Review Article

Focus on Mathematics Core Literacy, Design Unit-Class Teaching-Take "Equation of Conic Curve" as an Example

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Abstract

The new curriculum standard of senior high school mathematics points out that it is necessary to grasp the teaching content as a whole and promote the continuous and phased development of the core literacy of the subject. In order to promote the curriculum reform based on literacy, teaching should shift from focusing on isolated knowledge points to attaching importance to the overall design of the unit. This paper takes the equation of conic curve as an example, and determines the core tasks of the unit based on the analysis of the teaching content, learning situation, teaching emphasis and difficulty. Make clear the unit teaching goal and the learning goal of the first class of "ellipse", and then take "ellipse and its standard equation" as an example to design the teaching of specific class hours, hoping to provide useful reference for teachers and promote the reform of basic education.
Keywords: Core Literacy, Unit Teaching, High School Mathematics, Teaching Design.

QUOTE

"Mathematics Curriculum Standard for Ordinary Senior High Schools (revised in 2017) in 2020" (hereinafter referred to as the curriculum standard) points out in the implementation proposal that teachers "should establish a teaching consciousness oriented to developing students' core literacy in mathematics, and cultivate the core literacy in mathematics throughout the whole process of teaching activities [1]". Compared with the previous classroom teaching design, unit teaching design can link scattered knowledge together and establish a perfect knowledge system for students. Pay more attention to the improvement of students' core literacy. The important content of core literacy is to let students creatively solve problems in actual situations, and to let students think and solve problems with expert thinking has become the appeal of classroom innovation and teaching reform [2]. From the perspective of promoting teachers' professional development, Professor Lu Shihu pointed out that unit teaching design should have four characteristics: integrity, sequence, naturalness and creativity [3]. Professor Zhong Qiquan believes that, "Class-time Doctrine" treats teaching content as knowledge points in a fragmented way, lacking "overall outlook", and unit design is the basic unit of curriculum development. Teachers must pay attention to the connection before and after a certain class to break the shackles of "Class-time Doctrine" [4].

From the teacher's point of view, unit teaching design can help teachers better grasp the teaching content, but in practice, teachers often have such questions: from which angles should we analyze the unit teaching content? How to grasp the key points of learning situation analysis? How to set the core task of the unit? How to determine unit teaching objectives based on unit core tasks? Taking the equation of conic curve as an example, this paper discusses the unit-class teaching design based on cultivating students' core literacy in mathematics.

1. Analysis Unit Teaching Tasks

The premise and foundation of unit teaching design lies in the analysis of unit teaching tasks, which mainly covers the following four aspects, taking "the equation of conic curve" as an example for concrete analysis.

1.1 Unit Teaching Content Analysis

The content is selected from the third chapter "Equation of Conic Curve" in the elective compulsory course of Mathematics for Ordinary High Schools, Edition a of People's Education in 2019. This unit is

arranged in the order of ellipse, hyperbola, and parabola. These three curves are isomorphic in content, research process, and research method.

The knowledge structure diagram of the unit "Equation of Conic Curve" is shown in Figure 1:
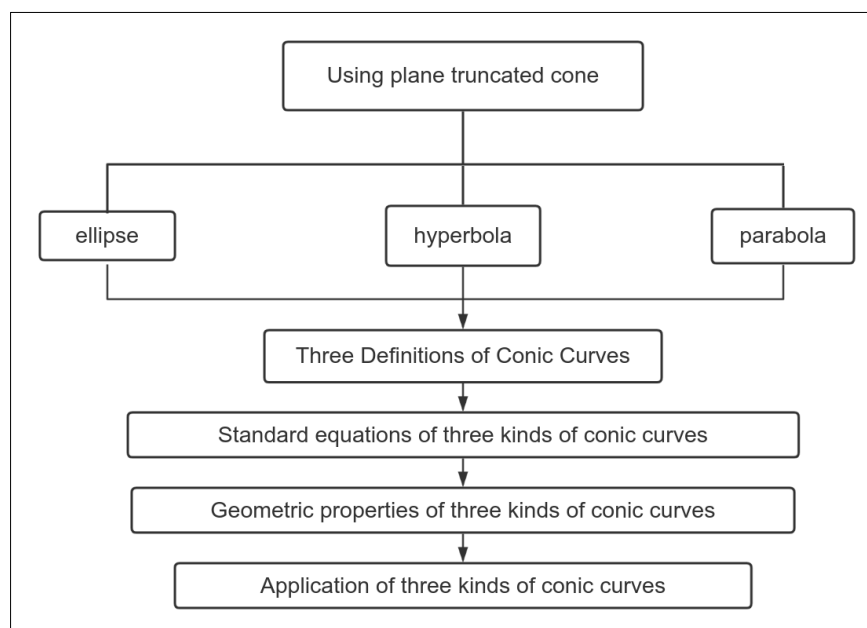
Figure 1: "Equation of Conic Curve" element knowledge structure diagram

1.1.1 Knowledge Content of This Unit

The introduction of this unit explains the origin of three cone sections: ellipse, hyperbola and parabola by changing the inclination angle of the plane. The development history of conic curve can also be fully utilized in teaching tof o stimulate students' interest in learning. This chapter follows the research thread "background-definition-establishment-equation-nature-application". In teaching, we should give full play to the initial role of ellipse demonstration class to help students master the research path of conic curve equation.

1.1.2 The Relationship between the Upper and Lower Positions of Knowledge in This Unit

The chapter "Equation of Conic Curve" is arranged after the content related to equations of space vectors and solid geometry, as well as straight lines and circles. Before that, students have mastered the basic skills of solving curve equation by coordinate method, which provides strong support for their subsequent derivation of conic curve equation. At the same time, students have a preliminary understanding of the three-step approach of solving geometric problems by algebraic method. The construction of this cognitive framework will help them to learn the relevant knowledge of conic curve in depth.

1.1.3: Unit Contains the Way of Thinking

According to students' life experience and cognitive basis, analogy runs through the whole process of this unit, from the analogy of the research path of linear equation to the research path of conic equation. At the same time, by using the thinking method of combining numbers and shapes, the plane rectangular coordinate system is established by observing geometric characteristics intuitively, and the geometric characteristics are expressed by algebraic method, so that three kinds of conic equations are deduced, which reflects the connection and integrity of mathematical knowledge.

1.1.4 Unit Focuses on Improving the Core Quality of Mathematics

By observing the generation process of curves obtained by a plane intersecting a cone, students' intuitive imagination literacy is enhanced. By summarizing the geometric characteristics of curves and abstracting the definition of curves, students' mathematical abstraction literacy is improved; through the derivation of curve standard equation, improve students' core literacy of mathematical operation and logical reasoning. By exploring the geometric properties of conic curves, students' abilities of intuitive imagination and mathematical operation are developed. Through the application of conic curve, students' core literacy of mathematical modeling is improved, and students can realize the important value that mathematics not only comes from life, but also can be applied to life.

1.2 Analysis of Learning Situation

On the one hand, students' existing cognitive basis should be analyzed, on the other hand, students' possible cognitive difficulties should be analyzed [5]. On this basis, the key points, difficulties and specific teaching strategies to break through the difficulties in unit teaching should be better determined.

1.2.1: Students' Cognitive Basis

In the study of the last chapter, students have systematically studied the coordinate method, and have a deep understanding of the coordinates of points in the plane rectangular coordinate system and related length calculation formulas. On this basis, students have certain spatial imagination ability, can abstract the properties of geometric figures, and are skilled in reasoning and operation. In addition, they have developed good study habits, mastered effective learning methods, and have a certain understanding of ellipses in daily life.

1.2.2: Difficulties That Students May Encounter

Although students have the foundation of learning this unit of knowledge, their abstract thinking and operation skills have not yet reached the ideal level. The cognitive challenges that students may face in this unit are mainly embodied in the abstract understanding of definitions. In the actual teaching process, teachers can dynamically show the generation process of curves with the help of information technology tools, such as Geogebra. In this way, students can intuitively observe and understand the geometric characteristics of curves. At the same time, the study of analytic geometry requires students' computing ability. In the process of deriving specific curves, teachers can guide students to observe the characteristics of equations first and simplify the computing process through specific analysis.

1.3: Analysis of Key Points and Difficulties in Unit Teaching

Key points of unit teaching: By observing the geometric characteristics of conic curve, selecting the appropriate plane rectangular coordinate system, and by means of the coordinate method and the combination of numbers and shapes, the geometric relationship is transformed into algebraic expression, and on this basis, the standard equation of the curve is derived.

Difficulties in unit teaching: this unit shows a high degree of consistency in research content and research path, so helping students build the research framework of this unit has become a difficult and key point in teaching; In addition, it is difficult for students to algebra geometric features and deduce specific equations.

1.4 Unit Core Task Analysis

First, the overall framework of analytic geometry learning is built on. Students will experience the exploration process from ' geometric features ' to ' standard equations ', then to ' geometric properties ' and ' applications ', so as to accumulate basic experience in analytic geometry learning [6]. Secondly, students will deeply understand the coordinate method and the thinking mode of combining numbers and shapes in studying geometric problems, and master the basic strategies of using algebraic methods to explore geometric problems. Finally, this unit will also focus on improving students' mathematical core literacy, and enhance their intuitive imagination ability by observing conic curves. In the process of deriving the standard equation of conic curves, it will promote the further development of students' mathematical operation ability and logical reasoning ability. In the practical application of explaining conic curves, such as the application of optical properties of ellipses in projectors, the architectural design of hyperbolic shapes, and the application of parabolic shapes in bridges and satellite antennas, the core literacy of mathematical modeling will be improved.

2. Determine the Unit Teaching Objectives
2.1 Unit Overall Teaching Objectives
Teaching Objective 1:

Based on the historical background of plane truncated cone, understand the origin of the name of conic curve and the internal relationship among them, go through the process from concrete to abstract, and develop mathematical abstraction and intuitive imagination; Through the observation of specific examples, we can perceive the wide application of conic curve in daily life and establish a preliminary perception of conic curve.

Teaching Objective 2:

To get the geometric figure of ellipse through hands-on operation, to perceive the geometric characteristics of ellipse, to abstract the definition of ellipse, to deduce the standard equation of ellipse, to master the geometric properties of ellipse, to summarize the basic ideas of exploring ellipse, to establish a big framework for studying conic curve, and to improve the core literacy of mathematical operation and logical reasoning.

Teaching Objective 3:

To learn the definition, standard equation and geometric properties of hyperbola and parabola by analogy with the exploration process of ellipse, and accumulate basic activity experience in exploring geometric problems.

2.2: Specific Class Learning Objectives
Learning Objective 1:

Through the drawing process of ellipse, observing the geometric features of ellipse, we can express the geometric features of ellipse with mathematical language, and then accurately define ellipse, develop the core literacy of intuitive imagination, and improve students' hands-on operation ability.

Learning Objective 2:

We can use the geometric characteristics of the ellipse to establish a suitable plane rectangular coordinate system, and we can deduce the standard equation of the ellipse according to the general steps of solving the curve equation, so as to further understand the basic idea of coordinate method and develop the core literacy of logical reasoning.

Learning Objective 3:

Go through the exploration process of ellipse, master the basic path of studying conic curve, learn to use the definition of ellipse to solve specific problems, and improve the core literacy of mathematical modeling.

3. Design Unit Learning Activities
3.1 Unit Learning Activity Framework

The unit "Equation of Conic Curve" is isomorphic in content, and it is developed in the logical order of "definition-standard equation-simple geometric properties-application". The basic framework of unit activity agrees with the research path according to the historical background of conic curve. Through intuitive observation and mathematical operation, the definition and standard equation of conic curve are obtained. By observing images and combining operations, the geometric properties of conic curve are deduced, and finally the learned knowledge is applied to improve the problem-solving ability and development.
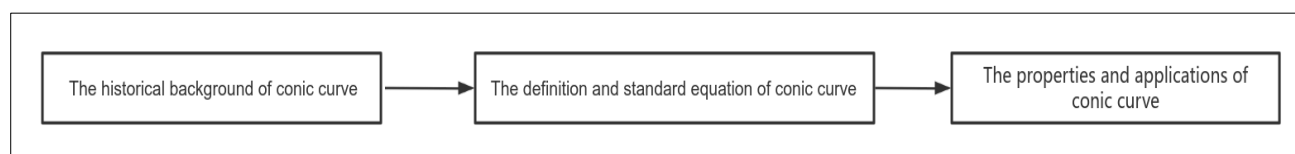
Figure 2: Basic framework of unit learning activities

Unit learning activities include three parts. By carefully decomposing the core tasks of each part, small tasks can be gradually achieved in the implementation of teaching activities. The completion of these small tasks will effectively promote the realization of the overall teaching objectives. See Table 1 for details of the core tasks of the unit.

Table 1: Unit Core Task Decomposition

Core task	Core task decomposition
Through the historical background of conic curve, the research ideas are unified.	Task 1: Teachers can use the history of mathematics to design the lead-in link of conic curve, and students can observe the geometric characteristics of conic curve according to the history of mathematics and get a preliminary understanding of its historical background. Task 2: Teachers can guide students to explore the development history of conic through group cooperation and inquiry, and establish a unified research framework.
Explore the definition of conic curve and derive its standard equation.	Task 1: Teachers should design a link for students to get the conic curve by hands, and use information technology to intuitively show students the generation process of conic curve, guide students to observe its geometric characteristics in specific operations and observations, guide students to express the characteristics of conic curve in mathematical language, and then summarize the definition of conic curve. Task 2: Establish an appropriate plane rectangular coordinate system through the geometric characteristics of conic curve, and derive the standard equation of conic curve according to its geometric relationship.
Explore the properties and comprehensive application of conic section.	Task 1: Guide students to guess the simple geometric properties of conic curve, including vertex, symmetry, eccentricity, etc., and verify the conjecture through specific operations, thus proving the conclusion. Task 2: Guide students to apply what they have learned to solve conic equation problems in life and mathematics situations, and summarize the basic methods, processes and steps from algebra and geometry.

3.2 "Ellipse and its Standard Equation" Class Learning Activity Design

The large unit teaching of conic curve can be divided into three small units: ellipse, hyperbola and parabola. The small unit of ellipse includes the definition, standard equation, properties and applications. The research among different small units is similar, so we take "ellipse and its standard equation" as an example to carry out the large unit teaching practice.

(1) Create a Situation and Introduce a Topic

Teachers can use multimedia to show that by changing the inclination angle of the plane, they can get three kinds of curves, namely ellipse, hyperbola and parabola. Combining with relevant mathematical historical materials, they can clarify the origin of the name of conic curve, and lay the foundation for the study of ellipse and its standard equation through the creation of situations and the introduction of topics [7].

(2) Actively Explore, Form the Concept

Teachers can follow the following steps in guiding students to explore: ① Introduce concepts from examples in life. For example, teachers can ask: "Have you ever found an ellipse in life? Please give specific examples. "Students are supposed to answer: Oval-shaped eggs, earrings, plates and reflections on the water

of the National Grand Theatre will also form ellipses. Teachers can show specific examples to make students perceive the shape of ellipses, thus deepening students' intuitive impression of ellipses. ② Draw ellipses by hands-on operation. Teachers can ask students to take out cardboard, pencils, strings and two thumbtacks prepared before class to perform specific operations according to the requirements on the big screen. And put forward the question: "Fix the two ends of the string on two pins on the cardboard, put on a pencil, pull in the rope, and move the pen tip. What is the figure drawn?" Students personally experience the process of ellipse generation and perceive its geometric features. ③ Through verification, the definition is obtained. Teachers can use Geogebra to show the process of ellipse generation, so that students can intuitively perceive the changes and invariants in this process, and then summarize the features to guide students to express the concept of ellipse with mathematical language. In this process, it is important to pay special attention to the points in ellipse definition, such as "in the same plane", which students easily ignore. At the same time, students can find out through specific operations that if the rope length is just right. If the length of the rope is less than the distance between the two focal points, no figure can be obtained. Through practical operation, it is emphasized that the constant in the definition must be greater than the distance between the two focal points, so as to deepen students' understanding and memory of the definition of ellipse. ④ Intuitive observation and summary of characteristics. Teachers can guide students to obtain simple geometric properties of ellipse through intuitive observation and guess verification, such as vertex, range, symmetry and eccentricity.

(3) Establish the System Reasonably and Deduce the Equation

The derivation of ellipse standard equation is a difficult point in this lesson. In this regard, teachers can first guide students to review the steps of solving curve equation by coordinate method, and follow this step to guide students to explore. ① Building a department. Teachers can first guide students to observe the geometric characteristics of ellipse and find that ellipse is an axisymmetric figure, and the line where the focus is located is an axis of symmetry of ellipse, so they can try to take the line where the focus is located as the X axis. Set up a coordinate system with the perpendicular bisector of the focal length as the Y axis. Set points. Let any point on the ellipse have the coordinate M(x,y), and the coordinates of the focal points F1 and F2 are (-c, 0) and (c, 0) .③. According to the geometric characteristics of the ellipse, list the equations: ④ Simplify. For the simplification of this formula, teachers can ask students to discuss in groups, and then show different simplification methods, such as Make students understand that the geometric characteristics should be observed before simplifying the formula, and adopting appropriate simplification methods can greatly simplify the operation process [8], and also cultivate students'

divergent thinking. After obtaining the equations from the algebraic point of view, teachers can guide students to observe the specific geometric meanings of A, B and C in the ellipse from the geometric point of view, and finally get the standard equation of the ellipse with the focus on the X axis. For the standard equation of the ellipse with the focus on the Y axis, teachers should guide students to formulate, Then, through observation, it is found that the difference between it and the equations listed in the elliptic equation with the focus on the X axis is only that the positions of X and Y are reversed, so the standard equation of the ellipse with the focus on the Y axis is. ⑤Verification. All the equations obtained in the solution process are the same solution deformation, and the whole derivation process is reversible. That is to say, the coordinates of all points on the ellipse satisfy the equation, and the coordinates corresponding to all the solutions of the equation are on the ellipse. Therefore, it is determined that this equation is the standard equation of the ellipse.

CONCLUSION

This paper illustrates the position and practical significance of unit teaching in senior high school mathematics teaching by exploring conic units. Although there are still many challenges in the implementation process, reasonable teaching design is helpful to establish a clear cognitive structure for students, so that students can better grasp knowledge and improve teaching effect. Taking "ellipse and its standard equation" as an example, this paper expounds in detail the whole process from situation creation to exploring definition to derivation of standard equation. It fully demonstrates the advantages of unit teaching in developing students' core literacy in mathematics, but the popularization and application of unit teaching mode still need to be explored and improved by teachers in practice.

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