

## An Interesting Diophantine Problem

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**Abstract:** We search for non-zero distinct integer triples  $(a_0, a_1, a_2)$  such that each of the expressions  $a_0 + a_1$ ,  $a_0 + a_2$ ,  $a_1 + a_2$ ,  $2(a_0 + a_1 + a_2)$  is a perfect square.

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### INTRODUCTION

Number theory, called the Queen of Mathematics, is a broad and diverse part of Mathematics that developed from the study of the integers. The foundations for Number theory as a discipline were laid by the Greek mathematician Pythagoras and his disciples (known as Pythagoreans). One of the oldest branches of mathematics itself, is the Diophantine equations since its origins can be found in texts of the ancient Babylonians, Chinese, Egyptians, Greeks and so on[7-8]. Diophantine problems were first introduced by Diophantus of Alexandria who studied this topic in the third century AD and he was one of the first Mathematicians to introduce symbolism to Algebra. The theory of Diophantine equations is a treasure house in which the search for many hidden relation and properties among numbers form a treasure hunt. Infact, Diophantine problems dominated most of the celebrated unsolved mathematical problems. Certain Diophantine problems come from physical problems or from immediate Mathematical generalizations and others come from geometry in a variety of ways. Certain Diophantine problems are neither trivial nor difficult to analyze [1-6]. In this context one may refer [9, 10].

In this communication we search for non-zero distinct integer triples  $(a_0, a_1, a_2)$  such that each of the expressions  $a_0 + a_1$ ,  $a_0 + a_2$ ,  $a_1 + a_2$ ,  $2(a_0 + a_1 + a_2)$  is a perfect square.

### METHOD OF ANALYSIS

Let  $a_0$ ,  $a_1$ ,  $a_2$  be three non-zero distinct integer such that

$$a_0 + a_1 = p^2 \tag{1}$$

$$a_0 + a_2 = q^2 \tag{2}$$

$$a_1 + a_2 = r^2 \tag{3}$$

$$2(a_0 + a_1 + a_2) = s^2 \tag{4}$$

Solving the system of equation (1) to (3), we have

$$\left. \begin{aligned} a_0 &= \frac{1}{2}(p^2 + q^2 - r^2) \\ a_1 &= \frac{1}{2}(p^2 + r^2 - q^2) \\ a_2 &= \frac{1}{2}(q^2 + r^2 - p^2) \end{aligned} \right\} \tag{5}$$

Substituting (5) in (4), we have

$$p^2 + q^2 + r^2 = s^2 \tag{6}$$

which is the well known space Pythagorean equation, whose solutions may be taken as

$$\left. \begin{aligned} p &= 2ab \\ q &= 2ac \\ r &= a^2 - b^2 - c^2 \\ s &= a^2 + b^2 + c^2 \end{aligned} \right\} \tag{7}$$

Substituting the above values of  $p, q, r, s$  in (5), the corresponding values of  $a_0, a_1, a_2$  are given by

$$\left. \begin{aligned} a_0 &= 3a^2b^2 + 3a^2c^2 - b^2c^2 - \frac{1}{2}(a^4 + b^4 + c^4) \\ a_1 &= a^2b^2 - 3a^2c^2 + b^2c^2 + \frac{1}{2}(a^4 + b^4 + c^4) \\ a_2 &= a^2c^2 - 3a^2b^2 - b^2c^2 + \frac{1}{2}(a^4 + b^4 + c^4) \end{aligned} \right\} \tag{8}$$

The values represented by  $a_0, a_1, a_2$  satisfy the conditions of the problem under consideration. Since  $a_0, a_1,$  and  $a_2$  are to be integers, it is observed that they are integers provided the parameters  $a, b, c$  are even or atleast one of  $a, b, c$  is even and the other two are odd. The above results are presented in the following table.

a	b	c	$a_0$	$a_1$	$a_2$	$a_0+a_1$	$a_0+a_2$	$a_1+a_2$	$2(a_0+a_1 + a_2)$
2	4	6	-736	992	1312	256	576	2304	3136
2	1	3	62	-46	82	16	144	36	196
3	4	5	226	350	674	576	900	1024	2500
1	3	4	-238	274	302	36	64	576	676

Note that the solutions (7) of (6) are three parametric solutions. However, we have four parametric solutions of (6) which are represented by

$$\left. \begin{aligned} p &= m^2 + n^2 - g^2 - h^2 \\ q &= 2(mg + nh) \\ r &= 2(ng - mh) \\ s &= m^2 + n^2 + g^2 + h^2 \end{aligned} \right\} \tag{9}$$

Substituting (9) in (5), we obtain the value of  $a_0, a_1,$  and  $a_2$  in four parameters. It is possible of choose the parameters so that  $a_0, a_1, a_2$  are integers.

A few examples are given below.

m	n	g	h	$a_0$	$a_1$	$a_2$	$a_0+a_1$	$a_0+a_2$	$a_1+a_2$	$2(a_0+a_1 + a_2)$
2	4	6	6	6944	-544	800	$80^2$	$88^2$	$16^2$	$120^2$
1	3	5	7	3272	824	-568	$64^2$	$52^2$	$16^2$	$84^2$
2	4	1	3	434	-334	350	$10^2$	$28^2$	$4^2$	$30^2$

**CONCLUSION**

In this paper we have presented two sets of non-zero integer triples such that the sum of any two as well as twice the sum of these in each set is a perfect square.

Since Diophantine problems are rich in variety, one may search for other patterns of integer triples under suitable constraints.

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