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A pattern search method with non-monotone strategy

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Abstract: In this paper, we propose and analyze a new non-monotone pattern search method for unconstrained optimization problems. Actually, we combined a new strategy of non-monotone technique with the traditional pattern search method. Then some properties of the new algorithm are analyzed. Theoretical analysis shows that the new proposed method has a global convergence under some mild conditions.
Keywords: unconstrained optimization, pattern search methods, non-monotone technique, global convergence.

INTRODUCTION

Consider the following unconstrained optimization problem:

$$\min f(x), \quad x \in R^n, \tag{1}$$

where $f : R^n \to R$, is continuously differentiable, but the information about the gradient of f is either unavailable or unreliable. There are lots of problems where derivatives are unavailable but we also want to do some optimization.

In such cases, direct search methods which neither compute nor approximate derivatives plays an important role [1-3]. They are designed to apply to the functions whose information of the derivatives is unavailable or unreliable and consequently to which all the classic derivative-based methods are useless. This kind of methods only uses the values of objective function and involves the comparison of each trial solution with the best previous solution.

However, due to the shrinking of available information, its implementation also becomes difficult and the convergence speed is slower than using derivative methods. Pattern search method is a special kind of direct search method; the method is put forward by Hooke and Jeeves [4] in 1961. It uses an independent objective function model to determine the current search direction by comparing the current iteration point of function value with pattern of each function value. The process of pattern search contains two moves: one is pattern move, the other is exploratory move. Pattern moves execute actual minimizing of the objective function and exploratory moves provide probable directions.

We would like to present some basic concepts we need. We use $\|\cdot\|$ and $\langle \cdot,\cdot \rangle$ to represent the Euclidean norm and inner product, respectively.

PATTERN SEARCH METHODS

Pattern search methods are characterized by the nature of the generating matrices and the exploratory moves algorithms. These features are discussed more fully in [2, 5, 6].

To define a pattern, we need two components, a basis matrix and a generating matrix.

The basis matrix can be any nonsingular matrix $B \in R^{n\times n}$. The generating matrix is a matrix $C_k \in Z^{n\times P_k}$, where $P_k > n+1$. We partition the generating matrix into components

$$C_k = [\Gamma_k \; L_k \; 0].$$

We require that $\Gamma_k \in M$, where M is a finite set of integral matrices with full row rank. We will see that Γ_k must have at least $n+1$ columns. The 0 in the last column of C_k is a single column of zeros.

A pattern P_k is then defined by the columns of the matrix $P_k = BC_k$. For convenience, we use the partition of the generating matrix C_k to partition P_k as follows:

$$P_k = BC_k = [B\Gamma_k \ BL_k \ 0].$$

Given $\Delta_k \in R, \Delta_k > 0$, we define a trial step s_k^i to be any vector of the form $s_k^i = \Delta_k BC_k^i$, where C_k^i is a column of C_k. Note that BC_k^i determines the direction of the step, while Δ_k serves as a step length parameter.

At iteration k, we define a trial point as any point of the form $x_k^i = x_k + s_k^i$, where x_k is the current iterate.

The following algorithm states the pattern search method for unconstrained minimization.

Algorithm 1

Let $x_0 \in R_n$ and $\Delta_0 > 0$ be given.

For $k = 0, 1, 2, \cdots$

1) Compute $f(x_k)$.

2) Determine a step s_k using an unconstrained exploratory moves algorithm.

3) If $f(x_k + s_k) < f(x_k)$, then $x_{k+1} = x_k + s_k$, otherwise $x_{k+1} = x_k$.

4) Update C_k and Δ_k.

Especially, Δ_k should be updated by the following standards.

Let $\tau \in Q, \tau > 1$, and $\{w_0, w_1, \cdots, w_l\} \subset Z, w_0 < 0$, and $w_i \ge 0, i = 1, \cdots, l$.

Let $\theta = \tau^{w_0}$ and $\lambda_k \in \wedge = \{\tau^{w_1}, \cdots, \tau^{w_l}\}$

a) If $f(x_k + s_k) \ge f(x_k)$, then $\Delta_{k+1} = \theta\Delta_k$.

b) If $f(x_k + s_k) < f(x_k)$, then $\Delta_{k+1} = \lambda_k\Delta_k$.

Obviously, the conditions on θ and $\wedge$ should ensure that $0 < \theta < 1$ and $\lambda_i \ge 1$ for all $\lambda_i \in \wedge$.

Non-monotone technique and our strategy

Recently, non-monotone techniques are widely used in the line search and trust region methods. In 1982, the first non-monotone technique that is the so-called watchdog technique was proposed by Chamberlain et al. [7] for constrained optimization to overcome the Maratos effect. Due to the high efficiency of non-monotone techniques, many authors are interested in working on the non-monotone techniques for solving optimization problems [8-11].

The general non-monotone form is as follows:

$$f_{l(k)} = f(x_{l(k)}) = \max_{0 \le j \le m(k)} \{f_{k-j}\}, \quad k = 0, 1, 2, \ldots \tag{2}$$

where $m(0) = 0, \ 0 \le m(k) \le \min\{N, m(k-1)+1\}$ and $N \ge 0$ is an integer constant.

Some researchers showed that utilizing non-monotone techniques may improve both the possibility of finding the global optimum and the rate of convergence [8, 9]. However, although the non-monotone technique has many advantages, it contains some drawbacks [9, 11]. To overcome those disadvantages, Ahookhosh et al. in [11] proposed a new non-monotone technique to replace (2). They define

$$R_k = \eta_k f_{l(k)} + (1-\eta_k) f_k, \tag{3}$$

where $\eta_{\min} \in [0,1)$, $\eta_{\max} \in [\eta_{\min}, 1]$ and $\eta_k \in [\eta_{\min}, \eta_{\max}]$.

Inspired by [5, 11], we use (3) to present a new non-monotone pattern research method. In this method, the algorithm adjusts the exploratory moves rules by using the non-monotone technique (3).

The rest of this paper is organized as follows. In Section 4, we introduce the algorithm of non-monotone pattern research method. In Section 5, we analyze the new method and prove the global convergence.

Non- monotone pattern search method

Usually, monotonic decline technology is commonly used in direct search method, to compare the current function value $f\left(x_{k+1}\right)$ with the value of a function iteration point $f\left(x_k\right)$. By using the technology of non-monotone descent, we design a new non-monotone pattern search method. Although this can lead to the iteration point function values sometimes increase, but due to the limitation of N, function value sequence presents the downward trend, and the pace of decline is faster, which improved the overall convergence. By means of adjusting the rules of exploratory moves, we can achieve monotonic decline technology. To be specific, if

$$f\left(x_k+s_k\right)<R_k, \text{ then } x_{k+1}=x_k+s_k. \tag{4}$$

Then, we can state our new algorithm.

Algorithm 2

Step1. Start with $x_0 \in R^n, \Delta_0>0, k=0, \varepsilon>0,$ non-singular matrix $B \in R^{n\times n}$, $0<\theta<1, \lambda \geq 1.$

$$C=\left[M,-M,0,0\right]\in Z^{n\times(2n+2)}, M \in R^{n\times n}$$

Step2. Check the stopping criteria. If $\Delta_k \leq \varepsilon$ or $\left\|x_k-x_{k-1}\right\| \leq \varepsilon$, then stop.

Step3. Pattern search iteration: $d_0=x_k, i=1$

3.1) If $i=n+2,$ then go to step2, else if $1\leq i\leq n,$ then $d=BC_k^i$, else if $i=n+2,$ then $C_k^{n+1}=x_k-d_0$, $d=BC_k^{n+1}$, else if $d=0,$ then $\Delta_{k+1}=\theta\Delta_k, C_{k+1}=C_k,$ $k=k+1$, go to step 2.

3.2) If $f\left(x_k+\Delta_k d\right)\leq R_k,$ go to step 3.3, else if $f\left(x_k-\Delta_k d\right)\leq R_k$, then $d=-d$, go to step 3.3, else $i=i+1$, go to step 3.1.

3.3) $x_{k+1}=x_k+\Delta_k d, C_{k+1}=C_k, k=k+1,$ go to step 4.

Step4. If $f\left(x_k+\Delta_k d\right)\leq R_k,$ then $\Delta_{k+1}=\theta\Delta_k,$ else $\Delta_{k+1}=\lambda\Delta_k$. Go to step 2.

Convergence analysis

In references [2], Torczon has discussed the convergence of basic pattern search method in detail. On this basis, we prove the global convergence of algorithm 2.

Lemma 1 Suppose that $\{x_k\}$ is generated by Algorithm 2, the level set $L_0=\{x\in R^n \mid f(x)\leq f(x_0)\}\subset \Omega,$ where $\Omega\in R^n$ is a closed, bounded set. Let $s_k=x_{k+1}-x_k$, if $s_k\neq 0,$ for all $k>0,$ there exist constant V_*, V^* such that

$$V_*\Delta_k \leq \left\|s_k\right\| \leq V^*\Delta_k.$$

Proof. The proof is similar to Lemma 4.1 in [5], we omit it for convenience.

Lemma 2 (See Lemma 4.2 in [5] and Theorem 3.2 in [2]) Suppose that $\{x_k\}$ is generated by Algorithm 2, the level set L_0 is a bounded set. Then any iterate $x_N\,(N>0)$ can be expressed by

$$x_N=x_0+\left(b^{r_{LB}}a^{-r_{UB}}\right)T^{-2}\Delta_0 B\sum_{k=0}^{N-1} z_k,$$

where

- $T, a, b \in \mathbb{N}, b/a=t$ and t is as defined in the Algorithm 1.
- r_{LB}, r_{UB} depend on N.
- $z_k \in Z^n, k=0,\dots,N-1.$

Lemma 3 Suppose that the level set L_0 is compact, then

$$\lim_{x\to\infty}\inf \Delta_k=0.$$

Proof. The proof is by contradiction.

A. Assume $0 < \Delta_{LB} \le \Delta_k$ for all k, then we have $\Delta_k = \tau^{r_k}\Delta_0 \ge \Delta_{LB}, r_k \in Z$. This means that the sequence $\{\tau^{r_k}\}$ is bounded away from zero.

B. By induction, we will show that $x_k \in L(x_0)$, for all $k \in N$. The result evidently holds for $k = 0$. Assume that $x_k \in L(x_0)$, then from definition of (4), we have that $x_{k+1} \in L(x_0)$. Thus, the sequence $\{x_k\}$ is contained in $L(x_0)$.

C. From $x_k \in L(x_0)$ and $x_k = x_{k-1} + \Delta_k B C_k^i$, we know $\{\Delta_k\}$ is bounded above.

D. Considering A and C, we have $\{\tau^{r_k}\}$ is a finite set. Let $r_{LB} = \min_{0 \le k < +\infty} \{r_k\}$, $r_{UB} = \max_{0 \le k < +\infty} \{r_k\}$, for all k, using Lemma 2, we get $x_k = x_0 + \left(\beta^{r_{LB}}\alpha^{-r_{UB}}\right)T^{-2}\Delta_0 B \sum_{i=0}^{k-1} z_i$, the rest proof is similar to Theorem 3.3 in [2].

Lemma 4 (See Lemma 4.4 in [5]) Suppose that the level set L_0 is compact, f is continuously differentiable on a neighborhood of L_0. Let $\Omega_\varepsilon = \left\{x_k \in L(x_0) \middle| \mathrm{dist}(x, x^*) \ge \varepsilon > 0\right\}$, $x_0 \in \Omega_\varepsilon$, if there exists $\delta > 0$ for all $k \ge 0$ that $x_k \in \Omega_\varepsilon, \Delta_k < \delta$, then the kth exploratory move is successful, i.e. $f\left(x_k + s_k\right) < R_k$, so we have $\Delta_{k+1} > \Delta_k$.

Theorem 5 Suppose that the sequence $\{x_k\}$ is generated by Algorithm 2 , then

$$\liminf_{k \to \infty} \|\nabla f(x_k)\| = 0.$$

Proof. Assume that $\liminf_{k \to \infty} \|\nabla f(x_k)\| \ne 0$. By Lemma 4 we know that there exists $\Delta_{LB} > 0$ such that for all k, $\Delta_k > \Delta_{LB}$. This contradicts Lemma3.

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