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## Simulation the Growth Process of Scoliosis Disease by Using Finite Element Method

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**Abstract:** Scoliosis deformity is characterized as a lateral deviation of the spine in three-dimensions accompanied by axial rotation of the vertebrae. In this paper by surveying on the mathematical model of spine movement, we simulate the growth of scoliosis disease of a patient by using finite element method for vertebrae T1-T6 of spine. For this purpose by using Newmark method, we first deal the numerical solution of the given mathematical model. We will also explain the growth process of this disease by using geometric drawing in Matlab environment. Moreover, numerical simulations based on the mentioned process are presented.

**Keywords:** Scoliosis deformity, axial rotation, vertebrae, mathematical model.

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### Introduction

The spine is one of the most important and indispensable structures in the human body. It allows complex motions whilst providing stability and protection for the spinal cord during a variety of loading conditions. However, it is also a very vulnerable part of our skeleton that is open to many medical problems, namely scoliosis.

Scoliosis deformity is characterized as a lateral deviation of the spine in three-dimensions accompanied by axial rotation of the vertebrae. It is classified into four categories: congenital, neuromuscular, degenerative and idiopathic. In idiopathic scoliosis, the etiology or cause of the disease remains unknown. This is the most common type of scoliosis involving 70-80% of the scoliotic patients. The current consensus is that a multi-factorial process is occurring. These include genetic, abnormal biomechanical forces, abnormal neurophysiologic process, connective tissue abnormality and biochemical changes during puberty [10].

Injury and disease may be studied using clinical studies, animal models, cadaveric models and mathematical models. Clinical studies are useful in assessing treatment outcomes. Animal and cadaveric models may be used to find the external response to external loading such as flexion/extension, right and left lateral bending, and right and left axial rotation. Mathematical models such as the finite element method (FEM) may also be used to find the structural response to external loading, but have a more important function in establishing the internal response such as stress and strain in response to external loading [13].

Since spine acts like a mechanical structure, so it is possible to explain the spine movement by using physical relations. In this regard, the first numerical model was proposed in by Gardner-Morse and was based on nonlinear FEM [2, 3]. These models are limited by unknown patient-specific mechanical properties, and most of these models presented several convergence difficulties due to the complexity of the material laws, displacements fields, and boundary conditions [11]. Recent works that aiming at modeling the behavior of spinal structures, are based on differential algebraic equations (DAEs) arising from multibody system dynamics and linear finite element analysis [8, 9]. Most of these models have been characterized based on the pioneer experimentally work of Panjabi [7].

In this paper, at first we will simulated scoliosis disease by using FEM for vertebrae T1-T6 of spine, and then by using Newmark method, we deal with numerical solving of given mathematical model. The growth process of this disease by using geometric drawing in Matlab environment is presented. Finally, we present some numerical conclusions based on the mentioned process.

**Spine movement equations**

The theory of elasticity consists of a set of differential equations that describe the state of stress, strain and displacement of each point within an elastic deformable body. Based on this theory, first spine movement equations are expressed to be able then a simulating is given for vertebrae T1-T6 of spine which are under forces. A relationship between the deformation of the object and the exerted forces can be established by synthesizing those equations. The equation governing the resulting deformation from external forces is:

$$M\ddot{U} + D\dot{U} + KU = F \tag{1}$$

where  $U$  is the  $3n$ -dimensional nodal displacement vector;  $\dot{U}$  and  $\ddot{U}$  are the velocity and acceleration vectors respectively;  $F$  is the external force vector;  $M$  is the  $3n \times 3n$  mass matrix;  $D$  is the damping matrix;  $K$  is the stiffness matrix;  $n$  is the number of nodes in the FEM. The salient features this matrices can be dispersed and its symmetric are noted [12].

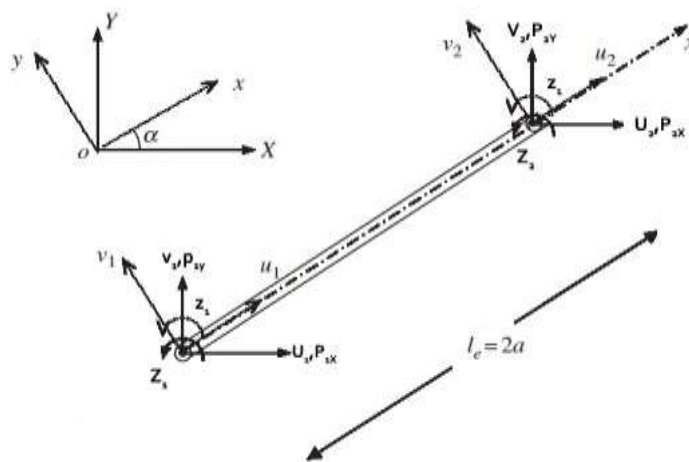
**Simulation method for vertebrae T1-T6**

The constructing an accurate finite element spine model is difficult because of the complexity of spine. Thus in this simulation of vertebrae T1-T6 spine by FEM, vertebrae T1-T6 as a finite element structure, where in the motion segments are represented as connected deformable 2-dimension (2D) beam element. Its features include [4]:

- i. The nodes of the beam elements are the vertebral centroids.
- ii. The elastic lengths of the beam elements correspond to the distance between the centroids of the successive vertebral bodies.
- iii. The geometrical parameters (crosssectional area, moment of inertia and length of the element) and the modulus of elasticity vary from one element to another.
- iv. Each element is homogeneous and isotropic.

Since a two-dimensional analysis is carried out in the scoliotic plane, torsional moments are not considered and axial rotations of the vertebral bodies are not accounted. Hence each 2D beam element has three degrees of freedom at each node, namely two orthogonal displacements and rotation in the scoliotic plane.

The spine has a non-linear structure; its constitutive (modulus) property is nonlinear and it undergoes large deformations. Since our FEM has invoked linear elasticity and small displacements, we have to apply small forces to the spine incrementally and monitor the deformations (or nodal displacements) at each incremental level. Since one of the goals of this paper is geometrical demonstration of deformed spine, so for instance, we demonstrate one of 2D beam element in local and global coordinates in Figure1 in which [5]:



**Fig-1: 2D beam element in local and global coordinates**

- $(X, Y)$  illustrates global coordinate system.
- $P_X$  and  $P_Y$  illustrate nodal forces in global coordinate in the directions  $X$  and  $Y$  respectively.
- $U$  and  $V$  illustrate nodal displacement vector in the directions  $X$  and  $Y$ , respectively.
- $Z$  illustrates rotation in the  $(X, Y)$ -plane.
- $(x, y)$  illustrates local coordinate system.

- $p_x$  and  $p_y$  illustrate nodal forces in local coordinate in the directions x and y, respectively.
- u and v illustrate nodal displacement vector in the directions x and y, respectively.
- z illustrates rotation in the (x,y)-plane.

### Numerical solution method of spine movement system

For solving equation system (1), we need first to calculate coefficient matrices such as K, C and M for the patient.

Regarding to achieve simulation by the help of 2D beam elements, these factors are depended to parameters such as length, cross sectional area, stiffness, density element and so on. These parameters are determined from experimental informations in papers and provided informations from CT of spine. Boundry condition is considered for equation system as below [4]:

- 1- T6 vertebral movement on (X,Y)-plane has been assumed to be fixed at all time steps.
- 2- T1 vertebral just can move along Y axis at all time steps.

We use direct integration of Newmark method for solving this equation system. Unconditional stability of this method is one of the dominant features that against the other methods like central difference, Wilson- $\theta$  and so on [5]. This method is based on two main idea:

- i. Instead of solving equation at all times, we devide the ideal time into separate intervals and solve the equations at these sub-intervals.
- ii. The way of changes of displacement, velocity and acceleration in each time step is clear.
- iii. Our next aim in this section is to present Newmark algorithm.

### Newmark algorithm:

The newmark integration can also be understood to be an extension of the linear acceleration method. The following assumptions are used [5]:

$$\dot{U}^{t+\Delta t} = \dot{U}^t + ((1-\delta)\ddot{U}^t + \delta\ddot{U}^{t+\Delta t})\Delta t \quad (2)$$

$$U^{t+\Delta t} = U^t + \dot{U}^t\Delta t + ((\frac{1}{2}-\alpha)\ddot{U}^t + \alpha\ddot{U}^{t+\Delta t})\Delta t^2 \quad (3)$$

Where  $\alpha$  and  $\delta$  are parameters as  $\delta=1/2$  and  $\alpha=1/4$ . To obtain integration accuracy and stability, we use of these parameters.

For obtaining the displacements, velocities and accelerations at time  $t+\Delta t$ , the equations system (1) at time  $t+\Delta t$  are also considered as below:

$$M\ddot{U}^{t+\Delta t} + D\dot{U}^{t+\Delta t} + KU^{t+\Delta t} = F^{t+\Delta t} \quad (4)$$

Solving from (3) for  $\dot{U}^{t+\Delta t}$  in terms of  $U^{t+\Delta t}$ , and then substituting for  $\dot{U}^{t+\Delta t}$  in to (2), we obtain equations for  $\dot{U}^{t+\Delta t}$  and  $\ddot{U}^{t+\Delta t}$ , each in terms of the unknown displacements  $U^{t+\Delta t}$  only. These two relations for  $\dot{U}^{t+\Delta t}$  and  $\ddot{U}^{t+\Delta t}$  are substituted into (4) to solve for  $U^{t+\Delta t}$ , after which, using (2) and (3),  $\dot{U}^{t+\Delta t}$  and  $\ddot{U}^{t+\Delta t}$  can also be calculated. In what follows, we present Newmark algorithm which was first presented in [10].

### Algorithm 1: Newmark algorithm

**Step1:** compute stiffness K, mass matrix M, and damping matrix C [5].

**Step2:** Initialize  $U^0$ ,  $\dot{U}^0$  and  $\ddot{U}^0$ .

**Step3:** Select time step size  $\Delta t$ , and parameters  $\delta \geq 0.5$ ,  $\alpha \geq 0.25(0.5 + \delta)^2$

**Step4:** Compute the matrix stiffness  $K^\wedge$  as  $K^\wedge = K + \frac{M}{\alpha \Delta t^2} + \frac{D\delta}{\alpha \Delta t}$

**Step5:** Calculate effective loads  $(F^\wedge)^{t+\Delta t}$  as:

$$(F^\wedge)^{t+\Delta t} = F^t + (\frac{M}{\alpha \Delta t^2} + \frac{D\delta}{\alpha \Delta t})U^t + (\frac{M}{\alpha \Delta t} + D(\frac{\delta}{\alpha}-1))\dot{U}^t + (M(\frac{1}{2\alpha}-1) + \frac{D\Delta t}{2}(-2 + \frac{\delta}{\alpha}))\ddot{U}^t$$

**Step6:** Compute the displacements vector as below:

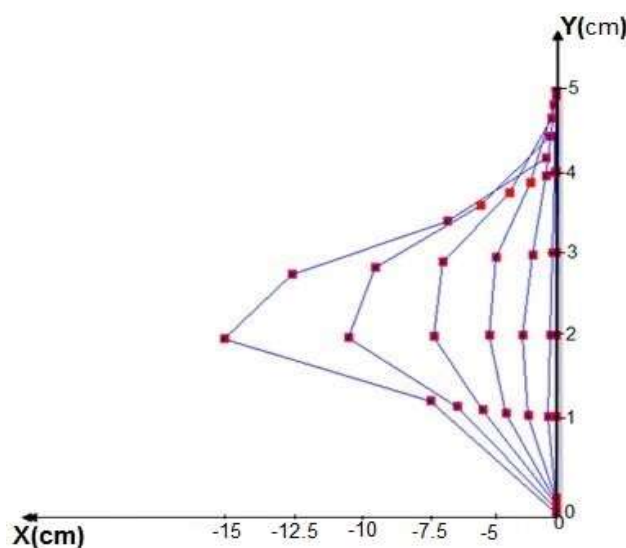
$$K^\wedge U^{t+\Delta t} = F^{t+\Delta t}$$

**Step7:** Calculate accelerations and velocities at time  $t + \Delta t$  :

$$\ddot{U}^{t+\Delta t} = \frac{1}{\alpha \Delta t^2} (U^{t+\Delta t} - U^t) - \frac{1}{\alpha \Delta t} \dot{U}^t - \left(\frac{1}{2\alpha} - 1\right) \ddot{U}^t$$

$$\dot{U}^{t+\Delta t} = \dot{U}^t + \Delta t (1 - \delta) \ddot{U}^t + \delta \Delta t \ddot{U}^{t+\Delta t}$$

In what follows, we show the growth procedure diagram of scoliosis disease during two years interval. To implement the algorithm, we use of some parameters, which are presented in [1, 6]. Drawing the diagrams in Figure 2 show the growth process of this disease on the sequence from right to left in repetitions of 2, 5, 8, 10, 15 and 20. Indeed in this algorithm, each 5 repetitions considered as a six month period.



**Fig-2: The growth process of scoliosis disease**

## RESULT AND DISCUSSION

In this paper, the growth process of scoliosis for vertebrae T1-T6 of spine was studied and simulated by using FEM. the proposed mathematical model solved numerically by using Newmark algorithm. Although the use of FEM increase the computing time, but it increases the accuracy of computation as well which in much more important. Despite of the high freedom degree of the model, the solution procedure causes to have a correct procedure for the growth process of scoliosis patient. The obtained results of the geometrical drawing of the scoliosis process of growth indicate that the variation of spine condition in 2-dimension space, watches the facts of this disease. In future, this drawing must be done in 3-dimension in order to find variation of the ribcage as well as spine.

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