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On Generalized (σ, τ)-i-n-Derivations in Near-Rings

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Abstract: In the present paper, we introduce the notion of generalized (σ, τ)-i-$n$derivation in near-ring N and investigate a property involving generalized (σ, τ)-i-n-derivation of a prime near-ring N, which forces N to be a commutative ring. Additive commutativity of a prime near-ring N satisfying certain identities involving generalized (σ, τ)-i-n-derivation has also been obtained.

Keywords: Prime near-ring, derivation, i-n-derivation, generalized (σ; τ)-i-n-derivation and commutativity
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INTRODUCTION

A nonempty set N equipped with two binary operations '+' and '.' is called a left nearring provided that $(N, +)$ is a group (not necessarily abelian), $(N,.)$ is a semigroup and $x(y + z) = xy + xz$ for all $x, y, z \in N$. A left near-ring N is called zero symmetric if $0x = 0$ for all $x \in N$ (recall that in a left near-ring N, $x.0 = 0$ holds for all $x \in N$). N is called a prime near-ring if $xNy = \{0\}$ implies $x = 0$ or $y = 0$. The symbol Z will denote the multiplicative center of N, that is, $Z = \{x \in N \; xy = yx$ for all $y \in N\}$. For any $x, y \in N$ the symbols $[x, y] = xy - yx$ and $(x, y) = x + y - x - y$ stand for multiplicative commutator and additive commutator of x and y respectively. Let X and Y be nonempty subsets of N. By the notation $[X, Y]$, we mean a subset of N defined by $[X, Y] = \{[x, y] \; j \; x \in X, y \in$ Y$\}$. *Throughout* this paper by the term "near-ring N", we mean a zero symmetric left near-ring. For terminologies concerning near-rings, we refer to G. Pilz [1].

An additive map $d: N \longrightarrow N$ is called a derivation if $d(xy) = d(x)y + xd(y)$ (or equivalently $d(xy) = xd(y) + d(x)y$) holds for all $x, y \in N$. The concept of derivation has been generalized in several ways by various authors [2-8]. Very recently the authors [5, 6] extended the above notion of derivation by introducing the notions of generalized n-derivation and (σ, τ)-n-derivation, where n is a positive integer.

Let n be a fixed positive integer. An n-additive (i.e., additive in each argument) mapping $D: N \times N \times \cdots \times N \longrightarrow N$ is called an *n-derivation* if the relations $D(x_1, x_2, \cdots, xi_{-1}, xix'i, xi+_1, \cdots, xn)$
$= D(x_1, x_2, \cdots, x_{i-1}, x_i, x_{i+1}, \cdots, xn)x'i + xiD(x_1, x_2, \cdots, xi_{-1}, x'i, xi+_1, \cdots, xn)$ hold for all $x_1, x_2, \cdots, xi_{-1}, xi, x'i, xi+_1, \cdots, xn \in N$, $i = 1, 2, 3, \cdots, n$ [4].

An n-additive mapping $F: N \times N \times \cdots \times N \longrightarrow N$ is called a *left generalized n-derivation* of N with associated n-derivation D if the relations $F(x_1, x_2, \cdots, xi_{-1}, xix'i, xi+_1, \cdots, xn)$
$= D(x_1, x_2, \cdots, xi_{-1}, xi, xi+_1, \cdots, xn)x'i + xi\, F(x_1, x_2, \cdots, xi_{-1}, x'i, xi+_1, \cdots, xn)$
hold for all $x_1, x_2, \cdots, xi_{-1}, xi, x'i, xi+_1, \cdots, xn \in N$, $i = 1, 2, 3, \cdots, n$.

An n-additive mapping $F : N\times N\times\cdots\times N$ -! N is called a *right generalized n-derivation*of N with associated n-derivation D if the relations $F(x_1, x_2, \cdots, xi_{-1}, xix'i, xi+_1, \cdots, xn)$
$= F(x_1, x_2, \cdots, xi_{-1}, xi, xi+_1, \cdots, xn)x'i + xiD(x_1, x_2, \cdots, xi_{-1}, x'i, xi+_1, \cdots, xn)$
hold for all $x_1, x_2, \cdots, xi_{-1}, xi, x'i, xi+_1, \cdots, xn \in N$, $i = 1, 2, 3, \cdots, n$. Lastly an n-additive mapping $F : N \times N \times \cdots \times N \longrightarrow N$ is called a *generalized n-derivation* of N with associated n-derivation D if it is both a right generalized n-derivation as well as a left generalized n-derivation of N with associated n-derivation D [5].

An n-additive (i.e., additive in each argument) mapping $D: N \times N \times \cdots \times N \longrightarrow N$ is called a *(σ, τ)-n-derivation* of N if there exist functions $\sigma, \tau : N \longrightarrow N$ such that the relations $D(x_1, x_2, \cdots, xi_{-1}, xix0i, xi+_1, \cdots, xn)$
$= D(x_1, x_2, \cdots, xi_{-1}, xi, xi+_1, \cdots, xn)\sigma(x'i)+\tau(xi)D(x_1, x_2, \cdots, xi_{-1}, x'i, xi+_1, \cdots, xn)$ hold for all $x_1, x_2, \cdots, xi_{-1}, xi, x'i, xi+_1, \cdots, xn \in N$, $i = 1, 2, 3, \cdots, n$ [4].

An n-additive mapping $F: N \times N \times \cdots \times N \longrightarrow N$ is called a *left generalized* (σ, τ)-*n-derivation* of N if there exists a (σ, τ)-n-derivation D and the relations $F(x_1, x_2, \cdots, x_i-_1, x_ix'_i, x_i+_1, \cdots, xn)$
$= D(x_1, x_2, \cdots, xi-_1, xi, xi+_1 \cdots, xn)\sigma(x'i) + \tau(xi)F(x_1, x_2, \cdots, xi-_1, x'i, xi+_1, \cdots, xn)$
hold for all $x_1, x_2, \cdots, xi-_1, xi, x'_I, xi+_1, \cdots, xn \in N$, $i = 1, 2, 3, \cdots, n$.

An n-additive mapping $F: N \times N \times \cdots \times N \longrightarrow N$ is called a *right generalized* (σ, τ)-*n-derivation* of N if there exists a (σ, τ)-n-derivation D and the relations $F(x_1, x_2, \cdots, xi-_1, xix'i, xi+_1, \cdots, xn)$
$= F(x_1, x_2, \cdots, xi-_1, xi, xi+_1 \cdots, xn)\sigma(x'i)+\tau(xi)D(x_1, x_2, \cdots, xi-_1, x'i, xi+_1, \cdots, xn)$ hold for all $x_1, x_2, \cdots, xi-_1, xi, x'i, xi+_1, \cdots, xn \in$, $i = 1, 2, 3, \cdots, n$. Here we say that F is a left generalized (σ, τ)-n-derivation (resp. right generalized (σ, τ)-n-derivation) of N with associated (σ, τ)-n-derivation D. Finally an n-additive mapping $F: N \times N \times \cdots \times N \longrightarrow N$ is called a *generalized* (σ, τ)-*n-derivation* of N if there exists a (σ, τ)-n-derivation D and F is both a left generalized (σ, τ)-n-derivation as well as a right generalized (σ, τ)-n-derivation of N with associated (σ, τ)-n-derivation D of N [6].

The literature on near-ring N contains a number of theorems asserting additive or multiplicative commutativity of N. Bell & Mason [6, 9], Ashraf *et al*. [2-6], G¨olbasi [10-12] etc. have proved several results on commutativity of addition and multiplication of prime near-rings which admit derivations, generalized derivations, (σ, τ)-derivations and n-derivations. Our aim in this paper is to study the commutativity of addition and multiplication of a prime near-ring which admits a generalized (σ, τ)-i-n-derivation. In fact our theorems extend, generalize and unify several results obtained earlier.

PRELIMINARY RESULTS

Now we introduce a weaker family of derivations in near-ring N. Of course this family generalizes the notions of n-derivations, generalized n-derivations, (σ, τ)-n-derivations and generalized (σ, τ)-n-derivations discussed above.

Let n be a fixed positive integer and i be an integer with $1 \leq i \leq n$. An n-additive (i.e., additive in each argument) mapping $D: N \times N \times \cdots \times N \longrightarrow N$ is called a *i-n-derivation* of N if the relation $D(x_1, x_2, \cdots, xi-_1, xix', xi+_1, \cdots, xn)$
$= D(x_1, x_2, \cdots, xi-_1, xi, xi+_1, \cdots, xn)xi'+xiD(x_1, x_2, \cdots, xi-_1, x'i, xi+_1, \cdots, xn)$ holds for all $x_1, x_2, \cdots, xi-_1, xi, x', xi+_1, \cdots, xn \in N$. From the above definition it is obvious that if D is a i-n-derivation of N for each i with $1 \leq i \leq n$, then D is an n-derivation of N and conversely. It can be also observed that every n-derivation is a i-n-derivation but its converse is not true.

An n-additive mapping $F: N \times N \times \cdots \times N \longrightarrow N$ is called a *right generalized i-nderivation* of N with associated i-n-derivation D if the relation
$F(x_1, x_2, \cdots, xi-_1, xix', xi+_1, \cdots, xn)$
$= F(x_1, x_2, \cdots, xi-_1, xi, xi+_1, \cdots, xn)\, x'i + xiD(x_1, x_2, \cdots, xi-_1, x'i, xi+_1, \cdots, xn)$
holds for all $x_1, x_2, \cdots, xi-_1, xi, x'i, xi+_1, \cdots, xn\ 2\ N$.

An n-additive mapping $F : N \times N \times \cdots \times N \longrightarrow N$ is called a *left generalized i-n-derivation* of N with associated i-n-derivation D if the relation
$F(x_1, x_2, \cdots, xi-_1, xix'i, xi+_1, \cdots, xn)$
$= D(x_1, x_2, \cdots, xi-_1, xi, xi+_1, \cdots, xn)x'i + xiF(x_1, x_2, \cdots, xi-_1, x'i, xi+_1, \cdots, xn)$ holds for all $x_1, x_2, \cdots, xi-_1, xi, x'i, xi+_1, \cdots, xn\ 2\ N$. Finally an n-additive mapping $F : N \times N \times \cdots \times N \longrightarrow N$ is called a *generalized i-n-derivation* of N with associated i-n-derivation D if it is both a right generalized i-n-derivation as well as a left generalized i-n-derivation of N with associated i-n-derivation D.

An n-additive (i.e., additive in each argument) mapping $D: N \times N \times \cdots \times N$ -! N is called a (σ, τ)-*i-n-derivation* of N if there exist functions $\sigma, \tau : N \longrightarrow N$ such that the relation $D(x_1, x_2, \cdots, xi-_1, xix'i, xi+_1, \cdots, xn)$
$= D(x_1, x_2, \cdots, xi-_1, xi, xi+_1, \cdots, xn)\sigma(x'i)+\tau(xi)D(x_1, x_2, \cdots, xi-1, x'i, xi+_1, \cdots, xn)$ holds for all $x_1, x_2, \cdots, xi-_1, xi, x'i, xi+_1, \cdots, xn \in N$.

An n-additive mapping $F : N \times N \times \cdots \times N \longrightarrow N$ is called a *left generalized* (σ, τ)-*i-n-derivation* of N if there exists a (σ, τ)-i-n-derivation D and the relation $F(x_1, x_2, \cdots, xi-_1, xix'i, xi+_1, \cdots, xn)$
$= D(x_1, x_2, \cdots, xi-_1, xi, xi+_1 \cdots, xn)\sigma(x'i) + \tau(xi)F(x_1, x_2, \cdots, xi-_1, x'i, xi+_1, \cdots, xn)$
hold for all $x_1, x_2, \cdots, xi-_1, xi, x'i, xi+_1, \cdots, xn \in N$.

An n-additive mapping $F : N \times N \times \cdots \times N \longrightarrow N$ is called a *right generalized* (σ, τ)-*i-n-derivation* of N if there exists a (σ, τ)-i-n-derivation D and the relation $F(x_1, x_2, \cdots, xi-_1, xix'i, xi+_1, \cdots, xn)$
$= F(x_1, x_2, \cdots, xi-_1, xi, xi+_1 \cdots, xn)\sigma(x'i)+\tau(xi)D(x_1, x_2, \cdots, xi-_1, x'i, xi+_1, \cdots, xn)$ hold for all $x_1, x_2, \cdots, xi-_1, xi, x'i, xi+_1, \cdots, xn \in N$. Here we say that F is a left generalized (σ, τ)-i-n-derivation (resp. right generalized (σ, τ)-i-n-

derivation) of N with associated (σ, τ)-i-n-derivation D. Finally an n-additive mapping $F: N \times N \times \cdots \times N \longrightarrow N$ is called a *generalized* (σ, τ)-*i-n-derivation* of N if there exists a (σ, τ)-i-n-derivation D and F is both a left generalized (σ, τ)-i-n-derivation as well as a right generalized (σ, τ)-i-n-derivation of N with associated (σ, τ)-i-n-derivation D of N.

We facilitate our discussion with the following lemmas which are essential for developing the proofs of our main results. Proofs of Lemmas 2*:*1 & 2*:*2 can be seen in [9, 13]. Throughout further discussion, σ and τ will represent automorphisms of N.

Lemma 2*:*1*:* Let N be a prime near-ring. If $Z \setminus \{0\}$ contains an element z for which $z + z \in Z$, then $(N, +)$ is abelian.
Lemma 2*:*2*:* Let N be a prime near-ring. If $z \in Z \setminus \{0\}$ and x is an element of N such that $xz \in Z$ or $zx \in Z$ then $x \in Z$*:*

In a left near-ring N, right distributive law does not hold in general, however, we can prove the following partial distributive properties in N.

Lemma 2*:*3*:* Let N be a near-ring admitting a generalized (σ, τ)-i- n-derivation F with associated (σ, τ)-i-n-derivation D of N. Then, $(D(x_1, x_2, \cdots, xi\text{-}1, xi, xi+_1 \cdots, xn)\sigma(x'i) + \tau(xi)F(x_1, x_2, \cdots, xi\text{-}_1, x'i, xi+_1, \cdots, xn))y$
$=D(x_1, x_2, \cdots, xi\text{-}_1, xi, xi+_1 \cdots, xn)\sigma(x'i)y + \tau(xi)F(x_1, x_2, \cdots, xi\text{-}_1, x'i, xi+_1, \cdots, xn)y$

for all $x_1, x_2, \cdots, xi\text{-}_1, xi, x'i, xi+_1, \cdots, xn \in N$*:*
Proof. For all $x_1, \cdots, xi, x'i, x''i, \cdots, xn \in N$,
$F(x1, \cdots, (xix'i)x''i, \cdots, xn) = F(x_1, \cdots, xix'i, \cdots, xn)\sigma(x''i)$
$+\tau(xix'i)D(x_1, \cdots, x''i, \cdots, xn)$
$= fD(x_1, \cdots, xi, \cdots, xn)\sigma(x'i)$
$+\tau(xi)F(x_1, \cdots, x'i, \cdots, xn)g\sigma(x''i)$
$+\tau(xi)\tau(x'i)D(x_1, \cdots, x''i, \cdots, xn)$
Also
$F(x_1, \cdots, xi(x'ix''i), \cdots, xn) = D(x_1, \cdots, xi, \cdots, xn)\sigma(x0ix00i)$
$+\tau(xi)F(x_1, \cdots, x'ix''i, \cdots, xn)$
$= D(x_1, \cdots, xi, \cdots, xn)\sigma(x'i)\sigma(x''i)$
$+\tau(xi)fF(x_1, \cdots, x'i, \cdots, xn)$
$\sigma(x''i) + \tau(x'i)D(x1, \cdots, x''i, \cdots, xn)g$
$= D(x_1, \cdots, xi, \cdots, xn)\sigma(x'i)\sigma(x''i)$
$+\tau(xi)F(x_1, \cdots, x'i, \cdots, xn)$
$\sigma(x00i) + \tau(xi)\tau(x'i)D(x_1, \cdots, xi, \cdots, xn)$

Combining the above two relations, we get
$fD(x_1, \cdots, xi, \cdots, xn)\sigma(x'i) + \tau(xi)F(x_1, \cdots, x'i, \cdots, xn)g\sigma(x''i)$
$= D(x_1, \cdots, xi, \cdots, xn)\sigma(x'i)\sigma(x''i) + \tau(xi)F(x_1, \cdots, x'i, \cdots, xn)\sigma(x''i)$
Since σ is an automorphism, replacing $x''i$ by $\sigma\text{-}1(y)$, where $y\ 2\ N$ we find that
$fD(x_1, \cdots, xi, \cdots, xn)\sigma(x'i) + \tau(xi)F(x_1, \cdots, x'i, \cdots, xn)gy$
$= D(x_1, \cdots, xi, \cdots, xn)\sigma(x'i)y + \tau(xi)F(x_1, \cdots, x'i, \cdots, xn)y$

Lemma 2*:*4*:* Let N be prime near-ring admitting a generalized (σ, τ)-i-n-derivation F with associated nonzero (σ, τ)-i-n-derivation D of N and $x\ 2\ N$*:*

- *(i)* If $xF(N, N, \cdots, N) = \{0\}$, then $x = 0$*:*
- *(ii)* If $F(N, N, \cdots, N)x = \{0\}$, then $x = 0$*:*

Proof. (*i*) Given that $xF(x1, \cdots, xix'i, \cdots, xn) = 0$ for all $x1, \cdots, xi, x'i, \cdots, xn \in N$.

This yields that $xfF(x_1, \cdots, xi, \cdots, xn)\sigma(x'i) + \tau(xi)D(x1, \cdots, xi', \cdots, xn)g = 0$*:* By hypothesis, we have $x\tau(xi)D(x_1, \cdots, x'i, \cdots, xn) = 0$. Since τ is an automorphism of N, this implies that $x\,ND(x_1, \cdots, xi', \cdots, xn) = \{0\}$. Now primeness of N and $D \neq 0$, provide us $x = 0$*:*

(*ii*) It can be proved in a similar way by using Lemma 2*:*3.

MAIN RESULTS
The main result of the present paper states as follows:
Theorem 3*:*1*:* Let N be a prime near-ring admitting a nonzero generalized (σ, τ)-i-$n$derivation F with associated (σ, τ)-i-n-derivation D of N*:* If $F(N, N, \cdots, N) \in Z$, then N is a commutative ring.

Proof. Here we divide the proof in two cases:
Case I: Assume that $D \neq 0$. We have for all $x_1, \cdots, xi, x'i, \cdots, xn \in N$
$F(x_1, x_2, \cdots, xi_{-1}, xix'i, xi_{+1}, \cdots, xn) = D(x_1, x_2, \cdots, xi_{-1}, xi, xi_{+1} \cdots, xn)\sigma(x'i) +$
$\tau(xi)F(x_1, x_2, \cdots, xi_{-1}, x'i, xi_{+1}, \cdots, xn) \in Z$: (3:1)
Hence
$fD(x_1, x_2, \cdots, xi_{-1}, xi, xi_{+1} \cdots, xn)\sigma(x'i)+\tau(xi)F(x_1, x_2, \cdots, xi_{-1}, x'i, xi_{+1}, \cdots, xn)g\tau(xi)$
$= \tau(xi)fD(x_1, x_2, \cdots, xi_{-1}, xi, xi_{+1} \cdots, xn)\sigma(x'i) +$
$\tau(xi)F(x_1, x_2, \cdots, xi_{-1}, x'i, xi_{+1}, \cdots, xn)g$:

By hypothesis and Lemma 2:3 we obtain $D(x_1, x_2, \cdots, xi_{-1}, xi, xi_{+1} \cdots, xn)\sigma(x'i)\tau(xi) = \tau(xi)D(x_1, x_2, \cdots, xi_{-1}, xi, xi_{+1} \cdots, xn)\sigma(x'i)$, putting $x'iy$ where $y \in N$ for $x'i$ in the preceding relation and using it again we get
$D(x_1, x_2, \cdots, xi_{-1}, xi, xi_{+1} \cdots, xn)\sigma(x'i)(\sigma(y)\tau(xi) - \tau(xi)\sigma(y)) = 0$
i.e,, $D(x_1, x_2, \cdots, xi_{-1}, xi, xi_{+1} \cdots, xn)N[\sigma(y), \tau(xi)] = \{0\}$. But primeness of N yields that for each fixed xi either $\tau(xi)$ 2 Z or $D(x_1, x_2, \cdots, xi_{-1}, xi, xi_{+1} \cdots, xn) = 0$ for all $x_1, x_2, \cdots, xn \in N$: If first case holds then $xi \in Z$. If second case holds then equation (3:1) takes the form
$F(x_1, x_2, \cdots, xi_{-1}, xix'i, xi_{+1}, \cdots, xn) = \tau(xi)F(x_1, x_2, \cdots, xi_{-1}, x'i, xi_{+1}, \cdots, xn)$
for all $x_1, \cdots, xi_{-1}, x'i, xi_{+1}, \cdots, xn \in N$: By hypothesis and Lemma 2:2, $\tau(xi) \in Z$ i.e., $xi \in Z$. Including both cases we obtain that $xi \in Z$ i.e., $N = Z$. Thus we conclude that N is a commutative near-ring. Since N 6= {0}, there exists 0 6= $p \in N = Z$ such that $p + p \in N = Z$. By Lemma 2:1 we find that N is a commutative ring.

Case II: Now let $D = 0$. Then under this condition relation (3:1) takes the form
$F(x_1, x_2, \cdots, xi_{-1}, xix'i, xi_{+1}, \cdots, xn) = \tau(xi)F(x_1, x_2, \cdots, xi-1, x'i, xi+1, \cdots, xn) \in Z$
for all $x1, \cdots, xi, xi', \cdots, xn \in N$. Using the hypothesis and Lemma 2:2, we infer that $\tau(xi) \in Z$ i.e., $xi \in Z$. This implies that $N = Z$: Now arguing in the similar lines as in the above Case I, we conclude that N is a commutative ring. 6

Corollary 3:1([6], Theorem 3:1). Let N be a prime near-ring admitting a nonzero generalized (σ, τ)-n-derivation F with associated (σ, τ)-n-derivation D of N: If $F(N, N, \cdots, N) \subseteq Z$, then N is a commutative ring.

Theorem 3:2: Let F_1 and F_2 be generalized (σ, τ)-i-n-derivations of prime near-ring N with associated nonzero (σ, τ)-i-n-derivations $D1$ and $D2$ of N respectively. If $[F_1(N, N, \cdots, N), F_2(N, N, \cdots, N)] = \{0\}$, then $(N, +)$ is abelian.

Proof. If both z and $z + z$ commute element wise with $F_2(N, N, \cdots, N)$, then $zF_2(x_1, x_2, \cdots, xi, \cdots, xn) = F_2(x_1, x_2, \cdots, xi, \cdots, xn)z$ and $(z + z)F_2(x_1, x_2, \cdots, xi, \cdots, xn) = F_2(x_1, x_2, \cdots, xi, \cdots, xn)(z + z)$ for all $x_1, x_2, \cdots, xi, \cdots, xn \in N$: In particular, $(z + z)F_2(x_1, x_2, \cdots, xi + x'i, \cdots, xn) = F_2(x_1, x_2, \cdots, xi + x'i, \cdots, xn)(z + z)$ for all $x_1, x_2, \cdots, xi, x'i, \cdots, xn \in N$: From the previous equalities we get $zF_2(x_1, x_2, \cdots, xi + x'i - xi - x'i, \cdots, xn) = 0$,i.e., $zF_2(x_1, x_2, \cdots, (xi, x'i), \cdots, xn) = 0$: Putting $z = F_1(y_1, y_2, \cdots, yn)$ we get $F_1(y_1, y_2, \cdots, yn)F_2(x_1, x_2, \cdots, (xi, x'i), \cdots, xn) = 0$: By Lemma 2:4($ii$) we conclude that $F_2(x_1, x_2, \cdots, (xi, x'i), \cdots, xn) = 0$: Putting $w(xi, x'i)$ in place of additive commutator $(xi, x0i)$ where $w \in N$ we have $F_2(x_1, x_2, \cdots, w(xi, x'i), \cdots, xn) = 0$: i.e., $D_2(x_1, x_2, \cdots, w, \cdots, xn)\sigma(xi, x'i) + \tau(w)F_2(x_1, x_2, \cdots, (xi, x'i), \cdots, xn) = 0$: Previous equality yields $D_2(x_1, x_2, \cdots, w, \cdots, xn)\sigma(xi, x'i) = 0$. By Lemma 2:4($ii$), we conclude that $\sigma(xi, x'i) = 0$: But σ is an automorphism, we obtain that $(xi, x'i) = 0$ and hence $(N, +)$ is abelian.

Corollary 3:2([6], Theorem 3:2). Let F_1 and F_2 be generalized (σ, τ)-n-derivations of prime near-ring N with associated nonzero (σ, τ)-n-derivations D_1 and D_2 of N respectively. If $[F_1(N, N, \cdots, N), F_2(N, N, \cdots, N)] = \{0\}$, then $(N, +)$ is abelian.

Theorem 3:3: Let F and G be generalized (σ, τ)-i-n-derivations of prime nearring N with associated nonzero (σ, τ)-i-n-derivations D and H of N respectively.

If $F(x_1, x_2, \cdots, xn)H(y_1, y_2, \cdots, yn) = -G(x_1, x_2, \cdots, xn)D(y_1, y_2, \cdots, yn)$ for all $x_1, x_2, \cdots, xn, y_1, y_2, \cdots, yn \in N$, then $(N, +)$ is abelian.

Proof. For all $x_1, x_2, \cdots, xn, y_1, y_2, \cdots, yn \in N$ we have,
$F(x_1, x_2, \cdots, xn)H(y_1, y_2, \cdots, yi, \cdots, yn) = -G(x_1, x_2, \cdots, xn)D(y_1, y_2, \cdots, yi, \cdots, yn)$.

We substitute $yi + yi'$ for yi in preceding relation thereby obtaining,

$F(x_1, x_2, \cdots, xn)\, H(y_1, y_2, \cdots, yi + yi, \cdots, yn) + G(x_1, x_2, \cdots, xn)\, D(y_1, y_2, \cdots, yi + yi0, \cdots, yn) = 0$ i.e., $F(x_1, x_2, \cdots, xn)H(y_1, y_2, \cdots, yi, \cdots, yn) +$
$F(x_1, x_2, \cdots, xn)H(y_1, y_2, \cdots, yi', \cdots, yn) + G(x_1, x_2, \cdots, xn)D(y_1, y_2, \cdots, yi, \cdots, yn) +$
$G(x_1, x_2, \cdots, xn)D(y_1, y_2, \cdots, yi, \cdots, yn)$: Using the hypothesis we get, $F(x_1, x_2, \cdots, xn)H(y_1, y_2, \cdots, yi, \cdots, yn) +$
$F(x_1, x_2, \cdots, xn)H(y_1, y_2, \cdots, yi', \cdots, yn) - F(x_1, x_2, \cdots, xn)H(y_1, y_2, \cdots, yi, \cdots, yn) -$
$F(x_1, x_2, \cdots, xn)H(y_1, y_2, \cdots, yi', \cdots, yn) = 0$ i.e., $F(x_1, x_2, \cdots, xn)H(y_1, y_2, \cdots, (yi, yi'), \cdots, yn) = 0$: Now using Lemma 2:4($ii$) we get $H(y_1, y_2, \cdots, (yi, yi'), \cdots, yn) = 0$. Replacing (yi, yi') by $w(yi, yi')$ where $w \in N$ in the previous relation and using it again we have $H(y_1, y_2, \cdots, w, \cdots, yn)\sigma(yi, yi') = 0$ for all $w, y_1, y_2, \cdots, yi, yi', \cdots, yn \in N$: By Lemma 2:4($ii$), we conclude that $\sigma(yi, yi') = 0$ and hence $(N,+)$ is abelian.

Corollary 3:3([6], Theorem 3:3). Let F and G be generalized (σ, τ)-n-derivations of prime near-ring N with associated nonzero (σ, τ)-n-derivations D and H of N respectively. If $F(x_1, x_2, \cdots, xn)H(y_1, y_2, \cdots, yn) = -G(x_1, x_2, \cdots, xn)D(y_1, y_2, \cdots, yn)$ for all $x_1, x_2, \cdots, xn, y_1, y_2, \cdots, yn \in N$, then $(N,+)$ is abelian.

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