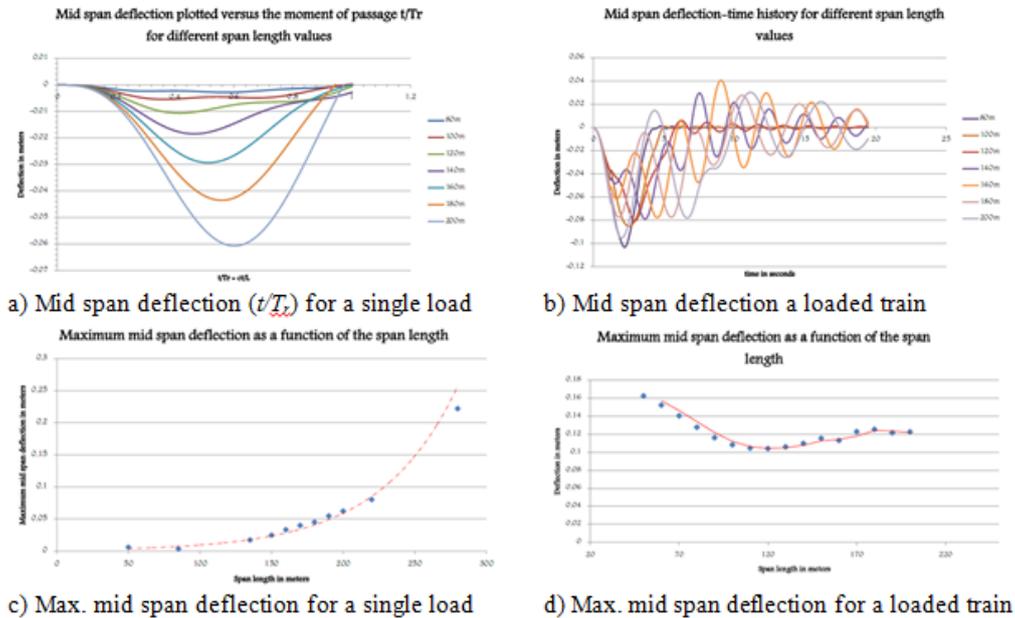


**Fig-4: Effect of irregularities on vertical deflections**

Comparing the mid span for a bridge without irregularities to a bridge with irregularities, we notice that the sole effect of irregularities does not have an adverse effect on the value of the deflections as the considered effect slightly oscillates harmonically around the curve without irregularities as seen in figures 4.c and 4.d. When we vary the depth of track regularity  $\bar{a}$ , this moderate amplification of the deflection remains lower and locally concentrated where the unevenness is located and might not have a major consequence on the bridge behavior except if the deterioration is amplified around the irregularity.

**The Span Length Parameter**

The mid span deflection of the bridge, as well as its maximal value, for different values of span length, for vehicles moving at a constant speed of 100km/h, are plotted in figures 5 with the normalized time abscissa  $t/T_r$ .

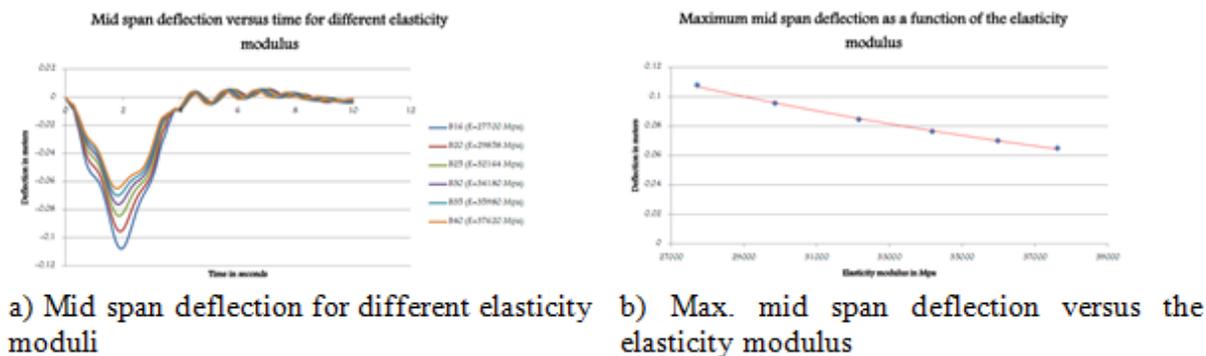


**Fig-5: Effect of span length on vertical deflections**

For both types of loadings, figures 5.a and 5.d show that the mid span deflection increases with the span length. However, we observe different rates increment for the building up of the deflection, the single load type having the greatest rate. The train load, being nearly evenly distributed over the bridge span, the input external energy of the load is also quasi evenly distributed in each section of the bridge, and resulted deflections are less than what the same point load value will produce (figures 5.c) at the bridge mid span. The rate of maximum mid span deflection tends to decrease with increasing values up to 120 m of the span length as it is seen in figure 5.d. For span length values above 120 m we observe a stagnation tendency and even a slight decrease to the maximum mid span deflection as the span value increases.

**The Elasticity Modulus Parameter**

The elasticity modulus of concrete materials depends on its resistance after 28 days. Considering six different values of this resistance, from 16MPa to 40MPa, that yield the elasticity modulus from 27700MPa to 37620MPa, the deflection of the bridge is given by figure 6, with the moment of inertia  $I$  of the cross section and the linear mass  $\mu$  remaining constant ( $I = 200m^4$  and  $\mu = 226940kg/m$ ). The dynamic deflection of the bridge decreases when the elasticity modulus increases both for the single point load and for the train load, a result that can be theoretical justified.



**Fig-6: Effect of the modulus of elasticity on vertical deflections**

### The Section Geometry Parameter

If the geometries of several bridges with the same cross sectional area  $S$  are different, then their behaviors toward the same loading will depend on their respective moments of inertia  $I$  of the cross section around a specified axis. In order to study the effect of the bridge geometry on its deflection, we fixe the cross section surface to  $S = 51,95m^2$  and three different geometries representing two types of bridges are derived from that same section: a simple slab with  $I = 69m^4$  ; a simple girder with  $I = 123m^4$  ; and a double girder with  $I = 294m^4$ . The modulus of elasticity and the linear mass remaining constant, with  $E = 28958 MPa$  and  $\mu = 226940kg/m$  the mid span deflection history is presented in figure 7. Increasing the moment section of inertia  $I$ , the mid span deflection as well as its maximal value decreases and makes the bridge stiffer.

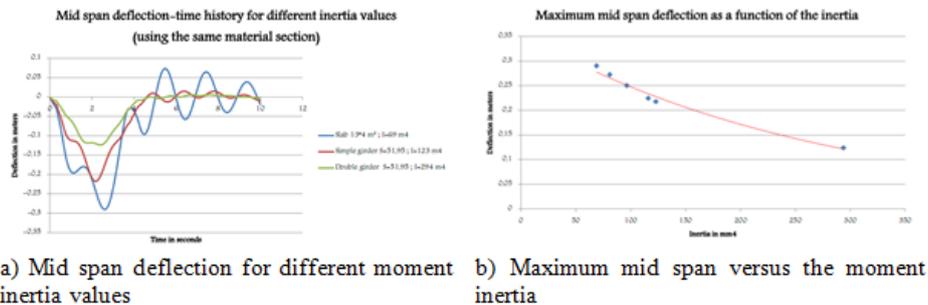


Fig-7: Effect of the Section Geometry on vertical deflections

### The Bridge Damping Parameter

The effect of structural damping of the bridge on mid span vertical deflection as a function of damping ratios ranging from 1% to 10 % is plotted in figure 8.a for the loaded train case. We notice that for low values of damping, the bridge continues to vibrate for a longer time than that observed for greater values of damping. In other words, the amplitude of vibrations also decreases more rapidly for greater values of damping. Changing the value of the train bogies suspension damping as depicted in figure 4.a ( $c_v$ ) and computing the maximum mid span deflection values, figure 8.b also shows that an increase of the suspension damping reduces the bridge response even if the degree of influence remains relatively marginal.

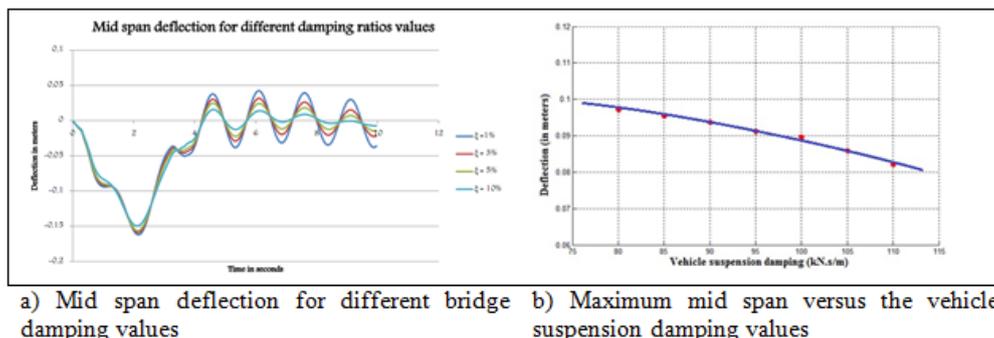


Fig-8: Effect of damping on vertical deflections

### CONCLUSIONS

A reduced coupled model of the load bridge interaction has been developed in this work to assess the effect of a certain number of parameters on the dynamic behavior of a railway bridge. These parameters included the load speed, track irregularities, the span length, the bridge geometry and material properties. The effect of these parameters, using well known engineering computational methods, on the bridge deflection has led to the following main conclusions:

- The deflection of a bridge roughly increases with the speed increment, and when the induced loading frequency is close to the bridge natural frequency, the effect of resonance might be noticed;
- The track irregularities have no significant influence on the bridge deflection even if appropriate remedial measures shall be taken for the passengers comfort ;
- Higher values of material properties, such as damping and elasticity, reduce the bridge vibration amplitude and the time necessary for the bridge to get back into its equilibrium position ;

Results obtained in the present work will be used by local engineers dealing with rough design and construction of simple bridges with sufficient accuracy. For bridges with complex mechanical and geometric features a more refined analytical theory dealing with nonlinear plate theories and sprung/un-sprung mass dynamic loading models in space is required.

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