OPEN ACCESS

Scholars Bulletin

Abbreviated Key Title: Sch Bull

ISSN 2412-9771 (Print) | ISSN 2412-897X (Online)

Scholars Middle East Publishers, Dubai, United Arab Emirates

Journal homepage: https://saudijournals.com

Subject Category: Mathematics

A Note on Lie Algebra of Killing Vector Fields of Locally Rotationally Symmetric Bianchi Type-I Spacetime in f(Q)-Gravity

Fakhar Alam[1*], Muhammad Ramzan[1], Muhammad Arslan Rafiq Khokhar[2], Sajid Ali[2], Muhammad Kashif[3], Sofia Batool[4]

[1]Department of Mathematics, The Islamia University of Bahawalpur, Pakistan

[2]Centre for Advanced Studies in Pure and Applied Mathematics (CASPAM), Bahauddin Zakariya University, Multan, Pakistan

[3]Department Mathematics & Statistics, University of agriculture, Mathematics & Statistics, 38000 Faisalabad, Pakistan

[4]Department of Mathematics, University of Narowal, Pakistan

DOI: https://doi.org/10.36348/sb.2024.v10i10.001 | **Received:** 02.11.2024 | **Accepted:** 05.12.2024 | **Published:** 10.12.2024

***Corresponding author:** Fakhar Alam

Department of Mathematics, the Islamia University of Bahawalpur, Pakistan

Abstract

The goal of this paper is to derive the Lie algebra of Killing vector fields for the locally rotationally symmetric Bianchi type-I spacetime within the framework of f(Q) gravity, where f(Q) gravity is a modified gravitational theory that extends General Relativity by introducing a function of the non-metricity tensor Q to explore alternative models of gravity. To achieve this, various algebraic methods and direct integration techniques are employed. Different metric functions are analyzed, and the associated Killing vectors are determined for each case. It is observed that the spacetime under investigation can support either4, 6, or 10 Killing vector fields.

Keywords: Lie Derivative, Differentiation, Straightforward Integration, Killing Vector Fields.

1. INTRODUCTION

The concept of Killing vector fields was introduced by the German mathematician Wilhelm Killing in the late 19th century. Killing's work focused on exploring the symmetries of a Riemannian or pseudo-Riemannian manifold. A vector field K is termed a Killing vector field if it satisfies the Killing equation $L_K g_{\alpha\beta} = 0$ [1, 2]. Locally rotationally symmetric (LRS) spacetimes are a subclass of Bianchi type models that exhibit specific symmetry properties. Bianchi type-I models are homogeneous but not necessarily isotropic, meaning they have spatial symmetry but not necessarily uniformity in all directions. Locally rotationally symmetric models are particularly interesting as they retain rotational symmetry around a specific axis while allowing for general homogeneity. These models are used to study cosmological scenarios where the universe's large-scale structure possesses certain symmetrical features [3, 4].

The concept f(Q) gravity represents a generalization of Einstein's General Theory of Relativity. Introduced by researchers in recent years, f(Q) gravity is a modification where the Einstein-Hilbert action is replaced by a more general function of the Q-tensor. The Q-tensor, which is related to the torsion of the spacetime manifold, extends the gravitational theory beyond the classical framework. Before f(Q) gravity, several theories of gravity were developed to address various aspects of gravitational phenomena. Notably, General Relativity (GR), formulated by Albert Einstein in 1915, remains the cornerstone of modern gravitational theory. GR describes gravity as the curvature of spacetime caused by mass and energy. Other theories, such as Scalar-Tensor Theories and f(R) gravity (where the action is a function of the Ricci scalar R), were developed to address specific limitations and anomalies within the framework of GR [5-7].

General Relativity, a refined theory of gravitation, characterizes gravity as a fundamental feature of spacetime's geometry. The theory connects the curvature of spacetime directly to the matter present through the Einstein field equations [8, 9]. The nonlinear nature of these field equations makes it challenging to identify exact solutions that accurately represent physical situations. To obtain exact solutions for Einstein's field equations and to classify them, it is essential to impose specific symmetry constraints. Among the most interesting symmetry constraints are Killing, homothetic, conformal, and self-similar vector fields. These symmetries offer essential insights into the physical characteristics of matter and the geometric properties of spacetime. Our universe permits matter to follow specific conservation laws under particular conditions. These conservation laws can also be explored by examining various symmetries. This paper focuses on the classification of the Lie algebra of Killing vector fields for Locally Rotationally Symmetric Bianchi type-I Spacetimes in the context of f(Q) gravity. It is important to note that a vector field K is classified as a Killing vector field if it satisfies the Killing equation, given by

$$\underset{K}{L} g_{\alpha\beta} = g_{\alpha\beta,\lambda}K^{\lambda} + g_{\lambda\beta}K^{\lambda} + g_{\alpha\lambda}K^{\lambda} = 0, \qquad (1)$$

In equation (1) $\underset{K}{L}$ denotes the Lie derivative of the metric tensor $g_{\alpha\beta}$ along the vector field K This operation measures how the metric tensor changes when one flows along the direction specified by the vector field

LRS Bianchi Type I Space Times to Solve Field Equations

The $f(Q)$ gravity action S is [10].

$$S = \int\left(\frac{f(Q)}{2} + L_m\right)d^4x\sqrt{-g}, \qquad (2)$$

Where g denotes the determinant of metric tensor and L_m is the matter Lagrangian. Metric tensor led to equations of motions upon variation of action, which are [10].

$$\frac{2}{\sqrt{-g}}\nabla_a\left(\sqrt{-g}\, f_Q P^a_{bc}\right) - \frac{1}{2} f g_{bc} + f_Q(P_{bai}Q^{ai}_c - 2Q_{aib}P^{ai}_c) = T_{bc} \qquad (3)$$

With respect to the non-metricity scalar Q, f_Q is the derivative of (Q), P_{bai} is the non-metricity tensor, P^a_{bc} is the super-potential tensor and T_{bc} denote the EMT. LRS Bianchi type-1 was considered as a background metric to solve Eq (3), which is [1].

$$ds^2 = -dt^2 + \alpha^2(t)dx^2 + \beta^2(t)\left[dy^2 + dz^2\right], \qquad (4)$$

The functions alpha = alpha (t) and beta = beta (t) are non-zero functions dependent on the cosmic time "t". When alpha = beta, the spacetimes described by equation (5) become an essential class of solutions in General Relativity, specifically representing the Friedmann–Lemaître–Robertson–Walker (FLRW) spacetimes. These spacetimes have been extensively examined within the field of cosmology. Comprehensive studies on FLRW cosmologies within the context of \(f(R) \) gravity can be found in various key reviews [11, 12]. This paper aims to classify the spacetimes defined by equation (5) using conformal vector fields (CVFs) in the framework of f(R) gravity. This classification will focus on spacetime components that lead to different significant forms of spacetime. Notably, when alpha = beta, the spacetimes in equation (5) reach the highest dimension for CVFs, as reported in [13]. Conversely, the spacetimes can also admit a minimum number of Killing vector fields (KVFs) [14].

$$K_1 = \frac{\partial}{\partial x}, K_2 = \frac{\partial}{\partial y}, K_3 = \frac{\partial}{\partial z} \text{ and } K_4 = z\frac{\partial}{\partial y} - y\frac{\partial}{\partial z} \qquad (5)$$

Translations along the x, y, and z directions of vector fields represents the conservation of linear momentum in each respective direction. Angular momentum is generated by the rotational symmetry of Killing vector fields (KVFs). The primary objective of this study is to derive such vector fields within the framework of f(R) gravity, focusing on the spacetimes described by equation [5]. For the matter content in this paper, a perfect fluid is assumed, with the EMT taking the standard form for such a fluid.

$$T_{bc} = (\rho + p)u_b u_c + p g_{bc}, \qquad (6)$$

Where p and ρ are pressure and density of the matter energy of the fluid element along the four-velocity vector u. The set of field equations have been found upon utilizing Eqs. (4) and (6) in Eq.(3), which are [15, 16].

$$\frac{1}{2} f - 2 f_Q \left[-2\frac{\dot{\alpha}\dot{\beta}}{\alpha\beta} + \left(\frac{\dot{\beta}}{\beta}\right)^2 \right] = \rho, \qquad (7)$$

$$f_Q \left[-2\frac{\dot{\alpha}\dot{\beta}}{\alpha\beta} - 2\left(\frac{\ddot{\beta}}{\beta}\right) - 2\left(\frac{\dot{\beta}}{\beta}\right)^2 \right] - 2\frac{\dot{\beta}}{\beta}\dot{Q} f_{QQ} - \frac{f}{2} = p, \qquad (8)$$

$$f_Q \left[-3\frac{\dot{\alpha}\dot{\beta}}{\alpha\beta} - \frac{\ddot{\alpha}}{\alpha} - \frac{\ddot{\beta}}{\beta} - \left(\frac{\dot{\beta}}{\alpha}\right)^2 \right] - \left(\frac{\dot{\alpha}}{\alpha} + \frac{\dot{\beta}}{\beta}\right)\dot{Q} f_{QQ} - \frac{f}{2} = p, \qquad (9)$$

Where the notation with the overhead dot (.) signifies the derivative with regard to the cosmic time ‘t’ are being considered. Additionally, the non-metricity scalar Q have been calculated for the spacetimes [17], which is

$$Q = -2\left(\frac{\dot{\beta}}{\beta}\right)^{2} - 4\frac{\dot{\alpha}\dot{\beta}}{\alpha\beta}. \qquad (10)$$

We are now using to solve equations (7) to (9). This approach helps to decrease complexities in above equations by involving constraints that restrict the component of the spacetime. Before adopting the above mentioned strategy, we use certain algebra techniques to simplify the equation:

$$\left[\frac{\dot{\alpha}\dot{\beta}}{\alpha\beta} + \frac{\ddot{\alpha}}{\alpha} - \frac{\ddot{\beta}}{\beta} - \left(\frac{\dot{\beta}}{\beta}\right)^{2} \right] f_Q + \left(\frac{\dot{\alpha}}{\alpha} - \frac{\dot{\beta}}{\beta}\right)\dot{Q} f_{QQ} = 0. \qquad (11)$$

Equation [11], was simplified for both linear and non-linear f(Q) gravity. First, let suppose f(Q) is a linear gravity [18].

$$f(Q) = c_1 Q + c_2, \qquad (12)$$

where $c_1, c_2 \in \mathbb{R}$. The case, when $c_1 = 1$ and $c_2 = 0$ leads to the GR. Choosing f(Q) as a linear, main advantage is that it reduces complexities of equations of motion (12), equation (11) takes the following form $c_1\left[\frac{\dot{\alpha}\dot{\beta}}{\alpha\beta} + \frac{\ddot{\alpha}}{\alpha} - \frac{\ddot{\beta}}{\beta} - \left(\frac{\dot{\beta}}{\beta}\right)^2 \right] = 0$. As $c_1 \neq 0$, therefore

$$\left[\frac{\dot{\alpha}\dot{\beta}}{\alpha\beta} + \frac{\ddot{\alpha}}{\alpha} - \frac{\ddot{\beta}}{\beta} - \left(\frac{\dot{\beta}}{\beta}\right)^2 \right] = 0. \qquad (13)$$

To get solutions of equation [13], we apply constraints on the space time components. We became able to calculate non-metricity scalar Q values by applying the observed values of spacetime components in equation [10]. The results were represented in the form of Table as shown in Table 1:

We now extend the linear case by selecting f(Q) to be a power law of the form:

$$f(Q) = cQ^m, \qquad (14)$$

Where $c, m \in \mathbb{R}$. The correspondent of linear or constant f(Q) are not of our interest, such as m = 1 or m = 0 respectively. The reason of taking f(Q) to be of the form [14], is that it simplifies the cosmological constant problem. Positive aspect of choosing equation (14) is that a minor deviation from the A cold dark matter is observed when redshift increases. It was also noted that f(Q) power law models are also capable of describing late time acceleration. When equation (14) is combined with equation (11), we get

$$\left[\frac{\dot{\alpha}\dot{\beta}}{\alpha\beta}+\frac{\ddot{\alpha}}{\alpha}-\frac{\ddot{\beta}}{\beta}-\left(\frac{\dot{\beta}}{\beta}\right)^{2}\right]Q+(m-1)\left(\frac{\dot{\alpha}}{\alpha}-\frac{\dot{\beta}}{\beta}\right)\dot{Q}=0. \qquad (15)$$

As the above equation [15], is highly non-linear and need some restriction on spacetime components, so we solve it for the following possibilities:

(A1) $\alpha=\alpha(t)$ and $\beta=\text{constant}.$

(A2) $\alpha=\text{constant}$ and $\beta=\beta(t).$

(A3) $\alpha=\alpha(t),\ \beta=\beta(t)$ and $\alpha=\beta^{n}$, where $n\in\mathbb{R}\setminus\{0\}.$

It is easy to observe that equation [15], is trivially fulfilled for the possibility (A1). It is important to note that, while evaluating possibilities (A2) and (A3), there are additional sub-possibilities since we have the ability to pick the multiple values of index 'm' in the confined region. By implementing such constraints on the spacetime components, for the possibilities (A2) and (A3) for the values of the non-metricity scalar Q and f(Q), are given in the Table 2:

It is important to note that in the setup of f(Q) gravity to solve motion equations, we have non-metricity scalar Q two assumed functional forms. The first scenario involves f(Q) being a linear function of Q, which, with suitable choices for the constants in Eq [12], reduces to General Relativity (GR). In the second scenario, we consider f(Q) gravity in the form of a power law, as expressed in Eq [14]. Astrophysical point of view power law gravity models seem more important. Some significant aspects of these models are as follows. Such models primarily result in a system of ordinary differential equations (ODEs), which can yield physically realistic outcomes. Notably, power law gravity models have shown promise in addressing the cosmological constant problem. These f(Q) gravity models are considered viable due to their consistency with cosmological observations and their successful performance in solar system tests. These laws also mimic the hypothesises about universe expansion. Positive aspect of choosing equation [14], is that a minor deviation from the A cold dark matter is observed when redshift increases [19]. It also has been observed that late time acceleration can also be described by power law of f(Q) models [20]. Additionally, the power law model highlights a stronger impact of anisotropy compared to the simple linear model. Considering these characteristics of the power law f(Q) gravity model, the outcomes of this study can be divided into two main categories: cases where the power of the non-metricity scalar (Q) is positive (as seen in cases (vii), (ix), (x), (xi), and (xii)) and a case where the power is negative (case (viii)). Physically, models with a positive power of the non-metricity scalar (Q) are considered viable for explaining the inflationary period. Conversely, models with a negative power of (Q) are associated with dark energy cosmological models.

A study of Killing motion in deduced classes of LRS Bianchi type-I metrics

Considering anisotropic spacetimes within the framework of symmetric teleparallel gravity can lead to various astrophysical consequences, offering distinct perspectives compared to the limitations of general relativity (GR) and other modified gravity theories. One persistent challenge in GR, for instance, is that,

Table 1: Solutions of Eq. (13) together with non-metricity scalar (Q)

Case No	metric components	Value of Q	Value of f(Q)
(i)	$\alpha = (\mathrm{m}_3 \mathrm{t} + \mathrm{m}_4)$ and $\beta =$ constant, $where\, m_3, m_4 \in \mathbb{R}\ (\mathrm{m}_3 \neq 0)$.	$Q = 0$.	$f(Q) = m_1 Q + m_2$, $where\, m_1, m_2 \in \mathbb{R}$.
(ii)	$\alpha = (\mathrm{m}_3 \mathrm{t} + \mathrm{m}_4)$ and $\beta = m_6 \dfrac{(\mathrm{m}_3 \mathrm{t} + \mathrm{m}_4)}{m_4}$, $where\, m_3, m_4\, m_6 \in \mathbb{R}\ (\mathrm{m}_3, m_6 \neq 0)$.	$Q = \dfrac{-6m_3^2}{(m_3 t + m_4)^2}$.	$f(Q) = m_1 Q + m_2$, $where\, m_1, m_2 \in \mathbb{R}$.
(iii)	$\alpha = \dfrac{m_5}{m_3}\sqrt{2m_3 t + m_4} + m_6$ and $\beta = \sqrt{2m_3 t + m_4}$, $where$ $m_3, m_4, m_5 \in \mathbb{R}\ (m_3, m_5 \neq 0)$.	$Q = \dfrac{-2m_3^2}{\beta^4} - \dfrac{4m_5 m_3^2}{m_5 \beta^4 + m_3 m_6 \beta^3}$.	$f(Q) = m_1 Q + m_2$, $where\, m_1, m_2 \in \mathbb{R}$.
(iv)	$\alpha = \left[(n+2)(m_{3t} + m_4)\right]^{\frac{n}{(n+2)}}$ and $\beta = \left[(n+2)(m_{3t} + m_4)\right]^{\frac{1}{(n+2)}}$, $where\, m_3, m_4 \in \mathbb{R}\ (m_3 \neq 0)$.	$Q = \dfrac{-2m_3^2(2n+1)}{\left[(n+2)(m_3 t + m_4)\right]^2}$.	$f(Q) = m_1 Q + m_2$, $where\, m_1, m_2 \in \mathbb{R}$.
(v)	$\alpha = -(\mathrm{m}_3\, t + m_4)^{-1}$ and $\beta = -(\mathrm{m}_3\, t + m_4)$, $where\, m_3, \mathrm{m}_4 \in \mathbb{R}\ (m_3 \neq 0)$.	$Q = \dfrac{2m_3^2}{(m_3 t + m_4)^2}$.	$f(Q) = m_1 Q + m_2$, $where\, m_1, m_2 \in \mathbb{R}$.
(vi)	$\alpha = cons\,\mathrm{tant}, \beta = \sqrt{2m_3 t + m_4}$, $where\, m_3, m_4 \in \mathbb{R}\ (m_3 \neq 0)$.	$Q = \dfrac{2m_3^2}{(m_3 t + m_4)^2}$.	$f(Q) = m_1 Q + m_2$, $where\, m_1, m_2 \in \mathbb{R}$.

Table 2: Solutions of Eq. (13) together with non-metricity scalar f(Q)

Case No.	Metric components	Value of Q	Value of f(Q)
(vii)	$\alpha = \text{constant} \text{ and } \beta = \left[\frac{2m_3t + 2m_4}{3}\right]^{\frac{3}{2}}, \text{where } m_3, m_4 \in \square \ (m_3 \neq 0).$	$Q = \frac{-9m_3^2}{2(m_3t + m_4)^2}.$	$f(Q) = cQ^2$
(viii)	$\alpha = \text{constant} \text{ and } \beta = \left[\frac{2m_3t + 2m_4}{-5}\right]^{-\frac{5}{2}}, \text{where } m_3, m_4 \in \square \ (m_3 \neq 0).$	$Q = \frac{-50m_3^2}{(2m_3t + 2m_4)^2}.$	$f(Q) = cQ^2$
(ix)	$\alpha = \text{constant} \text{ and } \beta = \left[-3(m_3t+m_4)\right]^{\frac{-1}{6}}, \text{where } m_3, m_4 \in \square \ (m_3 \neq 0).$	$Q = \frac{-m_3^2}{18(m_3t + m_4)^2}.$	$f(Q) = cQ^{\frac{1}{3}}$
(x)	$\alpha = (m_3t + m_4)^2 \text{ and } \beta = (m_3t + m_4), \text{where } m_3, m_4 \in \square \ (m_3 \neq 0).$	$Q = \frac{-10m_3^2}{(m_3t + m_4)^2}.$	$f(Q) = cQ^{\frac{3}{2}}$
(xi)	$\alpha = (m_3t + m_4)^{-2} \text{ and } \beta = (m_3t + m_4), \text{where } m_3, m_4 \in \square \ (m_3 \neq 0).$	$Q = \frac{6m_3^2}{(m_3t + m_4)^2}.$	$f(Q) = c\sqrt{Q}.$
(xii)	$\alpha = \sqrt{m_3t + m_4} \text{ and } \beta = (m_3t + m_4), \text{where } m_3, m_4 \in \square \ (m_3 \neq 0).$	$Q = \frac{-4m_3^2}{(m_3t + m_4)^2}.$	$f(Q) = cQ^{\frac{7}{4}}.$

Currently, experimental efforts are made for large scale dark components identification, which make up most of the energy-matter content, as new fundamental particles have not yielded definitive results. One reason for studying anisotropic spacetimes in f(Q) gravity is that, if spacetime anisotropy exists, it enhances the alignment of astrophysical systems for observational studies. Models based on f(Q) gravity appear to provide natural solutions to various issues related to the dark aspects of cosmology [21, 22]. Symmetric teleparallel gravity models in the context of anisotropic spacetimes may offer a path toward a quantum approach to gravity, addressing some of the limitations of General Relativity (GR). At astrophysical scales, one of the benefits of f(Q) gravity is that it does not inherently require Lorentz Invariance or the Equivalence Principle, unlike GR. However, a complex nonlinear system of coupled PDEs (partial differential equations) is formed by the equations of motion in f(Q) gravity within anisotropic spacetimes. There are several important reasons for considering conformal motions, a type of spacetime symmetry. Spacetime symmetries impose valuable constraints that simplify complex nonlinear equations. These constraints often emerge from conformal symmetry, which can transform partial differential equations (PDEs) into ordinary differential equations (ODEs), making them easier to solve. The presence of conformal symmetry thus simplifies computational efforts and facilitates the analysis of system dynamics. Additionally, studying conformal motions creates a natural link between geometry and matter through the Einstein field equations (EFEs) in symmetric teleparallel gravity. Conformal motions also give rise to conservation laws, aiding in the classification of spacetime. The classification established by the presence of conformal symmetry holds significance in both astrophysical and cosmological contexts. In astrophysics, conformal motion helps describe the internal structure of the gravitational field within compact objects. In the context of anisotropic spacetimes within symmetric teleparallel gravity, a key area of investigation involves exploring dark energy cosmological models and their relationship to astrophysical structures shaped by conformal symmetry. These structures include compact stars, gravastars, and black holes. In this section, the solutions presented in Table 2 are utilized to derive Killing vector fields (KVFs). The significance of KVFs lies in their role in generating conformal conservation laws. The generators of the conformal algebra are useful for characterizing metrics based on their conserved quantities, with the conformal factor being crucial in this process. For example, when the conformal factor vanishes during the calculation of Killing vector fields (KVFs), it results in isometries. Essentially, conservation laws are formulated by KVFs. KVF $\frac{\partial}{\partial t}$ the existence of time are linked with conservation of energy. Similarly, the translations along x, y and z directions are space-like KVFs $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\, and\, \frac{\partial}{\partial z}$ respectively. These KVFs provide results which give rise to the linear momentum along their respective directions, Likewise, a non-zero constant conformal factor gives rise to homothetic vector fields (HVFs). The class of metrics that exhibit this property is referred to as self-similar solutions in General Relativity. Conformal symmetry has also been employed to study various cosmological phenomena [23, 24]. To achieve similar productive outcomes, this paper uses equation (1) to examine the conformal motions of the derived LRS Bianchi type-1 models within the context of f(R) gravity.

CASE (I)

In this case, we have $\alpha = m_3 t + m_4, \beta = constant, where\ m_3, m_4 \in \mathbb{R}\ (m_3 \neq 0)$. Now, using the values of α and β in equation (1) the spacetime, after suitable proper recycling is altered to become

$$ds^2 = -dt^2 + (m_3 t + m_4)^2 dx^2 + dy^2 + dz^2. \qquad (16)$$

$$K^0_{,0} = 0, \qquad (17)$$

$$m_3 K^0 + (m_3 t + m_4) K^1_{,1} = 0, \qquad (18)$$

$$K^2_{,2} = 0, \qquad (19)$$

$$K^3_{,3} = 0, \qquad (20)$$

$$-K^0_{,1} + (m_3 t + m_4)^2 K^1_{,0} = 0, \qquad (21)$$

$$-K^0_{,2} + K^2_{,0} = 0, \qquad (22)$$

$$-K^0_{,3} + K^3_{,0} = 0, \qquad (23)$$

$$(m_3 t + m_4)^2 K^1_{,2} + K^2_{,1} = 0, \qquad (24)$$

$$(m_3 t + m_4) K^1_{,3} + K^3_{,1} = 0, \qquad (25)$$

$$K^2_{,3} + K^3_{,2} = 0. \qquad (26)$$

By utilizing the equations (17), (18), (19) and (20) then we get the following system of equations as

$$K^0 = A^1(x, y, z) \qquad K^1 = -\frac{m_3}{m_3 t + m_4} \int A^1(x, y, z)dx + A^2(t, y, z)$$

$$K^2 = A^3(t, x, z) \qquad K^3 = A^4(t, x, y) \qquad (27)$$

Where $A^1(x,y,z),\ A^2(t,y,z),\ A^3(t,x,z)$ and $A^4(t,x,y)$ are functions of integration to be determined. Each solution's result is summarized here, with details omitted for shortness. When Locally Rotationally Symmetric (LRS) f(Q) gravity models case (i) include ten Killing vector fields, the conditions are described as follows.

After the final calculation, the 10 Killing vector fields from Case (i) are as follows

$$\left.\begin{aligned}
K^0 &= yn_1e^{m_3x} + yn_2e^{-m_3x} + zn_3e^{m_3x} + zn_4e^{-m_3x} + n_5e^{m_3x} + n_6e^{-m_3x} \\
K^1 &= \frac{-1}{m_3t+m_4}\left[yn_1e^{m_3x} - yn_2e^{-m_3x} + zn_3e^{m_3x} - zn_4e^{-m_3x} + n_5e^{m_3x} - n_6e^{-m_3x}\right] + n_9 \\
K^2 &= tn_1e^{m_3x} + tn_2e^{-m_3x} + zn_7 + \frac{e^{m_3x}m_4\,\mathrm{n}_1}{m_3} + \frac{e^{-m_3x}m_4n_2}{m_3} + n_8 \\
K^3 &= tn_3e^{m_3x} + tn_4e^{-m_3x} - yn_7 + \frac{m_4}{m_3}n_3e^{m_3x} + \frac{m_4}{m_3}n_4e^{-m_3x} + n_{10}
\end{aligned}\right\}. \qquad (28)$$

The generator of Killing algebra are

$$K_1 = ye^{m_3x}\frac{\partial}{\partial t} - \frac{ye^{m_3x}}{m_3t+m_4}\frac{\partial}{\partial x} + te^{m_3x}\frac{\partial}{\partial y} + \frac{m_4e^{m_3x}}{m_3}\frac{\partial}{\partial y},$$

$$K_2 = ye^{-m_3x}\frac{\partial}{\partial t} + \frac{ye^{-m_3x}}{m_3t+m_4}\frac{\partial}{\partial x} + te^{-m_3x}\frac{\partial}{\partial y} + \frac{m_4e^{-m_3x}}{m_3}\frac{\partial}{\partial y},$$

$$K_3 = ze^{m_3x}\frac{\partial}{\partial t} - \frac{ze^{m_3x}}{m_3t+m_4}\frac{\partial}{\partial x} + te^{m_3x}\frac{\partial}{\partial z}, \quad K_4 = ze^{-m_3x}\frac{\partial}{\partial t} + \frac{ze^{-m_3x}}{m_3t+m_4}\frac{\partial}{\partial x},$$

$$K_5 = e^{m_3x}\frac{\partial}{\partial t} - \frac{e^{m_3x}}{m_3t+m_4}\frac{\partial}{\partial x}, \quad K_6 = e^{-m_3x}\frac{\partial}{\partial t} + \frac{e^{-m_3x}}{m_3t+m_4}\frac{\partial}{\partial x}, \quad K_7 = z\frac{\partial}{\partial y} - y\frac{\partial}{\partial z}$$

$$K_8 = \frac{\partial}{\partial y}, \quad K_9 = \frac{\partial}{\partial x} \ \ and\ K_{10} = \frac{\partial}{\partial z}$$

Now, Lie algebra of KVFs is what we have now founds, $[X_1, X_2] = X_1(X_2) - X_2(X_1)$ by putting the values of X_1 and X_2 we get the following value $[X_1, X_2] = \alpha X_3$

Tabular form can be constructed as follows:

Table 3

[,]	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
X_1	0	αX_3	βX_4	0	0	γX_5	δX_6	0	0	0
X_2	$-\alpha X_3$	0	εX_5	0	0	ςX_6	0	0	0	0
X_3	$-\beta X_4$	$-\xi X_5$	0	ηX_7	0	0	θX_8	0	0	0
X_4	0	0	$-\eta X_7$	0	ιX_9	0	0	0	0	0
X_5	0	0	0	$-\iota X_9$	0	κX_{10}	0	0	0	0
X_6	$-\gamma X_5$	$-\varsigma X_6$	0	0	$-\kappa X_{10}$	0	0	0	0	0
X_7	$-\delta X_6$	0	$-X_8$	0	0	0	0	0	0	0
X_8	0	0	0	0	0	0	0	0	λX_{10}	0
X_9	0	0	0	0	0	0	0	λX_{10}	0	0
X_{10}	0	0	0	0	0	0	0	0	0	0

Where $\alpha,\beta,\gamma,\delta,\varepsilon,\varsigma,\xi,\eta,\theta,\iota,\kappa$ and λ are constant.

CONCLUSIONS

In this research work we have studied the Killing vector fields along with their Lie algebra. We used the direct integration technique for finding the Killing vector fields of LRS Bianchi type-1 spacetime. Moreover we discussed the Lie algebra of Killing vector fields on different cases, i.e. Rectangular, Cylindrical and Spherical coordinates. In this study it is observed that the dimension of Killing vector fields is 15. Lie algebra of the KVFs is closed.

REFERENCES

1. Iqbal, S., Alam, F., Irfan, M., & Ramzan, M. (2024). Energy conditions in f(Q) gravity. *Phys. Rev. D 12*, 123.
2. Woodhouse, N. M. (1975). Killing tensors and the separation of the Hamilton-Jacobi equation. *Communications in Mathematical Physics*, *44*, 9-38.
3. De, A., Mandal, S., Beh, J. T., Loo, T. H., & Sahoo, P. K. (2022). Isotropization of locally rotationally symmetric Bianchi-I universe in f (Q)-gravity. *The European Physical Journal C*, *82*(1), 72.
4. Betschart, G., & Clarkson, C. A. (2004). Scalar field and electromagnetic perturbations on locally rotationally symmetric spacetimes. *Classical and Quantum Gravity*, *21*(23), 5587.
5. Heisenberg, L. (2024). Review on f (Q) gravity. *Physics Reports*, *1066*, 1-78.
6. Mandal, S., Wang, D., & Sahoo, P. K. (2020). Cosmography in f (Q) gravity. *Physical Review D*, *102*(12), 124029.
7. Mandal, S., Sahoo, P. K., & Santos, J. R. (2020). Energy conditions in f (Q) gravity. *Physical Review D*, *102*(2), 024057.
8. Einstein, A., & Rosen, N. (1935). The particle problem in the general theory of relativity. *Physical Review*, *48*(1), 73.
9. Einstein, A. (1917). On the special and general theory of relativity. *CPAE (English translation)*, *6*, 247-420.
10. Jiménez, J. B., Heisenberg, L., & Koivisto, T. (2018). Coincident general relativity. *Physical Review D*, *98*(4), 044048.
11. Jiménez, J. B., Heisenberg, L., Koivisto, T., & Pekar, S. (2020). Cosmology in f (Q) geometry. *Physical Review D*, *101*(10), 103507.
12. Anagnostopoulos, F. K., Basilakos, S., & Saridakis, E. N. (2021). First evidence that non-metricity f (Q) gravity could challenge ΛCDM. *Physics Letters B*, *822*, 136634.
13. Shabbir, G., Hussain, F., Jamal, S., & Ramzan, M. (2020). Existence of conformal vector fields of Bianchi type I space-times in f (R) gravity. *International Journal of Geometric Methods in Modern Physics*, *17*(08), 2050113.
14. Hall, G. S. (2004). Symmetries and Curvature Structure in General Relativity. *World Scientific Lecture Notes in Physics*, *46*.
15. De, A., Mandal, S., Beh, J. T., Loo, T. H., & Sahoo, P. K. (2022). Isotropization of locally rotationally symmetric Bianchi-I universe in f (Q)-gravity. *The European Physical Journal C*, *82*(1), 72.
16. Koussour, M., Shekh, S. H., & Bennai, M. (2022). Anisotropic nature of space–time in fQ gravity. *Physics of the Dark Universe*, *36*, 101051.
17. Sánchez, M. (1997). Structure of Lorentzian tori with a Killing vector field. *Transactions of the American Mathematical Society*, *349*(3), 1063-1080.
18. Solanki, R., De, A., Mandal, S., & Sahoo, P. K. (2022). Accelerating expansion of the universe in modified symmetric teleparallel gravity. *Physics of the Dark Universe*, *36*, 101053.
19. Capozziello, S., & D'Agostino, R. (2022). Model-independent reconstruction of f (Q) non-metric gravity. *Physics Letters B*, *832*, 137229.
20. Anagnostopoulos, F. K., Gakis, V., Saridakis, E. N., & Basilakos, S. (2023). New models and big bang nucleosynthesis constraints in f (Q) gravity. *The European Physical Journal C*, *83*(1), 1-11.
21. Finch, A., & Said, J. L. (2018). Galactic rotation dynamics in f (T) gravity. *The European Physical Journal C*, *78*, 1-18.
22. Mandal, S., Sahoo, P. K., & Santos, J. R. (2020). Energy conditions in f (Q) gravity. *Physical Review D*, *102*(2), 024057.
23. Kehagias, A., & Riotto, A. (2014). Conformal symmetries of FRW accelerating cosmologies. *Nuclear Physics B*, *884*, 547-565.
24. Hinterbichler, K., Hui, L., & Khoury, J. (2012). Conformal symmetries of adiabatic modes in cosmology. *Journal of Cosmology and Astroparticle Physics*, *08*, 017.